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Power Transmission to Flexible Receivers by Force Sources

by

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Contents

Abstract

This memorandum concerns how one might approximate simply and accurately the power transmission to flexible receivers by force sources (either discrete or line-distributed). Some existing methods of simplifying the power transmission predictions are first reviewed. Then a new technique – the "power mode" method – is described. By this theory the vibration power transmitted to a flexible receiver by N discrete point forces can be regarded as the power input by N independent power modes. As a result one may approximate the power by its mean value as well as an upper and a lower bound in a simple manner. It also has been shown that this approach can be used for the cases where both force and moment excitations are involved, provided the mobility matrices of the receiver structures are scaled properly. Similar results can be obtained by analogy for velocity source excitation cases. In a companion memorandum, the methods described in this report are developed to approximate the vibration power transmission to flexible receivers from general sources, where the interest concerns the power transmission between coupled subsystems.

1. Introduction

Any resiliently mounted machine will induce noises unavoidably to its surroundings partly by the way of airborne sound transmission and partly by vibration transmission. Techniques of the prediction of the transmitted vibration are less well developed than that of the airborne sound because of its greater complexity. It has been increasingly accepted that the power transmitted to the receiver by the machine, which is regarded as an equivalent quantity of the structure-borne sound, best quantifies the vibration transmission. Much of the fundamental theory of structure-borne sound power can be found in a classical book of Cremer and Heckl [1].

The time average power at a point is

$$P = \frac{1}{T}\mathrm{Re}\left\{\int_0^T F(t)^* v(t)\,dt\right\} = \frac{1}{T}\mathrm{Re}\left\{\int_0^T F(t) v(t)^*\,dt\right\} \tag{1.1}$$

where $F(t)$ is the instantaneous force, $v(t)$ the velocity, the asterisk the complex conjugated and T the time interval. When both the force and the velocity are assumed to have a harmonic time dependence, i.e., $F(t) = Fe^{j\omega t}$, $v(t) = ve^{j\omega t}$, equation (1.1) then can be re-written as

$$P = \frac{1}{2}\mathrm{Re}\left\{F^* v\right\} = \frac{1}{2}\mathrm{Re}\left\{F v^*\right\} \tag{1.2}$$

If M and Z represent the complex mobility and impedance at the point, respectively, by definition

$$v = MF, F = Zv \tag{1.3}$$

Equation (1.2) hence becomes

$$P = \frac{1}{2}|F|^2\,\mathrm{Re}\{M\} = \frac{1}{2}|v|^2\,\mathrm{Re}\{Z\} \tag{1.4}$$

Equation (1.4) shows that the power transmitted to a receiver depends not only on the strength of the excitation, but also on the properties of the receiver. This complicates the analysis considerably, since the receiver is often resonant, and its point mobility or impedance is not always known exactly.

Various methods are available to predict the vibration power transmission to flexible receivers by force/velocity source excitations, of which the frequency-response-function (FRF) method [2], the Finite Element (FE) method [3] and the Green Function method [1] are three of the most commonly used. These techniques are all quite general and can be used to predict exactly the vibration levels in mechanical structures. But for large number of excitation points (e.g. an approximation of line-couplings), the computational cost is generally very high. Under such circumstances these techniques may be too expensive to be practical. Besides, all the deterministic methods are based on the assumption of idealized mathematical models in which the properties of the structure and the excitation are known exactly. For many structures, however, the detailed physical properties required by such a model may not be available, or relevant, for the particular problem at hand. Therefore even if this cost is acceptable, in practical situations it might be impossible to predict the vibration response accurately. For example, as frequency increases the system response becomes increasingly sensitive to geometrical imperfections, so that even a very detailed deterministic mathematical model based on the nominal system properties may not yield a reliable response for high-frequency excitations. Under these circumstances, frequency/ensemble average technique is often being used to provide an understanding of broad features of the vibration levels and transmission, given only a relatively coarse and uncertain description of the structure and its excitation [1,4-5].

It is often more appropriate to approximate the main properties of the dynamic behaviour of a vibratory system rather than to precisely predict its detailed response, at the same time to reduce as many required details as possible. Of course the approximation should have acceptable accuracy as well as low computational cost.

There is a need, therefore, to simply and accurately approximate the vibration power transmitted to a receiver by force/velocity sources, which forms the subject of this memorandum.

Firstly some existing methods of simplifying the power transmission predictions are reviewed in the next two sections for discrete and line-distributed force excitation cases, respectively. They are mainly the multipole method [8-9], the mean emission method [10], the modal analysis method [1] as well as the Fourier Transform method [6-7]. When there is full geometrical dynamic symmetry of the excitation positions on the receiver, the power transmission prediction from a source to a receiver can be greatly simplified by the multipole method. The principle of this approach is to describe the source as a set of vibration "poles" and the receiver as a set of polar mobilities or impedances. Thus the vibration power transmitted by N forces can be regarded as the power transmitted by N independent "poles" of vibration. The mean emission method is to estimate the vibration power by a pair of formulae, one giving the mean power and the other the standard deviation. This technique only require the mean square force, mean point mobility and mean transfer mobility. However this method is only valid when the phases between excitation points can be assumed to be random. For line-distributed source excitations, The modal analysis [1] and Fourier Transform methods [6-7] have been found extremely useful. Both of these are based on the decomposition of distributed loads into a set of orthogonal functions and then superposition of the responses each produces. It will be shown later they are also very useful to approximate the power transmission between line-coupled subsystems.

A new technique – the "power mode" method – is then described. This extends the work of [11]. By this theory the vibration power transmitted to a flexible receiver by N discrete point forces can be regarded as the power input by N independent power modes. As a result one may approximate the power by its mean value as well as an upper and a lower bound in a simple manner. It also has been shown that this approach can be used for the cases where both force and moment excitations are involved, provided the mobility matrices of the receiver structures are scaled properly. Finally some numerical examples are presented.

Similar results can be obtained for velocity source excitation cases by analogy with the force excitation cases, with the mobility matrix of the receiver being replaced by the corresponding impedance matrix, although in this memorandum only force sources have been considered.

In a companion memorandum [12], the methods described in this report are developed to approximate the vibration power transmission to flexible receivers from general sources, where the interest concerns the power transmission between coupled subsystems.

2. Power transmission to flexible receivers by discrete force sources

2.1 Frequency Response Function (FRF) method

For discrete force source excitation cases, as shown in Figure 2.1, the transmitted power can be found using FRF-based mobility matrix method. It is given by

$$P = \frac{1}{2}\mathrm{Re}\left\{\mathbf{F}^H \mathbf{M}\mathbf{F}\right\} \qquad (2.1)$$

where $\mathbf{F}$ is the column vector of force amplitudes at various excitation points, and $\mathbf{M}$ the corresponding symmetric complex mobility matrix of the receiver structure, while superscript H denotes the conjugate transpose.

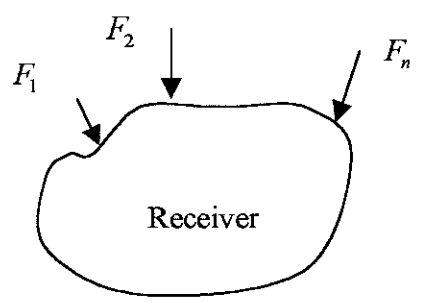


Figure 2.1

Equation (2.1) is a quite general expression for power transmitted to flexible receivers by discrete force excitations. The excitation forces can be bending forces/moments or in-plane forces/moments or both, and consequently the power transmission can be regarded as due to bending or in-plane motions of the structures, or both of them. Similar results can be obtained for velocity source excitation cases by analogy with the force excitation cases, with the mobility matrix of the receiver being replaced by the corresponding impedance matrix. In this study, therefore, only the force source cases have been concerned.

