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Mobility and Impedance Methods in Structural Dynamics: An Historical Review

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UNIVERSITY OF SOUTHAMPTON

INSTITUTE OF SOUND AND VIBRATION RESEARCH

SIGNAL PROCESSING & CONTROL GROUP

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by

P Gardonio and M J Brennan

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Authorised for issue by
Prof S J Elliott
Group Chairman

ABSTRACT

An historical review of the conception and evolution of impedance and mobility methods in structural dynamics is presented. As often happens in science, the concepts and ideas developed in one branch of science have inspired new approaches and theories in another branch. Indeed this was the case for mechanical mobility and impedance which has its origin in the field of electricity. The conception and formulation of both direct and inverse electromechanical analogies are described in this report together with the formulation of "ad hoc" impedance and mobility methods that were developed for mechanical systems. Also, the impedance, mobility and transmission matrix methods that evolved for flexible, distributed mechanical systems are discussed. Finally the analytical solutions of the various types of problems usually encountered in the study of structural vibration are presented in a consistent formulation based on the mobility/impedance terms. This historical review has been written with reference to a large number of published papers many of which have been listed in the reference and bibliography sections.

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1 INTRODUCTION

In this report a historical review is presented of the conception and evolution of *impedance* and *mobility* methods in structural dynamics. The task of reconstructing the origin of a methodology developed for the study of physical phenomena is challenging. It requires interpretation of documents written at the early stages of the formulation of a methodology, and thus it could be limited by the lack of material published in scientific journals. This is probably the case in this attempt to identify the milestones that led to the concepts of mechanical impedance and mechanical network theory. However the authors believe that they have included the key papers in the subject's development.

The report is organised as follows. Following this introduction, in section 2 the origin of the impedance method for mechanical systems is described. In section 3 the development of mechanical network theory is described together with its analogy to an electrical network. The way in which the analogy was developed into the mobility concept is the subject of section 4, and the extension of the approach to multi-degree-of-freedom systems is described in section 5. Finally in section 6 the analytical solutions of the various types of problems usually encountered in the study of structural vibration are presented in a consistent formulation based on the frequency response function parameter, i.e. the mobility/impedance terms.

During the preparation of this report effort has been made to collect as many documents as possible related to this subject. All the papers gathered have been listed into a bibliography for completeness. In this way the reader can find, for his own use, a list of references on specific topics related to impedance or mobility methods.

2 ORIGIN OF THE MECHANICAL IMPEDANCE CONCEPT

The early analysis of vibratory systems in terms of mechanical impedances was probably linked to the studies of electrical communication. Many documents indicate that a number of inventions such as the electromagnetic telegraph by Henry in 1830[1], the telephone by Bell in 1876, the phonograph by Edison in 1878 and the motion pictures by Edison in 1891 gave scientists a completely new set of problems to solve around the turn of the 20th century. Although primitive versions of these instruments were able to convert sound into mechanical and electrical signals and vice versa, they were not able to replicate speech or music. Both the electromechanical transducers and the electrical circuits used to build these new instruments were characterised by low efficiency and sharp resonances. Thus only a few octave bands of the acoustic signals were uniformly transmitted (Mason, 1940). Scientists, therefore, faced the challenges of increasing the number of uniformly reproduced octave bands, and increasing the efficiency of acoustic to electrical (and vice-versa) conversions. (Kennelly, 1923).

Under pressure from commercial development of the electric telegraph and the telephone at the turn of the 20th century, a detailed and systematic theory of electrical lines, which become known as the theory of electric networks or circuits, was developed. In 1884 Heaviside gave a definition of electric impedance (Heaviside, 1892)[2]. This, together with theorems by Kirchhoff, Thévenin and Norton, enabled the development of a new method for the systematic study of electrical networks using the principles of superposition, reciprocity and compensation. The main advantage of this new approach was that linear circuits could be studied without using differential equations (Guillemin, 1931, 1935, 1953). The first circuit studied with this new approach was the band-pass filter (Campbell, 1903, 1922). Campbell also applied the network theory to obtain a system with uniform transmission over a wide frequency range. By matching the termination impedances of several resonant elements he made a device which had high and uniform energy transmission efficiency over the widest possible frequency range. Telephone lines were finally improved by using carrier current systems with selective filters designed and optimised with the new network approach (Mason, 1940).

Professor Arthur G. Webster was the first to realise the possibility of using the impedance concept in the study of vibrating mechanical systems. In 1914, during a meeting of the Physical Society in Philadelphia, Webster read a paper (Webster, 1919) where, as shown in figure 1, he first defined the acoustic impedance for an oscillating system. In this system where a volume of air $\gamma(t) = \Gamma(\omega)\exp(j\omega t)$ periodically enters under the effect of a pressure equally periodic $p(t) = P(\omega)\exp(j\omega t)$, he defined the acoustic impedance as the complex ratio $Z_a(\omega) = P(\omega)/\Gamma(\omega)$. Following this he defined the impedance for a mass, spring and

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1 The invention of the modern ticker telegraph was the result of a process of successive ideas, new scientific discoveries and inventions that begun with the early notions of the "sympathetic telegraph", in the 16th century. It continued until Henry's electromagnetic telegraph (1830) and Morse's printing telegraph (1837) inventions (Hunt, 1954).

2 The early scientific publications of O. Heaviside are collected in the "Electrical Papers", London (1892). The first definition of impedance can be found on page 371 of volume I where Heaviside writes the following note: "*impedance is here and later substituted for apparent resistance. It is the ratio of the amplitude of the impressed force to that of the current when their variations are simply harmonic.*" (paper published by O. Heaviside (1884), The induction of currents in cores, *The electrician*). Also on page 64 of volume II Heaviside defines the impedance of an electrical circuit as follows: "*Let us call the ratio of the impressed force to the current in a line when electrostatic induction is ignorable the impedance of the line, from the verb impede. It seems as good a term as resistance, from resist*" (paper published by O. Heaviside (1886), Electromagnetic induction and its propagation, *The electrician*).

ACOUSTICAL IMPEDANCE, AND THE THEORY OF HORNS AND OF THE PHONOGRAPH

BY ARTHUR GORDON WEBSTER

DEPARTMENT OF PHYSICS, CLARK UNIVERSITY

Communicated, May 8, 1919*

The introduction more than thirty years ago of the term 'impedance' by Mr. Oliver Heaviside has been productive of very great convenience in the theory of alternating currents of electricity. Unfortunately, engineers have not seemed to notice that the idea may be made as useful in mechanics and acoustics as in electricity. In fact, in such apparatus as the telephone one may combine the notions of electrical and mechanical impedance with great advantage. Whenever we have permanent vibrations of a single given frequency, which is here denoted, as usual, by $n/2\pi$, the notion of impedance is valuable in replacing all the quantities involved in the reactions of the system by a single complex number. If we follow the convenient practice of denoting an oscillating quantity by e^{int} and taking its real part (as introduced by Cauchy) all the derivatives of e^{int} are obtained by multiplication by powers of in, or graphically by advancing the representative vector by the proper number of right angles.

If we have any oscillating system into which a volume of air X periodically enters under an excess pressure p, I propose to define the impedance by the *complex* ratio $Z = p/X$. If we call $dX/dt = I$ the current as in electricity, if we followed electrical analogy we should write $Z = pI$ so that the definition as given above makes our impedance lead by a right angle the usual definition. I believe this to be more convenient for our purposes than the usual definition and it need cause no confusion.

If we have a vibrating piston of area S as in the phonometer, we shall refer its motion to the volume $S\xi$ it carries with it and the force acting on it to the pressure, so that $F = Sp$. The differential equation of the motion is

$$m\frac{d^2\xi}{dt^2} + \kappa\frac{d\xi}{dt} + f\xi = F = Sp, \qquad X = S\xi, \qquad (1)$$

we have

$$Z_1 = (f - mn^2 + i\kappa n)/S^2, \qquad (2)$$

where m is the mass, κ the damping, f the stiffness. The real part of S^2Z, $f - mn^2$, is the uncompensated stiffness, which is positive in a system tuned too high, when the displacement lags behind the force, by an angle between zero and one right angle, negative when the system is tuned too low, when the

* This article was read in December 1914 at the meeting of the American Physical Society at Philadelphia, and has been held back because of the continual development of the experimental apparatus described in a previous paper in these PROCEEDINGS.

