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Pattern formation on the wall of acoustically driven gas bubble

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Abstract. An air bubble driven by ultrasound can become shape-unstable through a parametric instability. Above the critical driven pressure threshold for shape oscillations, which is minimal at the resonance of the breathing (radial) mode, regular patterns of surface waves are observed on the bubble wall. The existing theoretical models for bubble dynamics cannot explain the selection in the growth of the initial fluctuation distortions and the conditions for the realization of different shape structures. The proposed explanation is based on the consideration of a three-wave resonant interaction between the distortion modes. Corrections to the dynamical equations have been derived. Steady-state solutions of these equations describe the formation of a regular structure. A basic feature of pattern formation, which is applicable for the interpretation of preferred patterns of parametrically unstable Faraday ripples on the sphere, is that these structures have symmetry of point subgroups including the symmetries of Platonic solids. Our predictions are confirmed by images of patterns observed on the bubble wall.

Keywords: Bubble, Faraday ripples, symmetry breaking, pattern formation
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INTRODUCTION

The shape taken by an unstable fluid-fluid interface (liquid drops in liquids or gases, bubbles, interfaces between fluid layers) often exhibits sudden growth after threshold conditions are exceeded, and has a tendency to chaotic behaviour. This report addresses how this process results in the choice of a final stable shape. The predictions are tested against data obtained from air bubbles in water.

If a gas bubble of radius R_0 in a liquid of sound speed c is driven by an acoustic wave of low 'pump' frequency ω (such that $\omega R_0/c << 1$), then, at all amplitudes of that driving wave, the bubble undergoes spherically symmetric wall oscillation (i.e. a breathing mode pulsation). However, if the amplitude of the driving waves exceeds a well-defined threshold, then the nonlinear response of the gas bubble results in parametrically generated shape oscillations, superimposed upon the pulsation. The surface mode parametrically excited will be the one whose own natural frequency ω_l (where l is the order of the distortion mode) is closest to the subharmonic of the pump frequency, i.e. the mode for which $\omega_l \approx \omega/2$. The driving acoustic pressure which excites a surface mode will have a minimum at a frequency close to the breathing mode resonance $\omega \sim \omega_0$ (where $\omega_0(R_0)$ is the natural frequency of the breathing mode).

Experimental observations of patterns on the bubble wall have been reported in

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a number of publications (see review [1]), but elucidating the mechanisms for their realization is still an unsolved problem. However, an approach for Faraday waves on a plane surface has been derived [2]. The key element for the formation of a regular pattern in the parametrically excited Faraday ripples is a three-wave resonant interaction between the ripples. In this report, we present a systematic account of this three-wave resonant interaction between the distortion modes [3].

AMPLITUDE EQUATIONS

To account for the three-wave resonant interaction of distortion modes, we use the Hamiltonian formulation of the nonlinear bubble dynamics [4]. The slowly varying complex amplitudes of the breathing and distortion modes satisfy the equations similar to ones derived by [5], with accounting for the resonant triad interaction.

The occurrence of the three-wave resonance $\omega_{l_1} + \omega_{l_2} \approx \omega_{l_3}$ can be easily realized for $l >> 1$, to give $n' \approx 4^{1/3} l$. For simplicity, we shall account for only one triad, specifically the one which is nearest to the resonance condition.

The actual parametric instability occurs when one of the eigenvalues of the linear stability analysis [5] passes through zero at the threshold. If the conditions are sufficiently close to the threshold of this instability we can further reduce the description by eliminating "fast" variables [2]. The breathing mode and the high frequency (stable) distortion mode n' are fast phased to draw energy from the pumping and unstable modes l. The linear combination of the amplitudes corresponding to the nonzero eigenvalue is also fast. Thus, near the threshold it is possible to rewrite dynamical equations in terms of the slowly varying standing-wave amplitude B_{lm} corresponding to the zero eigenvalue. This yields [3]

$$\frac{dB_{lm}}{dt} = \lambda_1 B_{lm} - \frac{\partial}{\partial B^*_{lm}} \left(\Gamma_0 I_0^4(l) + \Gamma_{n'} I_{n'}^4(l) \right), \tag{1}$$

$$I_k^4(l) = \frac{1}{4} \sum_{m'=-k}^{k} \sum_{m''=-k}^{k} \sum_{m_1=-l}^{l} \sum_{m_2=-l}^{l} \sum_{s_1=-l}^{l} \sum_{s_2=-l}^{l} \begin{pmatrix} k & k & 0 \\ m' & m'' & 0 \end{pmatrix} \begin{pmatrix} l & l & k \\ m_1 & m_2 & -m' \end{pmatrix} \times \begin{pmatrix} l & l & k \\ s_1 & s_2 & -m'' \end{pmatrix} B_{lm_1} B_{lm_2} B_{ls_1} B_{ls_2},$$

The explicite form of constants $\Gamma_{n'}$ and Γ_0 is given in [3] and the average of the spherical harmonics over the sphere has been expressed in terms of the Wigner's 3j-symbols. Equation (1) is of gradient form and thus is covariant (i.e. it has the same structure/form in different coordinate systems). Though obtained for the bubble parametric instability, it has a general form describing bifurcation from spherical symmetry and can arise naturally in a large number of physical (and even biological) applications.