It can be seen a full description of the vibration power by this FRF technique needs the following information to be exactly known: (1) the source strength, (2) the point mobilities of the receiver at the excitation points, and (3) the transfer mobilities between these excitation points. Therefore very many terms need to be evaluated before a detailed and accurate prediction of power can be made, especially for large number of point excitations as well as distributed excitations. Besides this information may not be available to acceptable accuracy. This provides the motivation for engineering approaches which give a simple but accurate approximation to the transmitted power. Often also it is the frequency average power which is of interest. Some possibilities of simplifying the power prediction procedures by using fewer terms are described in the following text.

In many cases of practical interest, the study of vibration transmission from a source to a receiver usually only needs to consider the bending motions of the structures. Therefore no in-plane motion is included in the rest of the analysis.

2.2 Simplification and approximations for the vibration power

2.2.1 Special frequency range excitations

When the force sources are the same type and also uni-directional vibration is concerned, the transmitted power can be simply predicted for very low- and very high-frequency range excitations.

2.2.1.1 Power prediction for low-frequency range

The low-frequency range is defined here when the wavelengths of vibration waves (e.g. the flexural waves) in the receiver structure are large compared to the distances between the excitation points. The excitations are then in effect applied at the same point so that the corresponding point mobility and the transfer mobility are almost equal, i.e.,

$$M_{nn} \approx M_{mn} \approx M_{in} \tag{2.2}$$

Combining with equation (2.1), the power then can be approximated by

$$P \approx \frac{1}{2}\left(\left|\sum_{n=1}^{N} F_n\right|^2\right)\text{Re}\{M_{in}\} \tag{2.3}$$

Therefore an accurate prediction can be made knowing only the excitation forces and the real part of the point mobility of the receiver.

2.2.1.2 Power prediction for high-frequency range

Similarly, a simple estimate of transmitted power can be made for very high frequency limit where all excitation points are separately by a distance that is large compared to the wavelength. They can then be regarded as being independent, due to very short wavelengths of the flexural waves or heavy damping. In this case the transfer mobilities can be ignored and $\mathbf{M}$ becomes diagonal. The power then can be approximated by

$$P \approx \sum_{n=1}^{N} \frac{1}{2} |F_n|^2 \operatorname{Re}\{M_{nn}\} \tag{2.4}$$

Comparing with equation (2.1), great simplification also has been made since only the amplitudes of the excitation forces as well as the real parts of the point mobilities of the receiver need to be known. The same simplifications can also be made when the excitation forces are uncorrelated or statistically independent.

2.2.2 The multipole method

When there is full geometrical and dynamical symmetry of the excitation positions on the receiver, the power transmission prediction from a source to a receiver can be greatly simplified by the "multi-pole" method [6-7]. The principle of this approach is to describe the source as a set of vibration "poles" and the receiver as a set of polar mobilities or impedances. This technique is described below in detail.

Let $\mathbf{\Phi}$ be an $N \times N$ Hadamard matrix, i.e., a matrix of orthogonal functions and whose elements are ± 1. The matrix $\mathbf{\Phi}$ implies such relations

$$\mathbf{\Phi} = \mathbf{\Phi}^T, \mathbf{\Phi}^{-1} = \frac{1}{N}\mathbf{\Phi} \tag{2.5}$$

Let the set of force excitations $\mathbf{F}$ be weighted by $\mathbf{\Theta} = \frac{1}{\sqrt{N}}\mathbf{\Phi}$ to give a new set of polar forces $\mathbf{Q}$ defined by

$$\mathbf{Q} = \mathbf{\Theta F} \tag{2.6}$$

Accordingly the polar mobility matrix is defined by

$$\mathbf{D} = \mathbf{\Theta M \Theta} \tag{2.7}$$

The power transmitted to the receiver can hence be re-written as

$$P = \frac{1}{2}\operatorname{Re}\{\mathbf{Q}^H \mathbf{D Q}\} \tag{2.8}$$

Due to the geometrical symmetry of the excitation positions on the receiver, $\mathbf{D}$ is diagonal. The power then can be simply predicted by

$$P = \frac{1}{2}\sum_{n=1}^{N} |Q_n|^2 \operatorname{Re}\{D_{nn}\} \tag{2.9}$$

Equation (2.9) shows that the vibration power transmitted by N forces can be regarded as the power transmitted by N independent "poles" of vibration (e.g. monopole, dipole, etc). Thus the measurement and prediction of vibration power can be facilitated. At the same time it can be used quite easily to determine the dominant "poles" of the vibration behaviour of the structures. This technique is particularly effective if it is found that only one or two poles are significant with regard to power transmission. Then only a few of the terms in equation (2.9) contribute to the transmitted power. No moment excitations have been considered.

In the above the size of the Hadamard matrices are $N = 2^p, p = 1,2,3\ldots$. Although four or eight excitation points are relatively common in practical engineering, many other possible values of N may not be related by $N = 2^p$. However, since the polar mobilities and in particular that of the monopole, which is usually the most dominant source, are not very sensitive to the exact positions, the number of points used may not have to correspond to the exact number of excitations but to the convenient $N = 2^p$. This is particularly useful if there are a large number of excitation points and some of them are spaced within one half wavelength. But for higher frequencies where all the excitations points are spaced more than half of the wavelengths with each other, no such simplification can be made.

2.2.3 The mean emission method

For more general cases, i.e., the frequency range is neither very low nor very high, and the excitations and receiver structure are not symmetric, the simplifications of Section 2.2.1-2.2.2 cannot be made. However, it is also possible to find simple estimates of the transmitted power using fewer terms than equation (2.1) but at the cost of replacing the accurate prediction by an approximated power band.

In [10], equation (2.1) has been simplified to yield the following estimates of the mean power and the standard deviation

$$E(P) = \frac{N}{2}\left(\frac{1}{N}\sum_{n=1}^{N}|F_n|^2\right)\mathrm{Re}\left\{\frac{1}{N}\sum_{n=1}^{N}M_{nn}\right\} \tag{2.10}$$

$$\sigma(P) = \frac{1}{2}\left(\frac{1}{N}\sum_{n=1}^{N}|F_n|\right)^2\sqrt{\frac{N(N-1)}{2}}\left|\bar{M}_{tr}\right| \tag{2.11}$$

Here, $|\bar{M}_{tr}|$ is the mean transfer mobility of the receiver achieved by averaging all the transfer mobilities among the excitation points, and N the number of point-force excitations. It can be seen only the mean square force, mean point mobility and mean transfer mobility are required to estimate the vibration power. However this method is only valid when the phases between excitations can be assumed to be random. Furthermore the moment excitations can only be included when the cross-coupling effects between degrees of freedom can be ignored.

2.2.4 Simple estimates of the transmitted power

There is a strong desire in many situations to find simple but accurate approximate estimates of the transmitted power. The full expression of equation (2.1) may be too computationally expensive or the required data may not be known to sufficient accuracy. In most cases, however, the situation may not correspond to the low and high frequency limits of section 2.2.1, nor may the multipole approach be applicable, nor may the mean emission method be accurate enough (e.g. the forces may be in-phase, and relatively close compared to the wavelength). This provides the motivation of the development of a new technique, the power mode method, which is based on eigen-decomposition of the mobility matrix of the receiver. In Section 4 the power mode approach is described, together with its application to the estimation of transmitted power of some numerical examples.