Figure 1: *Front page of the paper in which Profesor Arthur Gordon Webster first introduced the concept of mechanical impedance (from Webster, (1919)).*

dash-pot mechanical oscillator system as the ratio of the periodic force applied on the mass $f(t)=F(\omega)\exp(j\omega t)$ over the equally periodic displacement of the mass $v(t)=V(\omega)\exp(j\omega t)$ so that $Z_m(\omega)=F(\omega)/V(\omega)$. Webster's decision to define the mechanical impedance as the ratio of a dynamic parameter over a kinematic quantity rather than the inverse quotient was probably because he wished to be consistent to the definition of electrical impedance (Le Corbellier, 1952). Electrical impedance represents the ratio between the e.m.f. across an electrical element and the current flowing through it. This could be seen as the ratio between the cause (e.m.f.), and its effect (current). Webster's definition of mechanical impedance represents the ratio between the cause of motion (force), and its effect (displacement). Indeed, in Webster's paper the equation of motion for a mass m, spring k and dash-pot c elements in parallel was derived using Newton's second law so that the typical second order differential equation $m\cdot d^2v(t)/dt^2+c\cdot dv(t)/dt+k\cdot v(t)=f(t)$ was obtained. Assuming harmonic motion for the force and displacement variables it was therefore straight forward to define the mechanical impedance as the complex ratio of the force to the displacement, so that the total impedance of the system was: $Z(\omega)=F(\omega)/V(\omega)=-m\omega^2+jc\omega+k$ (Webster, 1919).

3 DIRECT AND INVERSE ELECTROMECHANICAL ANALOGIES

Once the similarity between mechanical and electrical systems was realised many scientists began to analyse electromechanical transducers using network theory. Results from early studies appeared in patents, journal publications and books (Hähnemann and Hecht, Pt. I,II,III 1919-1920), (Wegel, 1921), (Pomey, 1921), (Lichte, 1926), (Harrison, 1929), (Norton, 1928), (Hanna, 1928) and (Franke, 1930). The first study of a purely mechanical system using network theory was probably carried out by Maxfield and Harrison (1925) on improving the design of a phonograph (system on the right hand side of figure 2).

Because many of the early results were released as patents, work on electromechanical systems using network theory was not immediately noticed by the wider scientific community. The method eventually became known as the "direct analogy", when papers and books dealing specifically with the analogy were published, (Nickle, 1925), (Maxfield and Harrison, 1926) and later on (Herbert, 1932), (Pawley, 1937), (Mason, 1942), (Olson, 1943) and (Bloch, 1945). The approach was based on the following methodology: first, the electrical circuit, which is analogous to the mechanical problem to be solved was drawn; second, the analogous electrical problem was solved using the electric network theory and third, the electrical answer was reworked into mechanical terms. Current and e.m.f. of the equivalent electrical circuit represented velocity and force parameters of the mechanical system respectively (Maxfield and Harrison, 1926). Equivalent network impedances were derived from the corresponding mechanical impedances by assuming appropriate conversion factors and connecting rules (Bloch, 1945).

Acoustical filters were first studied by Hershel in 1833, but the need for better design of the telephone electromechanical transducers, gave renewed drive to the design of efficient electromechanical and acoustical/mechanical systems. The electromechanical analogy and the concept of a mechanical filter (Maxfield and Harrison, 1926), (Harrison, 1929) and (Norton, 1928) were of considerable help in this endeavour. Stewart (1922), (1924) and (1925) showed that by combining resonators and tubes in an appropriate manner it was possible to obtain the same transmission frequency characteristics as electrical filters. Studies on acoustical and mechanical filters using the electromechanical analogy were subsequently carried out by Mason (1927), (1934), Cady (1929), Espenschied (1931), Lindsay and White (1932), Lindsay (1934), Lindsay *et al* (1934), Lindsay (1938), Lindsay (1939), Mason (1934) and Lakatos (1939). The origin of the electromechanical analogy was therefore strongly linked to the development of electrical network theory and electric wave filter systems. As often happens in science, the concepts and ideas developed in one branch of science were transferred to another branch. Campbell arrived at the design of the wave filter by analogy with the structural wave propagation along a string loaded with lumped masses at discrete intervals, which was first studied by Lagrange in the 18th century and later on by (Godfrey, 1898)[3]. Subsequently Campbell's idea gave a new stimulus to the study and design of vibratory mechanical systems, (Maxfield and Harrison, 1923), (Harrison, 1929) and (Norton, 1928).

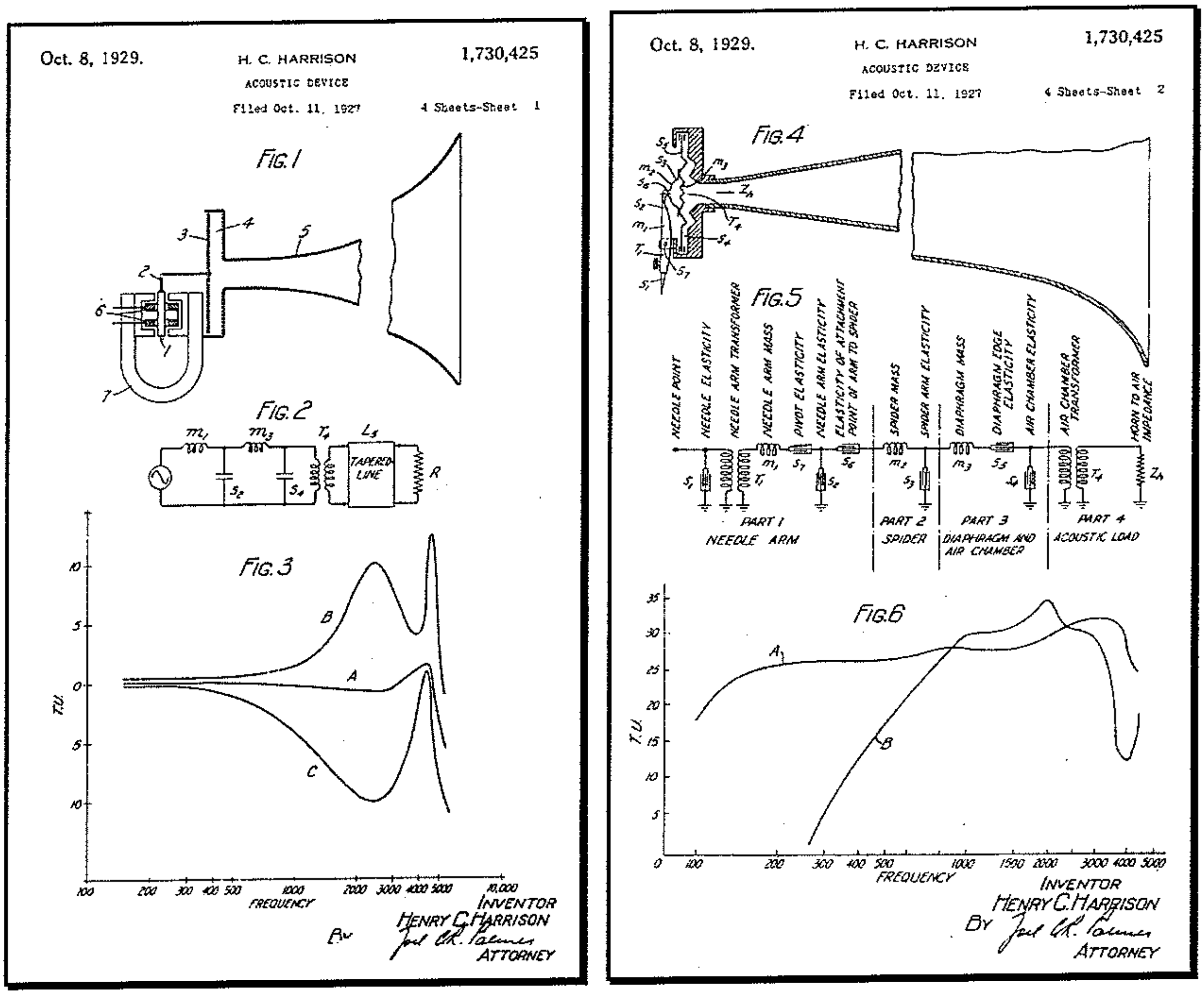

Figure 2: *First two pages of the patent written by H.C. Harrison showing two inventions: an electric loudspeaker (left) and a phonograph (right). Below the pictures of the two inventions, the schematic diagram of the equivalent electric circuit to the electro-mechanical (left) and purely mechanical systems (right) are shown, from (Harrison, 1929).*

[3] Campbell (1903), in his introduction about the structure of electrical loaded lines, writes: "*An interesting contribution to the general properties of this structure has been made by Mr. Charles Godfrey (1898) in a paper on wave propagation along a periodically loaded string and I am indebted to that aricle for equation (18) which furnishes a complete solution of the propagation*".