SOLUTIONS

A basic feature of the non-equilibrium phase transition is the spontaneous breakdown of symmetry: as the control parameter is changed, the stable steady state of the system,

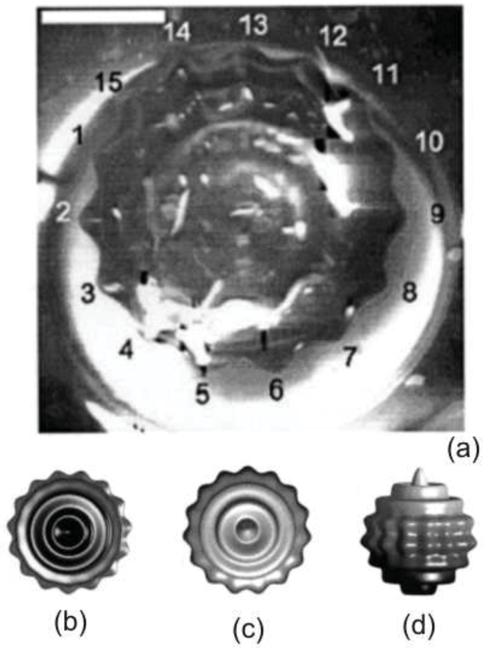

FIGURE 1. View from below an air bubble (a), restrained against buoyant rise by a glass rod, visible as the white circle "behind" the bubble (scale bar: 2mm). The bubble (of radius ∼2.5 mm) was driven at 1.297 kHz, with 83.5 Pa zero-to-peak acoustic pressure amplitude. The bubble shape corresponding to square pattern for these experimental conditions is shown in 2D projections: (b) - a view from below, (c) - a top view, (d) - a side view. Image from reference [6]. Photographs taken by P. Birkin and Y. Watson.

which is invariant under a symmetry group G, loses its stability and a new steady state appears which is invariant only under a subgroup of G. The set of B_{lm} forms the multi-component order parameter for the transition corresponding to the broken spherical symmetry. The number of components of the order parameter $2l+1$ is given by the dimension of the irreducible representation of the symmetry group. Thus, an important property of system (1) is that it is equivariant under the action of the original symmetry group of the sphere $O(3)$. $O(n)$ notation is used for the orthogonal group of degree n; $O(3)$ is the group of 3×3 orthogonal matrices with the group operation of matrix multiplication.

The symmetry of the preferred pattern is that for which the Lyapunov functional $F = -\lambda I_0^2(l) + \Gamma_0 I_0^4(l) + \Gamma_{n'} I_{n'}^4(l)$ is lowest [2, 3]. Apart from the trivial solution of $B_{lm} = 0$ for $m = 0, 1, 2, ...l$, there is a family of solutions differing in the total number of standing waves N for which $B_{lm} \neq 0$. For $N = 1$, there is only one standing wave B_{lm}. The deepest minimum of the Lyapunov function is achieved at $m = l$. By analogy with the structures formed by standing waves on a plane surface it is reasonable to name this type of pattern as "rolls". The isotropy group of this solution is $\mathbf{D}_{lh}$ of order $4l$ – prismatic symmetry. It has l-fold rotational symmetry, the 2-fold rotation axes perpendicular to the primary rotation axis and horizontal reflection plane.

For patterns formed by two standing waves $N = 2$ it is reasonable to assume that one of the partners in the formation of these structures will be mode $m_1 = l$, since its self-interaction is negligibly small. The system tries to avoid the modes that participate in a triad resonant interaction. The final step requires numerical evaluation of the Wigner's 3j-symbols and we have done this for the particular cases corresponding to the specific patterns observed by [6, 7]. For an $l = 15$ mode unstable on the wall of the bubble (of radius $\sim$2.5 mm) which was driven at 1.297 kHz, with 83.5 Pa acoustic pressure, the most effective resonant triad is formed with the $n' = 24$ mode. Calculations demonstrate that the deepest minimum of the Lyapunov function is realized for $m_2 = 0$. By analogy with the structures formed by orthogonal standing waves on a plane surface it is reasonable to name this type of pattern as "squares". Figure 1 illustrates the shape of this "square" pattern. The isotropy group of this solution is $\mathbf{C}_{lv}$ of order $2l$ which has pyramidal symmetry: it has l-fold rotational symmetry and a set of l mirror planes containing the axis. Finally, the pattern observed by [7] for an $l = 4$ mode has the symmetry of the orthohedron.

The derivation of angular dependence of scattering amplitude for combination components in the method of two frequency insonification, when the applied sound field contains two frequencies: one an imaging signal at ω_i which is set much higher than the resonant frequencies of any bubbles; and the other, the pump signal ω_p, tuned to the resonance frequencies is the final subject of this report. This provides prediction of the intensity of the combinative components for for typical structures corresponding to "rolls", "squares" and polyhedral patterns observed on the bubble wall.

CONCLUSIONS

To summarize, we have developed an asymptotic weak nonlinear theory (based on the third-order Hamiltonian) for pattern formation on a bubble wall driven near threshold by an acoustic field. The simplest solutions of the derived equations can explain the experimentally observed structures: rolls, squares, and octahedrons.

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