3. Power transmission to flexible receivers by line-distributed force sources

3.1 Deterministic approaches: The Green function and Finite Element methods

The Green Function and Finite Element methods are the most general of the techniques of estimating the power transmitted by distributed sources. The main principles of the Green Function method are described below. Further details can be found in [1].

If $F(\sigma_0)$ is the only force excitation applied at position σ_0 on the receiver and $v(\sigma)$ is the velocity response at σ, the general Green Function of the receiving structure can be expressed by

$$u(\sigma,\sigma_0)=\frac{v(\sigma)}{F(\sigma_0)} \tag{3.1}$$

Then under action of distributed force excitation the velocity response at σ can be written as

$$v(\sigma)=\int_{\sigma_0} u(\sigma,\sigma_0)F(\sigma_0)d\sigma_0 \tag{3.2}$$

When the force distribution $F(\sigma)$ is assumed to be time harmonic, the vibration power transmitted to the receiver can be written as

$$P=\frac{1}{2}\mathrm{Re}\left\{\iint_{\sigma} F^*(\sigma)u(\sigma,\sigma_0)F(\sigma_0)d\sigma_0 d\sigma\right\} \tag{3.3}$$

When the receiver is excited by N discrete forces, by equation (2.1) the transmitted power can be expressed by

$$P=\frac{1}{2}\mathrm{Re}\left\{\sum_{m=1}^{N}\sum_{n=1}^{N}F_m^* u_{mn}F_n\right\} \tag{3.4}$$

It can be seen that when $N\to\infty$, equation (3.4) becomes (3.3). Therefore equation (3.3) is in effect an extension of equation (2.1). It represents a unique and complete solution for the vibration power transmitted to a receiver by an arbitrary force source distribution. However the direct calculation of the power is usually rather complicated, especially when the structures have complex geometries and non-classical boundary conditions. In these cases finding the Green function can be quite computationally expensive. Under such circumstances, the Finite Element method is widely used.

The basic theory of the FE method [3] is to divide the structure into elements, and then, by the application of Lagrange's equation or otherwise, to obtain the differential equations of motion of the whole structure. This technique can be used either to find the Green functions of the structures, or to predict the structure's response directly. If F_n is the force excitation at the nth degree of freedom, and v_n is the corresponding velocity response, the power transmission can then be written as

$$P = \frac{1}{2}\text{Re}\left\{\sum_n F_n^* v_n\right\} \tag{3.5}$$

This technique can give accurate estimates provided at least 4~8 elements per wavelength are used. The amount of computation required is generally very large.

Both the Green function method and FE method can be prohibitively expensive to predict the power transmission to a general structure by arbitrary distributed sources. However as far as line-distributed force source excitation cases, as shown in Figure 3.1, the modal analysis and Fourier Transform methods can be used to predict the transmitted power in a straightforward way.

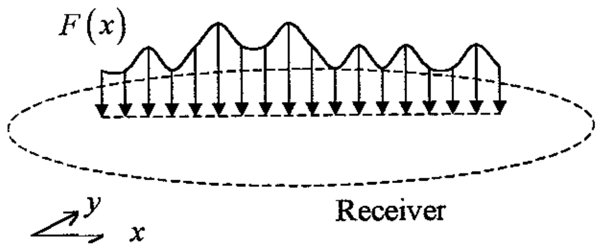


Figure 3.1

3.2 The modal analysis method

The modal analysis method states that the response of a system to any arbitrary excitation can be expressed in terms of the system's eigenfunctions (mode shape functions) and eigenfrequencies (natural frequencies).

Let $\phi_m(\sigma)$ denote the mth mass normalized modal shape function of the receiver structure, i.e.,

$$\int_{\sigma} m'\phi_n(\sigma)\phi_m(\sigma)d\sigma = \begin{cases} 1, m = n \\ 0, m \neq n \end{cases} \tag{3.6}$$

where m' is the mass density distribution of the receiver structure. Then the Green function in equation (3.1) can be obtained in a well-known form as

$$u(\sigma,\sigma_0) = j\omega \sum_{m=1}^{\infty} \frac{\phi_m(\sigma)\phi_m(\sigma_0)}{\omega_m^2(1+j\eta)-\omega^2} \tag{3.7}$$

where ω_m is the m th natural frequency of the structure and η the material loss factor.

Combining equation (3.7) with (3.3), the power transmitted to a receiver by a distributed force source can be written as

$$P = j\omega \sum_{m=1}^{\infty} \frac{\int_{\sigma} F^*(\sigma)\phi_m(\sigma)d\sigma \int_{\sigma} F(\sigma_0)\phi_m(\sigma_0)d\sigma_0}{\omega_m^2(1+j\eta)-\omega^2} \tag{3.8}$$

Equation (3.8) can then be re-written as

$$P = \frac{1}{2}\mathrm{Re}\left\{\sum_m |F_m|^2 \alpha_m\right\} \tag{3.9}$$

where F_m is the m th modal force given by

$$F_m = \int_{\sigma} F(\sigma)\phi_m(\sigma)d\sigma \tag{3.10}$$

and α_m is the mobility of the m th mode and is given by

$$\alpha_m = j\omega \frac{1}{\omega_m^2(1+j\eta)-\omega^2} \tag{3.11}$$

Equation (3.9) shows the modal analysis method is particularly efficient if the modal shape functions of the receiving structure along the excitation line are exactly known. In more general cases (e.g. non-uniform structures) where one cannot find the analytical expressions for the structure's mode shapes directly, some numerical methods, for example, the Finite Element method, may be useful to find the modal properties of the structure.

3.3 Fourier Transform method

The Fourier Transform method is particularly useful, when the receiver structures are uniform and infinite and where the excitation is applied along a straight line.

The one-dimensional Fourier Transform is defined by

$$G(\beta)=\int_{-\infty}^{+\infty} g(x)e^{-j\beta x}dx \tag{3.12}$$

$$g(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} G(\beta)e^{j\beta x}d\beta \tag{3.13}$$

The two-dimensional Fourier Transform is defined by

$$G(\beta,\gamma)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)e^{-j\beta x}e^{-j\gamma y}dxdy \tag{3.14}$$

$$g(x,y)=\frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(\beta,\gamma)e^{j\beta x}e^{j\gamma y}d\beta d\gamma \tag{3.15}$$

The principle of this method is to transform the vibration equation of the receiver as well as the force distribution into the wavenumber domain. This yields an algebraic equation for the response which can be solved analytically. Then the power transmitted can be obtained in the wavenumber domain, i.e.,

$$P=\frac{1}{2}\mathrm{Re}\left\{\int_{-\infty}^{+\infty} F(x)^* v(x)dx\right\}=\frac{1}{4\pi}\mathrm{Re}\left\{\int_{-\infty}^{+\infty} \bar{F}(k_x)^* V(k_x)dk_x\right\} \tag{3.16}$$

where $\bar{F}(k_x)$ is the Fourier transform of the distributed force $F(x)$

$$\bar{F}(k_x)=\int_{-\infty}^{+\infty} F(x)e^{-jk_x x}dx \tag{3.17}$$

and $v(x)$ and $V(k_x)$ are the velocity response along the line of the force excitation and the corresponding Fourier transform, respectively.

To demonstrate the application of this method, an example of a line-distributed force applying on an uniform and thin infinite plate is considered.