It is interesting to note that quite soon an electromechanical analogy was also used for the study and interpretation of oscillations and waves phenomena in electrical circuits. The electromechanical analogy turned out to be a good way of transferring electrical systems to more familiar mechanical ones whose interpretation was more immediate for many people. If an acoustic equivalent system was considered, it was possible to listen to the intensity of the excitation transmitted (Reich, 1932).

Once the electromechanical analogy had been established, scientists began to consider its limitations. Since the early studies on analogies it had been apparent that the variables used in the differential equations of electrodynamic or electromagnetic systems were not consistent with those of purely electric systems (Wegel, 1921) and (Corbeiller, 1929). It was concluded that elctrodynamic and electromagnetic systems network diagrams could not be directly transposed to an equivalent purely electrical network. Indeed the analogy based on Webster's definition of impedance for mechanical systems leads to some difficulties such as those listed below (Firestone, 1933).

1. The force transmitted "through" a mechanical element is replaced by the e.m.f. "across" the equivalent electrical element while the velocity "across" a mechanical element is replaced by the current "through" the equivalent electrical element; therefore the physical interpretation of the equivalent electric network variables with reference to the mechanical ones is inverted.

2. Mechanical elements in series are represented by electrical elements in parallel while mechanical elements in parallel are represented by electrical elements in series so that the composition of the analogue electrical circuit of a mechanical system differs from the procedure used to set the circuit chart of a purely electrical systems.

3. The resulting impedance of a series of mechanical elements is calculated as the reciprocal of the sum of the reciprocal impedance of the mechanical elements while the resulting impedance of a parallel of mechanical elements is given by the sum of each mechanical element impedance which is exactly the opposite formulation of that for the calculation of the total impedance of a series or parallel assembling of electrical elements.

4. The application of Kirchhoff's current law (sum of currents to a junction is zero) directly corresponds to the velocity law of mechanical systems (sum of velocity differences around a closed circuit is zero) while the application of Kirchhoff's voltage law (sum of e.m.f. around a mesh is zero) directly corresponds to the force law of mechanical systems (sum of force to junction is zero), so that, once more the physical explication of the analogue electric network is not simple and direct. On the contrary it requires a certain ability to compare a junction-type law with a mesh-type law and vice versa.

Darriues (1929) first mentioned the possibility of defining the analogy in a different way where force is equivalent to current rather than e.m.f. Two papers presenting a new analogy were subsequently published (Hähnle, 1932) and (Firestone, 1933), in which a new definition "bar impedance" was proposed. This was the ratio of a kinematic variable over a dynamic variable and gave a new electromechanical analogy for solving vibroacoustic problems. The new analogy was free from the limitations of 1 to 4 above and become known as the "inverse analogy". The transformation of the mechanical bar impedances to electric impedances used new conversion factors and connecting rules as described by Bloch (1945). Because the physical principles behind this new approach for the solution of vibratory problems were not trivial, the new analogy was not adopted immediately by the scientific community. It was based on an alternative formulation of Newton's second law where the dynamical behaviour of a system was derived from kinematic equations rather than force equilibrium equations (Trent, 1952)[4]. Starting from the known concept of duality between two electrical systems, Le Corbellier and Yeung (1952) showed that the duality principle held for mechanical systems. Duality is a topological transformation exchanging node pairs and meshes, and this type of transformation together with the electromechanical analogies provide a set of mathematical tools for the analysis of electrodynamic systems.

Table 1 summarises the main features of the direct and inverse electromechanical analogies. The first row shows two single degree of freedom mass-spring-damper systems. The one denoted $\mathbf{M_f}$ is a parallel connected system and is excited by a force. The one denoted $\mathbf{M_v}$ is a series connected system and is excited by a velocity source. Schematic diagrams of electrical networks shown below the mechanical systems are analogous or inversely analogous to the systems. The two mechanical systems depicted, and their direct or inverse analogue electric network pairs, are dual. The equation of motion of the mechanical system $\mathbf{M_f}$ (top left) can be derived using Newton's second law directly; the applied force is equal to the sum of the inertial force (which is proportional to the absolute acceleration of the mass), the damping force (which is proportional to the relative velocity across the viscous damper) and the elastic force (which is proportional to the relative displacement across the spring). For this system it is straightforward to derive the equivalent electrical circuit using the direct analogy where the forces from the three mechanical elements are represented by e.m.f. differences across an inductance, a resistor and a capacitor connected in series such that they balance the voltage generator e.m.f., $\mathbf{E_v}$ (centre left).

Alternatively the equation of motion of the mechanical system $\mathbf{M_v}$ (top right) can be derived using the principle of compatibility. The sum of the absolute displacement of the mass and the relative displacements of the dash-pot and spring elements is equivalent to that generated by the scotch-yoke motion generator. In this case it is easier to derive the equivalent electrical circuit by using the inverse analogy where the displacements of the three mechanical elements are represented by the e.m.f. differences across an inductance, a resistor and a capacitor connected in series in such a way to balance the voltage generator e.m.f., $\mathbf{E_v}$ (bottom right). Thus the choice of using the direct or inverse analogy depends on the topology of the mechanical system. Nevertheless Table 1 clearly shows that, as advocated by Firestone, the electrical networks derived with the inverse analogy have the same topological features of the mechanical systems from which they are derived. Alternatively, the electric networks derived with the direct analogy have the node pair and meshes exchanged with reference to the mechanical system from which are derived[5].

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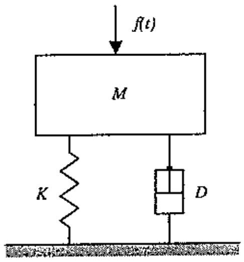
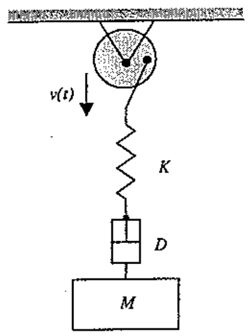
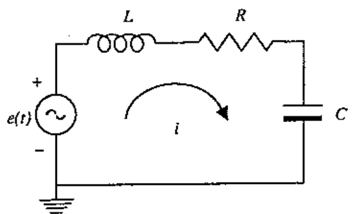
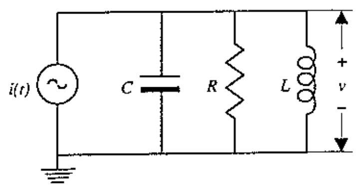
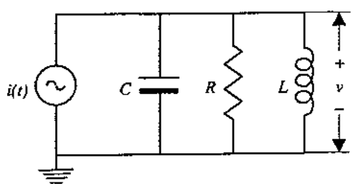
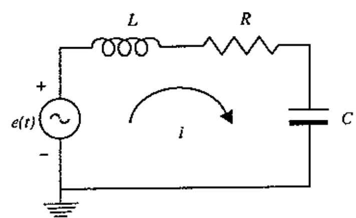
4 Le Corbeiller and Yeung (1952) on page 645 write: "*The reader may wonder how it is possible to derive the dynamical behaviour of a system from kinematic equations. This question arises because we are accustomed in dynamics to a mistaken emphasis on forces, traceable to a metaphysical attitude current in Newton's times. From this emphasis on force considered as the cause of motion, there followed in electricity an emphasis on electromotive force considered as the cause of the current flow. To one without such preconceptions, it should be clear that in Ohm's law E=IR, as well as in Newton's law f=ma, the general gas law PV=nRT, or any natural law whatsoever, there is no cause and no effect, there are only variable physical quantities permanently connected by a mathematical equation*".

5 Le Corbellier and Young (1952) wrote about this matter the following comment: "*The classical analogy makes the meshes of E_v correspond to the node-pairs of M_f, an awkward situation due to the topological innocence of our forebears*".

Table 1: *Electromechanical analogies. The factors ε_f, ε_v, δ_f, δ_v are the conversion factors from force or velocity variables to potential or current variables as from reference (Bloch, 1940).*

MECHANICAL SYSTEM M = mass (kg) K = stiffness (N/m) D = dissipation factor (N/ms^{-1}) *f(t) = F exp(jωt)* harmonic force (N) *v(t) = V exp(jωt)* harmonic velocity (m/s)	**M_f**  $M\frac{dv}{dt}+Dv+K\int vdt=f(t)$	**DUAL OF**	**M_v**  $\frac{1}{K}\frac{df}{dt}+\frac{f}{D}+\frac{1}{M}\int fdt=v(t)$
EQUIVALENT ELECTRIC CIRCUIT FROM **DIRECT ANALOGY** $f(t)\leftrightarrow e(t)$ $v(t)\leftrightarrow i(t)$ L = inductance (H) C = capacitance (F) R = resistance (Ω) *e(t) = E exp(jωt)* Potential (V) *i(t) = I exp(jωt)* Current (A)	**E_v**  $L\frac{di}{dt}+Ri+C\int idt=e(t)$ $L=M/\alpha^2 \quad C=D\alpha^2 \quad R=K/\alpha^2$ where: $\alpha=\varepsilon_f/\varepsilon_v$; $e=\varepsilon_f f$; $i=\varepsilon_v v$	**DUAL OF**	**E_i**  $C\frac{de}{dt}+\mathrm{Re}+L\int edt=i(t)$ $L=M/\alpha^2 \quad C=D\alpha^2 \quad R=K/\alpha^2$ where: $\alpha=\varepsilon_f/\varepsilon_v$; $e=\varepsilon_f f$; $i=\varepsilon_v v$
EQUIVALENT ELECTRIC CIRCUIT FROM **THE INVERSE ANALOGY** $f(t)\leftrightarrow i(t)$ $v(t)\leftrightarrow e(t)$ L = inductance (H) C = capacitance (F) R = resistance (Ω) *e(t) = E exp(jωt)* Potential (V) *i(t) = I exp(jωt)* Current (A)	**E_i**  $C\frac{de}{dt}+\mathrm{Re}+L\int edt=i(t)$ $L=D\beta^2 \quad C=M/\beta^2 \quad R=K/\beta^2$ where: $\beta=\delta_f/\delta_v$; $i=\delta_f f$; $e=\delta_v v$	**DUAL OF**	**E_v**  $L\frac{di}{dt}+Ri+C\int idt=e(t)$ $L=D\beta^2 \quad C=M/\beta^2 \quad R=K/\beta^2$ where: $\beta=\delta_f/\delta_v$; $i=\delta_f f$; $e=\delta_v v$

A detailed introduction to electromechanical analogies can be found in books (Mason, 1942), (Olson, 1943), (Beranek, 1954, Ch III), (Koenig and Blackwell, 1961), (Hurty and Rubinstein, 1964), and (Skudrzyk, 1968). An interesting paper by Miles (1943) compared the advantages and limitations of the two analogies.

4 IMPEDANCE AND MOBILITY METHODS FOR MECHANICAL SYSTEMS

Firestone hoped that the advantages introduced by the inverse analogy would slowly bring the direct analogy into disuse and subsequently the new definition of impedance would replace the one defined by Webster[6]. This was not the case, so Firestone (1938) presented a paper in which he discussed an approach to the vibratory problems based on a new parameter called mobility (ease of motion) which is the ratio between a kinematic and a dynamic parameter. A problem could now be formulated in terms of mechanical variables, and all the electric circuit laws, theorems and principles were defined for a purely mechanical network. Firestone summarised the main features of the mobility method as follows:

1. A set of conventionalized symbols with which the essential characteristics of a mechanical system can be set forth in the form of a schematic diagram.
2. The concept of the velocity *across* mechanical elements (velocity of one end of the element relative to the other end) as contrasted with the velocity of points in the system relative to ground; the advantage here is that the relationship between the velocity across an element and the force through it, depends only on the characteristics of the element itself and not on the characteristics of the rest of the system.
3. The use of complex numbers to represent simple harmonic velocities and forces, both the magnitude and phase of these quantities being represented by the absolute value and angle of the complex numbers.
4. The concept of "mobility" of an element (ease of motion), which is defined as the complex ratio of the velocity across an element to the force through the element; simple rules are developed for computing the mobility of series or parallel combinations of elements and the force can then be found simply as the velocity divided by the mobility, or the velocity can be found as the force multiplied by the mobility.

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6 Firestone on page 259 of his first paper on electromechanical analogies (Firestone, 1933) writes: "*It has seemed advisable to introduce a new term, bar impedance*" (subsequently called mobility) "*which is equal to velocity across / force through. It is natural that the old analogists, having arrived on the ground first, should have chosen to define impedance as force / velocity since that fitted in with the other assumptions they had made. But in the outhor's opinion, all of their assumptions were unwise and led to the left-handed result that while electrical impedances in series are additive, mechanical impedances in series must be added as the reciprocal of the sum of the reciprocals as was shown above. It would have been better if this new analogy had been thought of first, for in that case the quantity which we have been forced to call "bar impedance" would have been called impedance and would have been subject to the same laws of addition as are found in electrical circuits. It is now too late to change suddenly the unfortunate definition of impedance which has been so much used in the past, so it is recommended that the term "bar impedance" be used. Then if the new analogy should prove popular, the time may come when the old definition of impedance will have fallen into disuse, at which time the "bar impedance" may be shortened to "impedance" with the new definition*".

The formulation of the mobility method was completed by the description of force and velocity laws as the equivalent of Kirchhoff's current and voltage laws. Equivalent Thévenin and Norton theorems for force or velocity sources were also described together with the mechanical forms of the principles of reciprocity, of superposition and compensation. Thus the innovative elements of the mobility method proposed by Firestone were twofold: first, the formulation of the problem directly in terms of mechanical variables and second, the introduction of the mobility parameter which enables a more intuitive formulation of a mechanical network.

The first referenced works where the dynamics of a system was analysed using directly a mechanical formulation in terms of a mobilitiy–type parameter, e.g. , $(\text{kinematic variable})/(\text{dynamic variable})$ or in terms of impedance–type parameter, e.g. $(\text{dynamic variable})/(\text{kinematic variable})$, are those of Carter (1936), Duncan (1941) and (1943), Biot (1941, I,II), Manley (1943 I,II) and Miles (1943). Many other studies followed and an extensive list is reported in the bibliography in section 9.

In a later paper, Firestone (1956) proposed an extension of the mobility method. He reworked the direct and inverse analogies so that instead of drawing an analogous electrical network the acoustical or mechanical diagrams were drawn directly using acoustical and mechanical impedance or mobility symbols. Two comprehensive tables were presented where symbols for lumped acoustical or lumped mechanical elements were given with reference to either the impedance or mobility parameters. Solving of a network problem was then carried out with reference to acoustical or mechanical units. Firestone suggested the following procedure for drawing the schematic diagrams:

1. Choose your analogy, either for life or for the problem at hand, remembering that the mobility analogy is the most convenient for rod-connected systems while the impedance analogy is the most convenient for hydraulic tube-connected systems.