If it is assumed the line force is acting on the plate along the $y=0$ direction, the equation of motion of the plate is

$$D_p\nabla^4 w_p(x,y)+m_p\frac{\partial^2 w_p(x,y)}{\partial t^2}=F(x)\delta(y) \tag{3.18}$$

where D_p is the complex stiffness of the plate, m_p the mass per unit area of the plate and

$$\nabla^4 w_p(x,y)=\frac{\partial^4 w_p(x,y)}{\partial x^4}+2\frac{\partial^4 w_p(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 w_p(x,y)}{\partial y^4} \tag{3.19}$$

Applying the two-dimensional Fourier transform to (3.18), it follows that

$$D_p\left(k_x^2+k_y^2\right)^2 W_p\left(k_x,k_y\right)-m_p\omega^2 W_p\left(k_x,k_y\right)=\bar{F}\left(k_x\right) \tag{3.20}$$

From the above equation, the Fourier transform of the displacement is

$$W_p\left(k_x,k_y\right)=\frac{\bar{F}\left(k_x\right)}{D_p\left(k_x^2+k_y^2\right)^2-m_p\omega^2} \tag{3.21}$$

Then the inverse Fourier transform of the plate is

$$w_p\left(x,y\right)=\frac{1}{4\pi^2}\iint_\infty W_p\left(k_x,k_y\right)e^{jk_x x}e^{jk_y y}dk_x dk_y \tag{3.22}$$

From the above equation it follows that

$$w_p\left(x,0\right)=\frac{1}{4\pi^2}\iint_\infty W_p\left(k_x,k_y\right)e^{jk_x x}dk_x dk_y \tag{3.23}$$

The integral over k_y can be performed to give

$$W_p\left(k_x\right)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} W_p\left(k_x,k_y\right)dk_y \tag{3.24}$$

Substituting equation (3.21) into (3.24), the displacement response of the plate in the wave-number domain can be obtained as

$$W_p\left(k_x\right)=F\left(k_x\right)\alpha\left(k_x\right) \tag{3.25}$$

where

$$\alpha\left(k_x\right)=\frac{1}{4D_p k_p^2}\left(\frac{1}{\sqrt{k_x^2-k_p^2}}-\frac{1}{\sqrt{k_x^2+k_p^2}}\right) \tag{3.26}$$

is the line receptance of the plate in the wavenumber domain, and $k_p=\sqrt[4]{m_p\omega^2/D_p}$ is the wavenumber of the plate.

From equation (3.25)

$$V\left(k_x\right)=j\omega F\left(k_x\right)\alpha\left(k_x\right) \tag{3.27}$$

Combining this with equation (3.16), the power transmitted to the plate is

$$P=\frac{1}{4\pi}\mathrm{Re}\left\{\int_{-\infty}^{+\infty}\left|F\left(k_x\right)\right|^2 j\omega\alpha\left(k_x\right)dk_x\right\} \tag{3.28}$$

From equation (3.26), it can be seen that when $k_x \geq k_p$ then, ignoring the (small) damping of the plate

$$\mathrm{Im}\{\alpha(k_x)\} \approx 0 \tag{3.29}$$

Therefore equation (3.28) can be simplified as

$$P \approx \frac{1}{4\pi}\mathrm{Re}\left\{\int_{-k_p}^{+k_p} |F(k_x)|^2 j\omega\alpha(k_x)dk_x\right\} \tag{3.30}$$

Physically, the power transmitted by terms for which $k_x < k_p$ represents the relatively large amount of energy carried by the far-field propagating waves, and the power transmitted by $k_x > k_p$ terms represents the energy dissipated in the near-field and non-propagating terms.

4. Power transmission approximation by power-mode method

4.1 Power-mode theory

As shown in Section 2, there is a strong desire in many situations to find simple but accurate approximate estimates of the transmitted power, since the full expression of equation (2.1) may be too computationally expensive or the required data may not be known to sufficient accuracy. Although some specialist methods [8-10] have been developed to approximate the power transmitted from machine to flexible receivers, it is true to say more research is required before generally accepted and reliable methods are developed, due to the limitations of these existing techniques. This provides the motivation of the development of a new technique, the power mode method, which is based on eigen-decomposition of the mobility matrix of the receiver. It has some similarities to the multi-pole method [8-9] in that a set of force/velocity sources is transformed into a new set of power modal forces/velocities by weighting them by a set of orthogonal functions. (Some initial investigation can be found in [11].) There are two main advantages of this power mode approach. First, the transmitted power is often dominated by one or a few power modes, so that a simple approximation to the power transmission can be found. Secondly, it allows expressions for the bounds of the transmitted power to be developed in a simple manner as described in Section 4.4. The theory behind of the power mode method is described below.

The general expression for the power can be written as

$$P = \frac{1}{4}\mathrm{Re}\left\{\mathbf{F}^H\mathbf{V} + \mathbf{V}^H\mathbf{F}\right\} = \frac{1}{4}\mathrm{Re}\left\{\mathbf{F}^H\left(\mathbf{M} + \mathbf{M}^H\right)\mathbf{F}\right\} \tag{4.1}$$

Then the transmitted power becomes

$$P = \frac{1}{2}\mathbf{F}^H\,\mathrm{Re}\left[\mathbf{M}\right]\mathbf{F} \tag{4.2}$$

Equation (4.2) denotes that the power transmission is only related with the real part of the mobility matrix, while its imaginary part can be totally ignored.

Since $\mathrm{Re}[\mathbf{M}]$ is a real, symmetric and non-negative definite matrix, equation (4.2) is in a non-negative definite quadratic form. By matrix theories [13-14], $\mathrm{Re}[\mathbf{M}]$ can be decomposed into the form

$$\mathrm{Re}[\mathbf{M}] = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^T \tag{4.3}$$

where, $\mathbf{\Lambda}$ is a real and non-negative diagonal matrix comprising the eigenvalues of $\mathbf{M}$, and $\mathbf{\Psi}$ is an orthogonal matrix whose columns comprise the corresponding eigenvectors. The eigenvalues are arranged in descending order, i.e.,

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N \geq 0 \tag{4.4}$$

The eigenvalues satisfy the following relations [14]

$$\sum_{n=1}^{N} \lambda_n = \sum_{n=1}^{N} M_{nn} \tag{4.5}$$

$$\sum_{n=1}^{N} |\lambda_n|^2 = \sum_{m=1}^{N}\sum_{n=1}^{N} |M_{mn}|^2 \tag{4.6}$$

From equations (4.5) and (4.6), the mean value of λ_n and its standard deviation are found to be

$$\bar{\lambda} = \sum_{n=1}^{N} M_{nn} \Big/ N \tag{4.7}$$

$$\sigma = \sqrt{\frac{\|M\|_2}{N} - \frac{1}{N^2}\left(\sum_{n=1}^{N} M_{nn}\right)^2} \tag{4.8}$$

where $\|M\|_2$ is the second order norm of matrix $\mathrm{Re}[\mathbf{M}]$, i.e.,

$$\|M\|_2 = \sum_{m=1}^{N}\sum_{n=1}^{N} \left[\mathrm{Re}\{M_{mn}\}\right]^2 \tag{4.9}$$

Now let the set of forces $\mathbf{F}$ be weighted by $\mathbf{\Psi}$ so as to give a new set of "power modal" forces $\mathbf{Q}$ where

$$\mathbf{Q} = \mathbf{\Psi}^T \mathbf{F} \tag{4.10}$$

Obviously, the relation

$$\sum_{n=1}^{N} |Q_n|^2 = \sum_{n=1}^{N} |F_n|^2 \tag{4.11}$$

holds. Combining equations (4.2), (4.3) and (4.10), the input power to a structure can be rewritten as

$$P = \frac{1}{2}\mathbf{Q}^H \mathbf{\Lambda} \mathbf{Q} = \frac{1}{2}\sum_{n=1}^{N} |Q_n|^2 \lambda_n \tag{4.12}$$

Equation (4.12) indicates that the vibrational power transmitted to a structure by N forces can be regarded as the power input by N independent power modes, with the applied force $\mathbf{F}$ being transformed to a new set of power modal forces $\mathbf{Q}$, and the receiver described by a set of power modal mobilities λ_n, these being the eigenvalues of the real part of the mobility matrix.