2. Identify the functions performed by each part of the given structure.

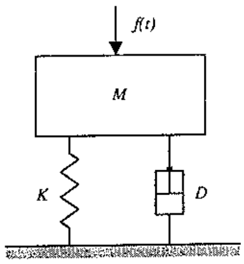
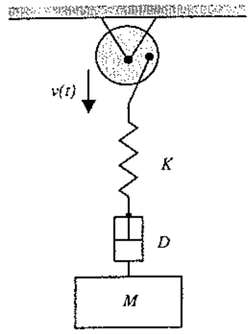
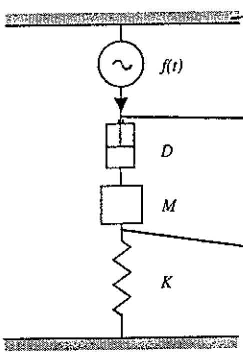
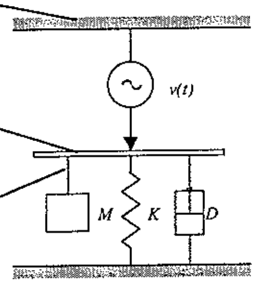
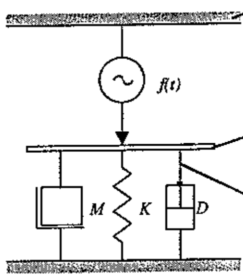
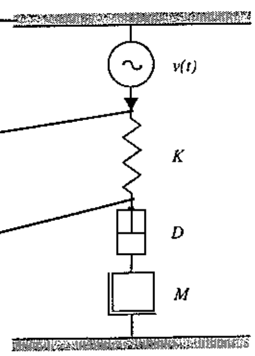
3. Choose the schematic symbols which represent these functions.

4. Identify the terminals of each element, coupler, and vibrator of the structure.

5. Connect in the schematic diagram by means of appropriate connectors and rigid or hydraulic junctions those terminals which are connected in the structure.

As with the electromechanical analogies, the choice of either the impedance or mobility methods is dependent upon the system of interest. Many mechanical systems can be modelled by rigidly connected lumped elements where the adjacent elements have the same velocity, or volume velocity in the acoustic case ($\mathbf{M}_v$ system in Table 1). For this type of system the mobility approach is convenient since the mechanical network can be drawn by inspection. When adjacent elements of a system have the same forces at their terminals, or pressures in the acoustic case, ($\mathbf{M}_f$ system in Table 1) it is easier to derive the impedance network. In certain cases it could be necessary to draw part of the system scheme with the mobility method and part with the impedance method. These two different networks are then linked by means of specific couplers (Bauer, 1953).

Table 2: *Impedance and Mobility diagrams.*

	M_f		M_v
MECHANICAL SYSTEM M = mass (kg) K = stiffness (N/m) D = dissipation factor (Nms^{-1}) $f(t) = F\,exp(j\omega t)$ harmonic force (N) $v(t) = V\,exp(j\omega t)$ harmonic velocity (m/s)	 $M\frac{dv}{dt}+Dv+K\int vdt=f(t)$	**DUAL OF**	 $\frac{1}{K}\frac{df}{dt}+\frac{f}{D}+\frac{1}{M}\int fdt=v(t)$
IMPEDANCE SCHEMATIC OR TUBING DIAGRAM FROM THE **IMPEDANCE METHOD** Z_{eq} total impedance Z_m impedance of the mass Z_c impedance of the dissipator Z_k impedance of the spring	 $Z_{eq}=Z_m+Z_c+Z_k$	**"Sky"** an ideal dynamic reference which maintains constant the force or pressure Z=0 or Y=∞. **Hydraulic junction** which ensures equal pressures (and forces if all tubes are of the same area) at all the terminals connected by the tree of tubes. **Tube connector** which is held stationary and it is filled with ideal fluid free from mass, viscosity or compressibility. **DUAL OF**	 $Z_{eq}=\frac{1}{1/Z_m+1/Z_c+1/Z_k}$
MOBILITY SCHEMATIC OR WIRING DIAGRAM FROM THE **MOBILITY METHOD** Y_{eq} total mobility Y_m mobility of the mass Y_c mobility of the dissipator Y_k mobility of the spring	 $Y_{eq}=\frac{1}{1/Y_m+1/Y_c+1/Y_k}$	**"earth"** an ideal kinematic reference which uses an inertial system as the reference for velocity or volume velocity Y=0 or Z=∞. **Rigid junction** which ensures equal velocities of all the terminals connected by the tree of rods. **Rod connector** which is a rigid element free from mass, friction or compliance. **DUAL OF**	 $Y_{eq}=Y_m+Y_c+Y_k$

The concept of an entirely mechanical network was well received by the scientific community working on mechanical problems. Table 2 shows the impedance (centre) and mobility (bottom) schematic diagrams of the two mechanical systems $\mathbf{M_f}$ and $\mathbf{M_v}$ considered in Table 1. The equivalent impedance and mobility schematic diagrams have been derived from the appropriate mechanical diagrams for either the impedance or mobility methods. The lines connecting the lumped elements have a different significance in the two diagrams. With the impedance method the connecting lines represent a "force junction" where there is equal force at the terminals of two adjacent elements. When the mobility method is employed the lines connecting the elements represent a "velocity junction" that ensures equal velocity at the terminals of adjacent elements. The diagrams obtained with the mobility method keep the same topological features of the mechanical systems from which they are derived, so in general the mobility approach is more intuitive. This is true except in a few cases where the system is such that the transmission of force is imposed at the junction between the elements (M_v in Table 2). In many cases, acoustical filters with or without side branches, are better represented in terms impedances.

In 1958 the American Society of Mechanical Engineers (ASME), organised a colloquium that presented the state of the art in the area of mechanical impedance and mobility methods. The papers presented during the colloquium were collected in a report (Plunket 1958), and this is one of the key milestones in the theory of impedance and mobility methods. In particular a paper was presented by Crandall (1958) which discussed the impedance and mobility approach for mechanical network analysis as suggested by Firestone but without any reference to an equivalent electric circuit. This was an important step in the consolidation of Firestone's ideas. However papers and reports of successive symposia on impedance and mobility related topics; see for example (Belsheim, 1962) highlight a certain confusion within the scientific community, particularly in the interpretation of mechanical impedance and mobility parameters[7]. It took some time before the ideas introduced by Firestone were assimilated and exploited, which was probably because there was no standard nomenclature for mechanical network representation. Indeed most of the papers collected in Plunket's or Belsheim's proceedings or in other publications the nomenclature differs. In 1963 The American National Standards Institute was the first to introduce a standard nomenclature: "Nomenclature and Symbols for Specifying the Mechanical Impedance of Structures" (ANSI, S2.6-1963 R 1971).

There are few books that introduce the main principles of impedance or mobility methods. Perhaps the most complete presentation is in chapter 10 of the "Shock and Vibration

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[7] For example, at the 30th Symposium on Shock, Vibration and Associated Environments, R. Plunkett gave the following answer to a question about the electromechanical analogy asked during the discussion of the paper he presented (Belsheim, 1962): "*I was afraid somebody was going to ask that question. The answer is yes and no. I do not like to think of this tool in terms of an analogy. Because if you are going to try and set up an analogy you can set it up in a number of different ways depending on the application. From my standpoint, I find it easier to think of it in terms of its primitive definition, in terms of a ratio between a sinusoidal force and the resulting sinusoidal motion. Now it depends, if you are going to call it admittance, it depends on weather you call your analogy between motion and voltage, or between motion and current. You can set up your analogy either way and this then determines whether you call it admittance or not, which is why I personally dislike the use of the word admittance. I think it is unfortunate that we have used the word impedance, that's bad enough. Does that successfully evade your question?*".

Handbook" edited by Harris and Crede (1961), entitled "*Mechanical Impedance*", by E.L. Hixson. A detailed description of mobility and impedance concepts of purely mechanical systems consisting of lumped elements is given. The schematic representation of the systems are derived using only the mobility approach even though both impedance and mobility formulae for the solutions of the systems are described.

5 IMPEDANCE, MOBILITY AND TRANSMISSION MATRIX METHODS FOR FLEXIBLE, DISTRIBUTED SYSTEMS

One of the first vibration problems studied using electromechanical analogies or impedance/mobility methods was vibration isolation. This type of problem consists of a vibration source connected to a receiving structure, via a mounting system. Simple impedance or mobility networks consisting of lumped bilateral elements (mass, spring and damping elements) allow the formulation of basic vibration isolation theory. However, three important features of isolator systems cannot be investigated: first, the distributed and flexible nature of the elements composing the whole systems (Harrison et al. 1952), (Skudrzyk, 1958, 1959), (Wright, 1958), and (Ruzicka and Cavanaugh, 1958); second, the effects due to a multiple mounting isolator or multiple connecting system (Soliman and Hallam, 1968) and (Sykes, 1971); and third the multiple degrees-of-freedom vibration transmission at connecting points (Neubert and Ezell, 1958) and (Sykes, 1971). Thus there was the need to extend the impedance/mobility approach to networks that included multiple terminal elements. Each terminal should represent a specific component of the kinematic (linear or angular velocity) or dynamic parameters (force or moment excitation). This was possible using a mechanical network theory where each element had $2\times(6\times n)$ terminals, where 6 was the number of allowable degrees-of-freedom at a junction, n was the number of element connection points and 2 takes into account both kinematic and dynamic parameters.

To solve problems of this type, mechanical engineers once more referred to work previously carried out by electrical engineers. They adapted the transmission matrix method developed for the study of electrical lines. This approach used block diagrams, where complicated electric circuits were simplified by means of single entities called "black boxes". These entities were characterised by relatively simple algebraic equations relating the input and output e.m.f. and current. Initial studies considered only a single pair of input and output terminals in which case the block diagram was called " a four pole network", which was linked to Campbell's study on electrical filters. In his paper "Cissoidal Oscillations" Campbell (1903) showed that passive four pole networks are equivalent to T or Π circuits. In this way he established the link between the classical network theory and the four pole networks. From this first study the theory of electric networks including four pole networks evolved. Several papers and books were published in the period 1920's-1950's (Campbell, 1922), (Le Corbellier and Lange, 1923), (Streker and Feldtkeller, 1929), (Feldtkeller, 1930), (Feldtkeller and Streker, 1930), (Konig, 1930), (Feldtkeller and Kautter, 1931), (Selach, 1931), (Gearwald, 1932), (Brillouin, 1936), (Darlington, 1939), (Pipes, 1940), (Peterson, 1948), (Reed, 1955) and (Weber, 1956).

The first study of a mechanical system by means of four pole networks was probably carried out by Mason (1927), for an acoustical filter, and by MacLean (1940) for an electromechanical system. Many other studies on specific problems followed these first examples of mechanical four pole network studies (Peterson and Bogert, 1950). However it was some time before a four pole theory was systematically developed for purely mechanical systems. Molloy (1957, 1958) presented two similar papers where he gave definitions of four-pole mechanical elements and presented four-pole matrices of lumped elements (mass, spring, damper) and some distributed elements (cantilever beam, rubber in shear). He also presented the connecting rules for systems with series and parallel elements and considered both force and velocity generators. The study of four-pole networks for mechanical systems followed the same course of mechanical network studies. In the beginning, mechanical systems were converted into equivalent electrical networks using either the direct or inverse analogy. Later, however, mechanical systems were drawn in a way similar to the impedance and mobility methods (Molloy, 1957, 1958), (Harris and Crede, 1961) and (Snowdon, 1971).

The mechanical four-pole network theory made analysis easier and in some cases made the study of systems composed of distributed elements possible. Exact solutions for the flexible elements was facilitated by the use of T or Π circuits where the input and output impedances and the velocity and forces ratios between the two terminals were derived analytically (Wright, 1958) and (Hixson, 1961). When engineers began to study vibration isolation systems with multiple mounts they found that to have an accurate analysis, multiple degrees-of-freedom models were required. In many cases all six kinematic and dynamic parameters at the connecting points were needed so that coupling between different types of vibration (flexural, longitudinal, shear, torsional vibrations) could be taken into account. Neither the T or Π impedance/mobility networks nor the four pole networks were suitable for modelling systems with multiple input and output parameters, so the impedance/mobility and four poles network theories were developed so that multiple degrees-of-freedom systems could be studied. In place of bilateral or four-pole elements, black box elements with multiple terminals were introduced (Rubin, 1964, 1966) and (O'Hara 1966). These block elements were described in terms of matrix relations. O'Hara defined mobility as: "*a tensor (or a tensor component) which operationally describes the effects upon the resultant velocity (or several velocities) of the application of a force or an array of forces*". He also defined the impedance as: "*a tensor (or a tensor component) which operationally describes the effects upon the resultant velocity (or several forces) of the application of a velocity or an array of velocities*". The diagonal terms of these tensors were defined as the driving point mobilities and driving point impedances respectively. The off-diagonal terms were called transfer or cross mobilities and impedances respectively. Rubin (1964, 1966) extended the four-pole theory to a multiple terminal approach for mechanical systems, where the input and output variables were related by a transmission matrix. Rubin also presented the relationship between mobility, impedance and transmission matrices. It is important to note that both O'Hara and Rubin worked out their formulations directly in terms of mechanical variables rather than in terms of analogue electrical parameters since, at that time, the ideas of mobility and impedance methods were widely accepted by the scientific community.

The multiple terminal block diagram approach allowed the possibility of analysing complicated networks by means of the input and output parameters of each element only. As a consequence papers began to be published where mobility, impedance or transmission matrix elements were given using either measured data or analytical formulae. In particular, mobility, impedance or transmission functions were derived for simple structures as rods, shafts, beams, plates and cylindrical shells. These type of studies are still underway, particularly with reference to line or surface excitations for the so called "line" or "strip" mobilities.

Regretfully, there are few books dealing with the matrix representation of impedance/mobility and transmission theory. The most complete work is probably by Neubert (1987). Bishop and Johnson's book (1960) gives the driving point and transfer mobilities/impedances for one-dimensional flexible systems for longitudinal, torsional and flexural vibration. A general modal formulation for the calculation of plate and shell mobilities, longitudinal, shear and out-of-plane flexural vibration is presented in Soedel's book (1993). Finally, Cremer, Heckl and Ungar (1988) give mobility or impedance formulae for one- and bi-dimensional distributed elements of infinite extent. A good introduction to transmission matrix theory is given in books by Pestel and Leckie (1963) and Hatter (1973).

6. STUDY OF VIBRATION PROBLEMS USING IMPEDANCE OR MOBILITY PARAMETERS

The vibration analysis of a mechanical systems is usually classified as *"structure borne sound"* and *"mechanical or seismic vibration"* (Cremer et al, 1988). Structure borne sound is concerned with mechanical waves in solid bodies that are related to sound radiation phenomena. This type of vibration is defined for periodic motions occurring in a frequency range between 16 and 16000 Hz: the audible frequency range. In the majority of cases the study of structure borne sound is related to problems of noise control, radiation of sound in water and "sonic" structural fatigue. Studies of musical instruments or materials engineering are more rare. In contrast, a mechanical vibration problem is defined for periodic motion at frequencies lower than 16 Hz and it is concerned with the kinematics and dynamics of the systems itself. In many cases vibrations of mechanical systems occurring at relatively low frequencies, below a few hundred Hertz, are also treated as seismic vibrations. Typical mechanical vibration problems are concerned with studies of machine or ground vibration isolation, rotors unbalance, vibration absorbers and neutralisers. In general mechanical vibration is modelled with lumped elements (springs, rigid masses and dissipative elements). In contrast, structure borne sound problems require more detailed models that account for the wave propagation phenomena in the elements composing the system.

Impedance and mobility methods can be used for both structure borne sound and mechanical vibration studies. In the following subsections a concise review is given for the response of a generic linear mechanical system due to three types of excitations: first, harmonic or periodic; second, transient and third, stationary random excitations. This survey highlights how impedance or mobility parameters can be used to build up a model that can be used for the analysis of any type of stationary excitation.