It can be seen that the power modes of a structure are analogous to the "radiation modes" [16] sometimes used to describe the power radiated by a vibrating surface into a surrounding acoustic medium.

Conceptually equation (4.12) is very helpful. However it does not provide any practical advantages over equation (4.2) since one needs to know $\mathbf{F}$ and $\mathrm{Re}[\mathbf{M}]$ to find $\mathbf{Q}$ and $\mathbf{\Lambda}$. In [11], it had been suggested to approximate power using an upper and a lower bound, but no further results had been published.

4.2 Power transmission approximations

4.2.1 Low-frequency range power transmission approximation

The low-frequency range here is defined as that for which $kl \ll 1$ (k is the wavenumber and l the space between the excitation points), or the frequencies are below the critical frequency of the receiver structure. In this range,

$$M_{nn} \approx M_{mn} \approx M_{in} \tag{4.13}$$

From equation (4.3), it follows that

$$\lambda_1 \approx \sum_{n=1}^{N} \mathrm{Re}\{M_{nn}\}, \lambda_{2,3,\ldots,N} \approx 0 \tag{4.14}$$

$$\boldsymbol{\psi}_1 \approx \frac{1}{\sqrt{N}} [1 \quad 1 \quad \cdots \quad 1]_{1\times N}^T \tag{4.15}$$

Substituting equation (4.15) into (4.10), the first-order power modal force can be obtained as

$$|Q_1|^2 \approx \frac{1}{N} |F_1 + F_2 + \cdots + F_N|^2 \tag{4.16}$$

Equation (4.12) becomes

$$P \approx \frac{1}{2}|Q_1|^2 \lambda_1 \approx \frac{1}{2}\left(\left|\sum_{n=1}^{N} F_n\right|^2\right)\sum_{n=1}^{N}\mathrm{Re}\{M_{nn}\}\Big/N \tag{4.17}$$

Equation (4.17) shows that in the low-frequency range the vibrational power can be regarded as transmitted by the first-order power mode only, i.e., one power mode dominates. This is similar to monopole in the multipole approach.

4.2.2 High-frequency range power transmission approximation

At very high frequencies the point mobility tends to be larger than the transfer mobility so that the points appear to be 'uncoupled'. In this case

$$\lambda_n \approx M_{nn},\ Q_n \approx F_n \tag{4.18}$$

Equation (4.12) becomes

$$P \approx \sum_{n=1}^{N} \frac{1}{2}|Q_n|^2 \lambda_n \approx \sum_{n=1}^{N} \frac{1}{2}|F_n|^2 \mathrm{Re}\{M_{nn}\} \tag{4.19}$$

Equation (4.19) denotes that each power mode contributes to the total power transmission at high frequencies.

It also can be seen that equations (4.17) and (4.19) is the same result as those derived from the mobility matrix in Section 2.2.1

4.2.3 Mid-frequency range power transmission approximation

For the mid-frequency range (between the low- and high-frequency ranges), no simplifications such as (4.14) and (4.18) can be made since the transfer mobility is comparable to the point mobility. Equation (4.12) seems have no practical advantages over equation (4.2). However, since generally a small number of eigenvalues tend to be very much larger than all the others, a very good estimation can be obtained by a truncated form using only L terms ($L < N$), i.e.,

$$P = \sum_{n=1}^{N} \frac{1}{2}|Q_n|^2 \lambda_n \approx \sum_{n=1}^{L} \frac{1}{2}|Q_n|^2 \lambda_n \tag{4.20}$$

Conceptually equation (4.20) is very concise. But it is not simple to approximate correctly the few eigenvalues and the corresponding eigenvectors. Practically it would be of more value to approximate the power by its mean value as well as an upper and a lower bound which can be derived from the power-mode theory.

4.3 Mean value of power transmission

Equation (4.12) can be further simplified by the mean value averaged over all the power modes rather than the individual values. The mean square power modal force can be obtained from equation (4.11) as

$$|\bar{Q}|^2 = \frac{1}{N}\sum_{n=1}^{N}|F_n|^2 \tag{4.21}$$

The mean power modal mobility is given by equation (4.7). The mean value of the transmitted power, when averaged over all the power modes, can then be written in terms of the mean square force and the mean point mobility as

$$E(P) = \frac{N}{2}\left(\frac{1}{N}\sum_{n=1}^{N}|F_n|^2\right)\left(\frac{1}{N}\sum_{n=1}^{N}\mathrm{Re}\{M_{nn}\}\right) \tag{4.22}$$

Equation (4.22) is in agreement with the result of reference [10].

For the special case where $M_{11} = M_{22} = \ldots = M_{NN} = M_{in}$,

$$E(P) = \frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)\mathrm{Re}\{M_{in}\} \tag{4.23}$$

Equation (4.23) is equivalent to the power input to an infinite structure by multi-point forces at high frequencies. Thus the transmitted power converges to this mean value as frequency increases.

4.4 Approximate bounds for the transmitted power

Upper and lower bounds for the transmitted power can be derived from some properties of the power modes. In [11], the upper and lower bounds of the input power to a structure are given by

$$\frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)\lambda_N \le P \le \frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)\lambda_1 \tag{4.24}$$

Combining with equations (4.7) and (4.8), the power bounds can be approximated by

$$\frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)(\bar{\lambda}-\sigma) \le P \le \frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)(\bar{\lambda}+\sigma) \tag{4.25}$$

The advantage of equation (4.25) is that no knowledge of the force source distribution is required. However, for structures with low mode-count (e.g. low and mid-frequency vibration), $(\bar{\lambda}-\sigma)$ tends to be much smaller than $(\bar{\lambda}+\sigma)$, especially at the resonant frequencies, due to the strong correlation between the excitation points. As a result, the power band will be too broad to be of practical value. However, in this case, the lower bound of equation (4.25) may be replaced by the first power mode approximation as

$$\frac{1}{2N}\left|\sum_{n=1}^{N}F_n\right|^2(\bar{\lambda}+\sigma)\le P\le\frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)(\bar{\lambda}+\sigma) \qquad (4.26)$$

Accordingly for very flexible and/or heavily damped structures which do not show resonant behaviour (e.g. high frequency vibration), the power then can be approximated by

$$\frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)\min_n\left[\mathrm{Re}\{M_{nn}\}\right]\le P\le\frac{1}{2}\left(\sum_{n=1}^{N}|F_n|^2\right)\max_n\left[\mathrm{Re}\{M_{nn}\}\right] \qquad (4.27)$$

It also has been mentioned in Section 2 that the transmitted power can be approximated as lying in the band [10] given by

$$P=\frac{1}{2}\left(\sum_{n=1}^{N}|F_n|\right)^2\left(\frac{1}{N}\sum_{n=1}^{N}\mathrm{Re}\{M_{nn}\}\right)\pm\frac{1}{2}\left(\frac{1}{N}\sum_{n=1}^{N}|F_n|\right)^2\sqrt{\frac{N(N-1)}{2}}\left|\bar{M}_{tr}\right| \qquad (4.28)$$

where, $|\bar{M}_{tr}|$ is the mean transfer mobility of the receiver.