6.1. Time-varying response to harmonic, periodic and transient deterministic excitations

The instantaneous response $x(P_1,t)$ at a point P_1 of a linear mechanical system excited by a force $f(P,t)$ acting at the same, $P = P_1$, or different point $P = P_2$ of the system is given by the integral

$$x(P_1,t) = \int_0^\infty F(P_2,t-\tau)I(\tau)d\tau\,, \tag{6.1}$$

where $I(\tau)$[8] is the impulse response of the system (Thomson, 1993). This expression is valid

[8] The variable τ represents time prior to the instant t so that the instant t is given by τ=0 and the instant infinity in the past, t=−∞, is given by τ=∞. Therefore the response calculated with equation (6.1) can be interpreted as the superposition of the responses at the instant t due to a continuous sequence of impulse responses $F(t-\tau)I(\tau)\delta\tau$ calculated between an instant in the infinite past (τ=∞) and the instant t (τ=0).

for any type of time history of the excitation (harmonic, periodic, transient or random type) and can be used to study systems composed by both lumped and flexible elements. The response $x(P_1,t)$ could be a displacement, velocity or acceleration parameter as long as the impulse response $I(\tau)$ is expressed with the appropriate dimensions. When the excitation acting on the system is deterministic and harmonic the system response calculated with equation (6.1) is characterised by a "*transient*" part where the response is composed by an harmonic function that decays exponentially with time[9] and by an harmonic function with constant amplitude. After a few initial oscillations the first component of the response can be neglected so that the "*steady-state*" response is then given only by the harmonic function with constant amplitude. If the harmonic excitations have the form $f(t)=\text{Re}\{F(\omega)\exp(j\omega t)\}$, where ω is the circular frequency (rad/s) and $F(\omega)$ is complex, then the steady state response is also harmonic so that the displacement could be written as: $x(t)=\text{Re}\{X(\omega)\exp(j\omega t)\}$ where $X(\omega)$ is complex. The displacement $x(t)$ is related to the harmonic excitation via a linear relation of the following form (Thomson 1993):

$$x(t)=\text{Re}\left\{\alpha(\omega)\cdot F(\omega)e^{j\omega t}\right\}, \tag{6.2}$$

where

$$\alpha(\omega)=X(\omega)/F(\omega) \tag{6.3}$$

is a complex function of frequency called the "*frequency response function*" (FRF) of the system that, in this particular case, is the system receptance (Bishop and Johnson 1960, Ewins 2000). The relationship between receptance and mobility is described in section 6.3.

In many cases the excitation acting on the mechanical system is periodic, with period T, but not harmonic. In this case the excitation can be regarded as a summation of an infinite number of harmonic functions, whose circular frequency is an integer multiple of the fundamental one $\omega_n=n\omega_o=2n\pi/T$, by means of a *Fourier series* $f(t)=\sum_{n=-\infty}^{+\infty}F(\omega_n)\exp(j\omega_n t)$ (Wylie and Barrett, 1982). Because the system is linear, the steady-state response can be calculated by superposing the harmonic responses due to each of the exciting harmonic forces $x(\omega_n,t)=\text{Re}\{\alpha(\omega_n)\cdot F_n\exp(j\omega_n t)\}$, so that (Ewins 2000, Wylie and Barrett, 1982):

$$x(t)=\text{Re}\left\{\sum_{n=-\infty}^{+\infty}\alpha(\omega_n)F(\omega_n)e^{jn\omega_o t}\right\}. \tag{6.4}$$

Even if the time variation of the excitation is not periodic the response of a linear system can be treated as a superposition of an infinite continuous number of harmonic motions. In this case the Fourier series of the excitation is replaced by an integration of harmonic functions by means of a *Fourier integral* $f(t)=\int_{-\infty}^{+\infty}F(\omega)\exp(j\omega t)d\omega$ (Wylie and Barrett, 1982). The Fourier integral represents a continuous function in the frequency domain also called *Fourier*

[9] The exponential decay of the transient oscillation is proportional to the loss effects in the system. For example, in a lightly damped system, with damping ratio ζ=0.05, the amplitude is reduced of 50% (i.e. 3 dB) after 2 cycles while in a heavily damped system, with damping ratio ζ=0.5, the amplitude is reduced of 50% during the first half cycle.

spectrum since it provides a measure of the intensity of the force f in the frequency interval between ω and $\omega + d\omega$. In this case the system response is not periodic and depends on the forcing variation in time. However, similarly to the transient excitation, the response can also be expressed as the Fourier integral $x(t) = \int_{-\infty}^{+\infty} X(\omega)\exp(j\omega t)d\omega$ where, as shown by Strasberg (1958), $X(\omega) = \alpha(\omega) \cdot F(\omega)$. Thus, the response of the system is once more given in terms of the frequency response function as follows (Ewins 2000, Wylie and Barrett, 1982):

$$x(t) = \mathrm{Re}\left\{ \int_{-\infty}^{+\infty} \alpha(\omega) \cdot F(\omega) e^{j\omega t} d\omega \right\} \tag{6.5}$$

This brief survey shows that the frequency response function is a key element for the derivation of the forced response of a mechanical system excited by harmonic, periodic or transient deterministic excitations. In general, most of the vibration problems are studied in the frequency domain rather than in the time domain by plotting, as a function of frequency, the modulus and phase of the frequency response function. These two plots provide, for a certain frequency range, the amplitude and phase shift of the steady state harmonic response when the system is excited by a unit harmonic force at each frequency. With these two plots it is therefore straightforward to derive the response relative to any harmonic or periodic excitation. For the response to a transient excitation a third plot is used which gives the *Fourier transform* of the excitation: $F(\omega) = \int_{-\infty}^{+\infty} f(t)\exp(-j\omega t)dt$. For the particular case of shock excitations, i.e. transient excitation of brief duration with respect to the period of the fundamental natural frequency of the system the response of the system is usually examined with reference to the *response spectrum*. The response spectrum is given by the maximum peak response of a single degree of freedom oscillator excited by the given transient force as a function of the natural frequency of the oscillator itself (Thomson, 1993). Two types of shock spectra are most used: first the "*initial shock spectrum*" and second the "*residual shock spectrum*". The first one represents the maximum response while the shock pulse is still acting and the second one represents the maximum response after the pulse has occurred.
In conclusion it can be seen that the frequency response function occupies a central role in the vibration studies independently to the type of problem studied. Indeed engineers often derive from measurement or computer simulations the frequency response function in order to characterise the dynamics of a system under study.

6.2. Time-average response to random stationary excitations

In real operational conditions the excitations acting on the elements of a mechanical system are affected by many factors as for example changes of the working conditions (alteration of the primary loads due to external disturbances) or modifications of the intrinsic properties of the system (modification of the physical properties of the elements of the system due to variation of temperature or static loads or alteration of the characteristics of the connecting surfaces between elements of a mechanism due to abrasion or chemical deterioration phenomena). When these random variations of the operational conditions are significant, the time history of the excitation is not deterministic. The response of a system to a random excitation is also random, but of modified type (Mains, 1962), and can not be derived with the formulation described in the previous section. A new approach is therefore required which takes into account the statistical properties of the excitation and response time histories, and consequently the statistical properties of the response of the system.

When random phenomena are studied no attempt is made to specify the amplitude and phase of the response instant by instant. In contrast the response is determined in terms of averages or mean value parameters which are given by the following three relations (Strasberg, 1958):

$$\overline{x(t)} = \langle x(t) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} x(t)dt \tag{6.6}$$

$$\overline{x^2(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} [x(t)]^2 dt \tag{6.7}$$

$$\text{r.m.s.} = \left(\overline{x^2(t)} \right)^{1/2} = \left\{ \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} [x(t)]^2 dt \right\}^{1/2} \tag{6.8}$$

that are respectively the "*mean-value*", "*mean-square*" and "*root-mean-square*" (rms). The mean-square parameter is of particular significance for the vibration of a system. For example, the mechanical power dissipated by a vibrating member is proportional to the mean-square value of the oscillating strain for any type of wave deformation. As can be deduced from equations (6.6) to (6.8), these average parameters are function of the starting time t_1 and of the averaging time T. Therefore, they are meaningful only for a particular class of random time histories whose integration is independent of the starting time t_1 and of the averaging time T. Random phenomena whose time history has such characteristics are called "*stationary*". It is important to point out that there are many phenomena that are non-stationary. Considering for example a transient excitation of short duration, a shock excitation, its mean-value is dependent on the averaging time. In fact, it decreases as the averaging time is increased.

The statistical parameters that are more suited to relate the random excitation and response of a linear system are two quantities called respectively "*autocorrelation function*" and "*power spectral density,*" (PSD). The autocorrelation function is derived for a record $x(t)$ by multiplying the ordinates of the record at the instant t and $t+\tau$ and determining the average value:

$$\text{autocorrelation} = \langle x(t) \cdot x(t+\tau) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} x(t)x(t+\tau)dt \ . \tag{6.9}$$

For a stationary process, i.e. a process whose probability distributions are invariant with reference to time, then the three statistical quantities given above in equations 6.6-6.8 are constants independent of the starting time t_1. Under these conditions the autocorrelation function depends only on the quantity τ and in this case it is represented by the symbol $R_{xx}(\tau)$. For the limiting case of $\tau = 0$ the autocorrelation function equals the mean square value $R_{xx}(0) = \overline{x^2(t)}$.