Equations (4.25) and (4.28) are quite similar: both of them approximate the power by the point- and transfer mobility of the receiver, as well as the magnitudes of the excitations. Their accuracy and efficiency will be investigated by numerical examples in Section 4.6.

4.5 Moment excitations

In the study of power transmission from a machine source to a flexible supporting structures, usually only the translational motion normal to the surface of the seating has been considered [8-11], this often being the dominant mechanism. However, it is known that in many case of practical interest, vibration sources apply moments as well as forces. The power transmitted by moment excitation is generally greater at high frequencies [15]. Therefore it is essential to consider also moment excitation.

The power mode method can also be used for the cases in which both force and moment excitations are involved, provided the mobility matrix of the receiver structure is scaled properly.

If the source excitation $\mathbf{F}$ is formed partly by a set of forces and partly by a set of moments, and $\mathbf{M}$ the corresponding the mobility matrix of the receiver composed of force and moment point mobility and transfer mobilities, the transmitted power can be written in the same form as equation (4.2).

Let $\mathbf{M}$ be written as

$$\mathrm{Re}[\mathbf{M}] = \mathbf{\Lambda}_c \mathbf{M}_c \mathbf{\Lambda}_c \tag{4.29}$$

where, $\mathbf{\Lambda}_c$ is a diagonal matrix with

$$\Lambda_{c,nn} = \sqrt{\mathrm{Re}\{M_{nn}\}} \tag{4.30}$$

and $\Lambda_{c,nn}$ and $\mathrm{Re}\{M_{nn}\}$ are the nth diagonal elements of $[\mathbf{\Lambda}_c]$ and $\mathrm{Re}[\mathbf{M}]$, respectively. Substituting equation (4.29) into (4.2), one may obtain

$$P = \frac{1}{2}\mathbf{F}_c^{\,H}\mathbf{M}_c\mathbf{F}_c \tag{4.31}$$

where

$$\mathbf{F}_c = \mathbf{\Lambda}_c\mathbf{F} \tag{4.32}$$

$$\mathbf{M}_c = \mathbf{\Lambda}_c^{-1}\,\mathrm{Re}[\mathbf{M}]\,\mathbf{\Lambda}_c^{-1} \tag{4.33}$$

Then all the results derived above will hold, provided one replaces $\mathbf{F}$ by $\mathbf{F}_c$ and $\mathrm{Re}[\mathbf{M}]$ by $\mathbf{M}_c$. This procedure in effect scales the individual generalised forces by a factor equal to the square root of the real part the input mobility.

4.6 Numerical examples

The response of a plate in bending is not only relatively simple to model analytically but is also physically representative of many real receiving structures. For an initial investigation, numerical examples of thin perspex plates (finite and infinite) excited by discrete force sources at arbitrary points are therefore developed. For simplicity, the finite plate is chosen to be rectangular and simply supported so that its modal properties is easily to know. Three points on the plate are chosen as the excitation positions. The properties of the plates are listed in Table 4.1. The results show running frequency averages taken over a frequency band of width equal to the mean modal spacing, so as to illustrate the broad features of the power transmission. The exact results are found using the FRF method compared to the various results and approximations using power modes.

4.6.1 Power modal mobilities of the receiver structure

In the power mode theory, the receivers can be described as a set of power modal mobilities. To show the relations between these values, the three power modal mobilities of the rectangular plate (against frequency) are plotted in Figure 4.1. It can be seen that at low frequencies, the first-order power mobility tends to be much larger than the others, and as frequency increases, these values get closer, which implies the effects of the higher order power modes increase. When the rectangular plate is extended to be infinity, the three power mobilities, as shown in Figure 4.2, are much closer than in Figure 4.1. Thus it can be seen that for resonant structures a small number of power modes dominate, and for non-resonant structures, more power modes are significant.

4.6.2 Power modes and total power transmission

By the description of power mobilities, the power transmitted to a structure by N forces can then be regarded as the power transmitted by N independent power modes, where the set of forces is defined by a new set of power modal forces. To investigate the relations of the total power transmission to the individual power modes, the power transmitted to the plate by the point-forces as well as the corresponding power modes are calculated and plotted in Figure 4.3 and Figure 4.4 for finite and infinite plate receivers, respectively. It can be seen, as we might expect from Figure 4.1 and Figure 4.2, that for the rectangular plate, the first power mode is generally larger than the other two, especially for the low mode-count area, and for the infinite plate, the effects of higher order power modes become significant. From equation (4.12), it can be seen that each individual power mode depends on not only the corresponding power mobility but also the magnitude of the power modal force. By the expression of (4.10), it can be seen that these power modal forces are determined by the excitation forces, their excitation positions and the relative phases. It is quite possible that a small power modal mobility corresponds with a large power modal force, or a large power mobility corresponds with a small power modal force. Thus at some frequencies, the lower order power modes can transmit less power than the higher ones, as in Figure 4.3 and Figure 4.4

4.6.3 Power approximation by its mean value and power bounds

It was shown in the previous sections that the transmitted power can be simply predicted by its mean value and upper and lower bounds. The related approximations are given in equations (4.22), (4.25) or (4.26). Figures 4.5 and 4.6 show the comparisons of the mean and exact power transmitted to the finite and infinite plates, respectively. It can be seen that the mean power expression in equation (4.22) can be used to approximate the power transmission quite well. The upper- and the lower-bounds expressed in equations (4.25) and (4.26) give the limits of this transmitted power, as shown in Figure 4.7 and Figure 4.8. It can be seen that for a structure with low mode-count, the lower bound expressed in (4.26), which is the approximation of the first power mode, is generally closer to the exact value than equation (4.25), whereas for high mode-count or non-resonant structures, equation (4.25) is more appropriate.

Figure 4.9 shows the predicted power by the mean emission method [10] in a form of power bounds given by equation (4.28) for the rectangular plate example. It can be seen the bigger errors occur in the low mode count frequency range. This is because the mean emission method is derived on the assumption that the value of the relative phase angles, which is a combination of the phase differences of the force excitations at the individual points and the phases of the complex transfer mobility between these points, are random and equally likely. However, this "random phase" assumption does not hold for the low mode-count area. Therefore the mean emission method is not suitable for the low mode-count structures. For high mode-count or non-resonant structures, the mean emission method is a good way to predict the transmitted power.

4.6.4 Simultaneous force/moment excitations

The power mode method can also be applied to the cases in which both force and moment excitations are involved, provided the mobility matrices of the receiver structures are scaled appropriately. The influences of the moment excitations are investigated here by assuming the discrete point sources are simultaneously co-located force/moment excitations. Figure 4.10 is the comparison of the power transmitted to the rectangular plate by its mean value approximation and the exact result, and Figure 4.11 shows the approximations of upper and lower bounds. It can be seen that the all the conclusions obtained by the power mode theory hold.

Similar results can be obtained for velocity/rotation source excitation cases by analogy with the force/moment sources, with the mobility matrix of the receiver being replaced by the corresponding impedance matrix.