The power spectral density is given by the Fourier transform of the autocorrelation function:

$$S_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau \ . \tag{6.10}$$

Therefore, the autocorrelation function can be expressed in terms of the power spectral density with the following Fourier integral

$$R_{xx}(\tau) = \int_{-\infty}^{+\infty} S_{xx}(\omega) e^{j\omega\tau} d\omega \ . \tag{6.11}$$

Equations (6.10) and (6.11) are referred as the *Wiener-Khintchine* relations. Considering the limiting case of $\tau = 0$ for which the autocorrelation function equals the mean square value $R_{xx}(0) = \overline{x^2(t)}$, then, using equation (6.11) it turns out that the mean square value of a time history is given by the sum over all frequencies of $S_{xx}(\omega)d\omega$

$$\overline{x^2(t)} = R_{xx}(0) = \int_{-\infty}^{+\infty} S_{xx}(\omega) d\omega \ , \tag{6.12}$$

$S_{xx}(\omega)$ can therefore be interpreted as the distribution of the mean-square over the frequency range.

Considering a linear system, the mean-value of the random response due to a random excitation can be derived using the general expression of the response of a linear system given in equation (6.1) so that (Crandall and Mark, 1963):

$$\overline{x(P_1,t)} = E\left[\int_0^{\infty} F(P_2, t-\tau) I(\tau) d\tau\right] = \overline{F(P_2,t)} \cdot \alpha(0) . \tag{6.13}$$

Therefore the ratio between the mean value of the response and excitation is given by the frequency response function for $\omega = 0$. The mean-square value of the response can be calculated in terms of the power spectral density of the random excitation with the following equation (Crandall and Mark, 1963):

$$\overline{x^2(P_1,t)} = R_{xx}(0) = \int_{-\infty}^{+\infty} S_{xx}(\omega) d\omega = \int_{-\infty}^{+\infty} |\alpha(\omega)|^2 S_{FF}(\omega) d\omega \ . \tag{6.14}$$

In fact, for a stationary random process, the excitation and response power spectral densities are related by the squared modulus of the frequency response function:

$$S_{xx}(\omega) = |\alpha(\omega)|^2 S_{FF}(\omega) \ . \tag{6.15}$$

This brief introduction to the analysis of systems excited by random forces also highlights the central role of the frequency response function $\alpha(\omega)$ for the derivation of the statistical parameters characteristics of a random phenomena.

6.3. Frequency Response Function forms

The frequency response function $(\text{kinematic parameter/dynamic parameter})$ can assume different formats depending on the kinematics variable used to describe the response of the mechanical system: displacement, velocity or acceleration. For each of the defined frequency response functions it is possible to derive the inverse function $(\text{dynamic parameter/kinematic parameter})$, which could also be defined as a frequency response function.

Response Parameter	Direct or Standard Frequency Response Function		Inverse Frequency Response Function	
	Formula	Name	Formula	Name
displacement	$\alpha(\omega)=\left.\frac{X(P_1,\omega)}{F(P_2,\omega)}\right\|_{F(P\neq P_2)=0}$	Receptance Admittance Compliance Dynamic flexibility	$K(\omega)=\left.\frac{F(P_1,\omega)}{X(P_2,\omega)}\right\|_{X(P\neq P_2)=0}$	Dynamic stiffness
velocity	$Y(\omega)=\left.\frac{\dot{X}(P_1,\omega)}{F(P_2,\omega)}\right\|_{F(P\neq P_2)=0}$	Mobility	$Z(\omega)=\left.\frac{F(P_1,\omega)}{\dot{X}(P_2,\omega)}\right\|_{\dot{X}(P\neq P_2)=0}$	Mechanical impedance
acceleration	$A(\omega)=\left.\frac{\ddot{X}(P_1,\omega)}{F(P_2,\omega)}\right\|_{F(P\neq P_2)=0}$	Inertance Accelerance	$M(\omega)=\left.\frac{F(P_1,\omega)}{\ddot{X}(P_2,\omega)}\right\|_{\ddot{X}(P\neq P_2)=0}$	Apparent mass

Table 3: *Frequency Response Functions.*

Table 3 summarises the three standard definitions of frequency response functions and the equivalent inverse functions. It is important to emphasise that the definition of the direct frequency response function parameters assumes that only one degree-of-freedom (terminal) of the bilateral or multiple terminal element is excited. In contrast the definition of the inverse frequency response functions assumes that all the degrees-of-freedom characteristic of the bilateral or multiple terminal element are set to zero except that accounted in the inverse frequency function. When the kinematic and dynamic parameters correspond to the same degree-of-freedom of the system, i.e. to the same terminal, then the terms listed in table 3 have the prefix "*driving point*" while if the two parameters correspond to two different terminals, i.e. to two different degrees-of-freedom, then the terms listed in table 3 have the prefix "*transfer*". For example the ratio between velocity and force can be a "*driving point mobility*" or a "*transfer mobility*" depending weather the velocity and force are defined with reference or not to the same terminal, i.e. degree-of-freedom.

In general it is suggested to avoid the use of the inverse definition of frequency response function since this type of parameter is prone to generate some confusion. When bilateral elements are considered the direct and inverse frequency response functions are the inverse of each other. However, this simple relation does not hold when multiple terminal elements are used in which case the direct and inverse frequency response functions relative to the same pair of terminals are not the inverse of each other (O'Hara, 1966). Also, the direct frequency response function is invariant with respect to number of terminals of the element. In contrast the inverse frequency response function depends on the number of terminals accounted in the model of the system so that when one or more terminals are added or removed from the model of a system all the inverse frequency response functions have to be calculated again (O'Hara, 1966).

The American National Standard Association published in 1961 a document (*Nomenclature and Symbols for Specifying the Mechanical Impedance of Structures*, S2.6-1963 (R 1971)) with a list of recommended terminology. However there is still some confusion since in many scientific papers and books different terms are used to define the same parameter. For example the ratio between displacement and force can be defined as receptance, admittance, compliance and dynamic flexibility. Moreover quite often it is possible to encounter documents where either mobility or admittance terms are used to define the ratio between a linear or angular velocity and a force or moment excitation respectively (Belsheim, 1962; Plunket, 1958).

7. CONCLUDING REMARKS

This report presents a historical review of the concèption and evolution of *impedance* and *mobility* methods in structural dynamics. The large number of papers and reports collected for the review, which are listed in the bibliography section, have indicated that the origin of the mechanical impedance concept was inspired by the work done by scientists at the end of the 19^{th} century for electrical systems. Starting from the concept of electric impedance Professor Arthur G. Webster first defined the mechanical impedance during a meeting of the Physical Society held in Philadelphia in 1914. Since at that time the well established theory that was already available for electrical networks has been used to convert the mechanical systems into their electrical analogues. Two theories arose which were the direct and the inverse electromechanical analogies. In 1938 Firestone presented a paper describing an "*ad hoc*" network theory for mechanical systems. This was a major step forward in the formulation of a new network theory that is now day known as mobility-impedance approach in structural dynamics. Firestone's theory has been completed by extending the mobility and impedance relations to multi-terminal structural elements using mobility, impedance and transmission matrices.

At present, there is still research in progress for the definition and characterization of mobility/impedance parameters of distributed flexible mechanical systems. In particular a lot of work is being carried out for the analysis of systems composed by one- or by-dimensional flexible systems (beams, plates and shells) connected along line or surface junctions.

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This bibliography contains a large list of papers which are somehow related to impedance or mobility methods in dynamics. The list has been organised in seven sections

1) Books,
2) Proceedings Of Conferences And Reports,
3) Mobility And Impedance Methods: General Theory,
4) Distributed One- And Bi-Dimensional Systems, Structural Power And Mobilities,
5) Structural Vibrations Isolation (Matrix Methods For Coupled Systems),
6) Characterisation Of Structure-Borne Sound Sources Using Mobility/Impedance Parameters,
7) Experimental Measurement Of Frequency Response Functions.

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