Table 4.1 Parameters of the numerical examples

Structures	Rectangular plate (simply supported), and its extended infinity
Dimension sizes	Thickness=0.01m; Length=2m; Width=0.9m (for the rectangular plate only)
Material properties	Young's modulus=4.4e9 N/m^2; Density=1152kg/m^3; Loss factor=0.05; Poisson's ratio=0.38
Excitation positions	$(x_1,y_1)=(0.37,0.45)$; $(x_2,y_2)=(0.89,0.45)$; $(x_3,y_3)=(1.37,0.45)$
Excitation forces	$F_1=1$, $M_{x1}=0.2$, $M_{y1}=0$; $F_2=2e^{j\pi/3}$, $M_{x2}=0.3e^{j2\pi/3}$, $M_{y2}=0$; $F_3=0.5e^{-j\pi/4}$, $M_{x3}=0.2j$, $M_{y3}=0$

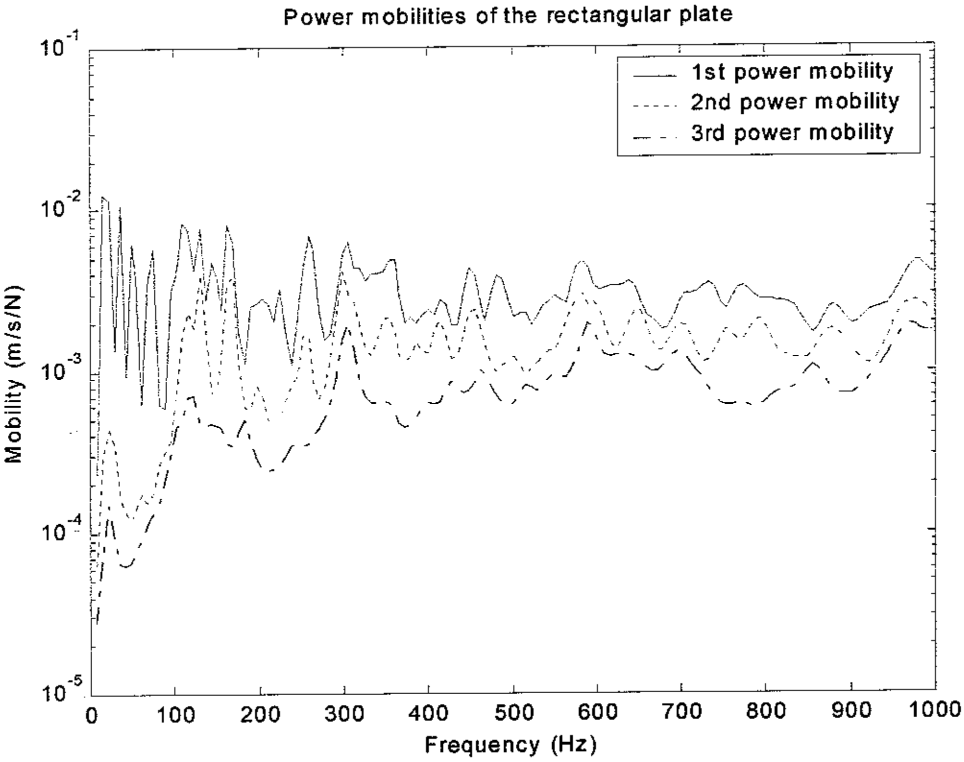


Figure 4.1

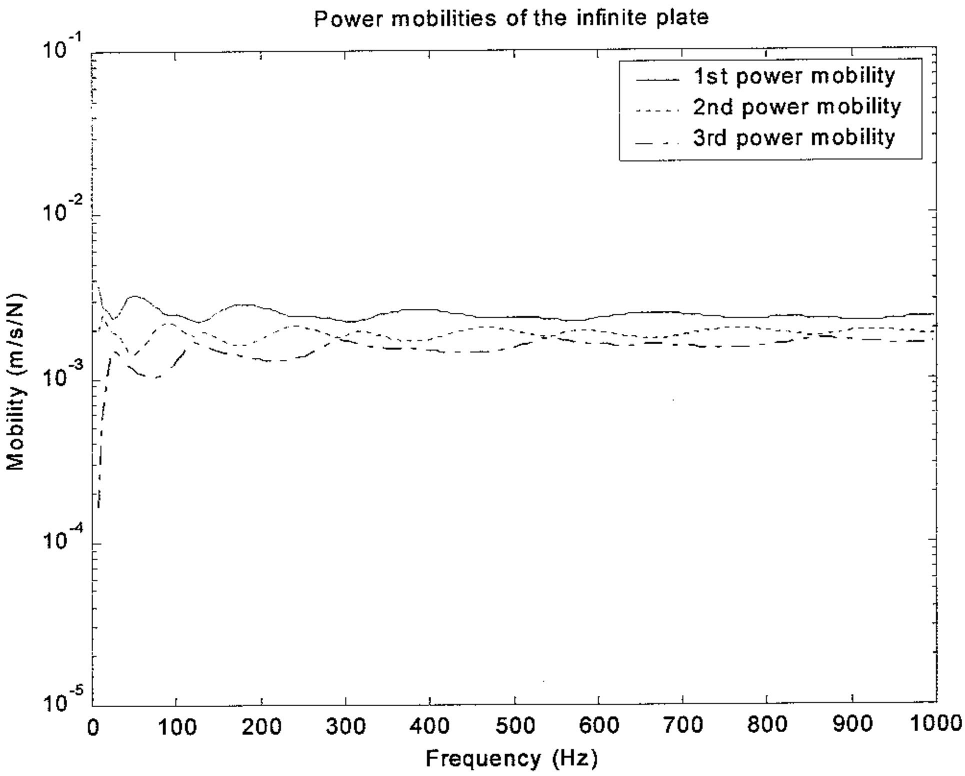


Figure 4.2

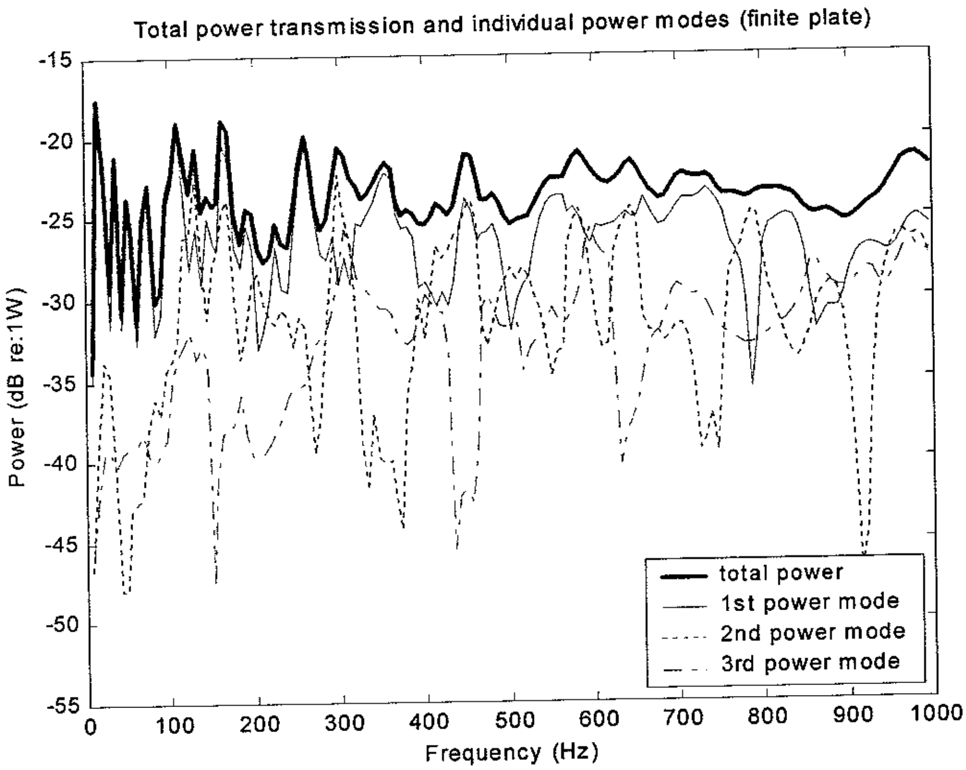


Figure 4.3

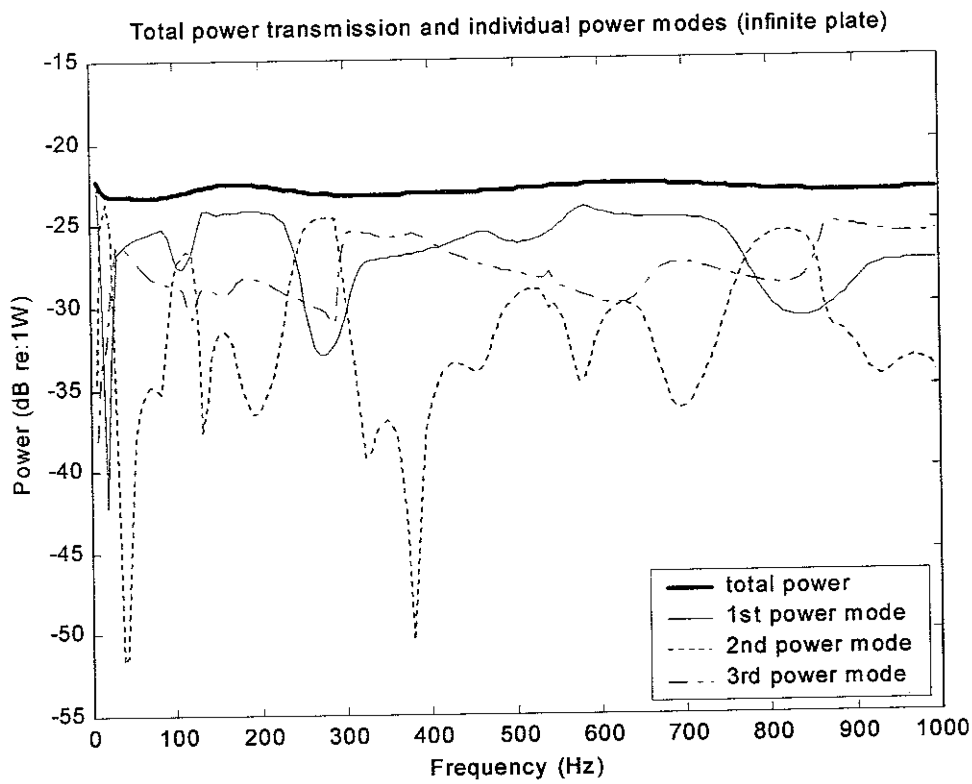


Figure 4.4

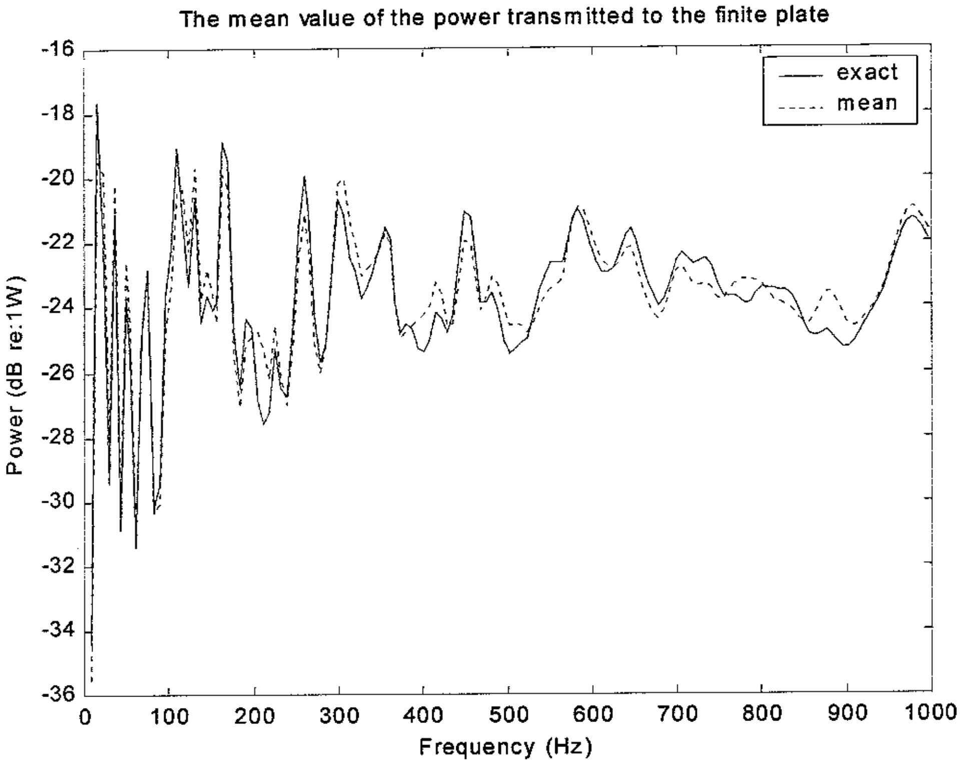


Figure 4.5

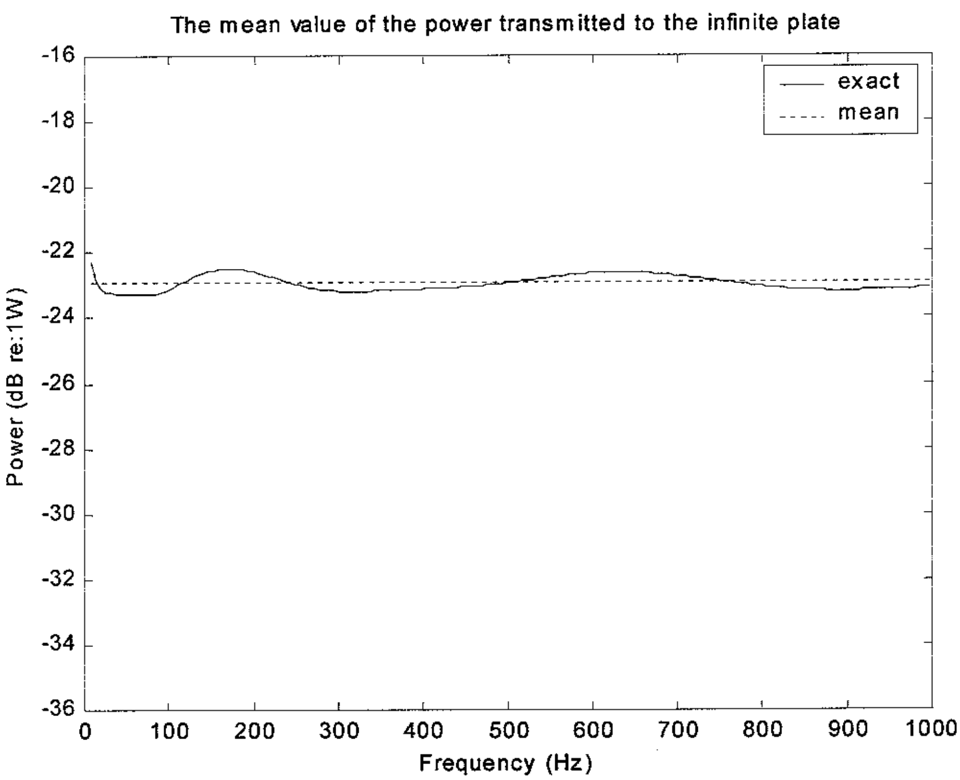


Figure 4.6

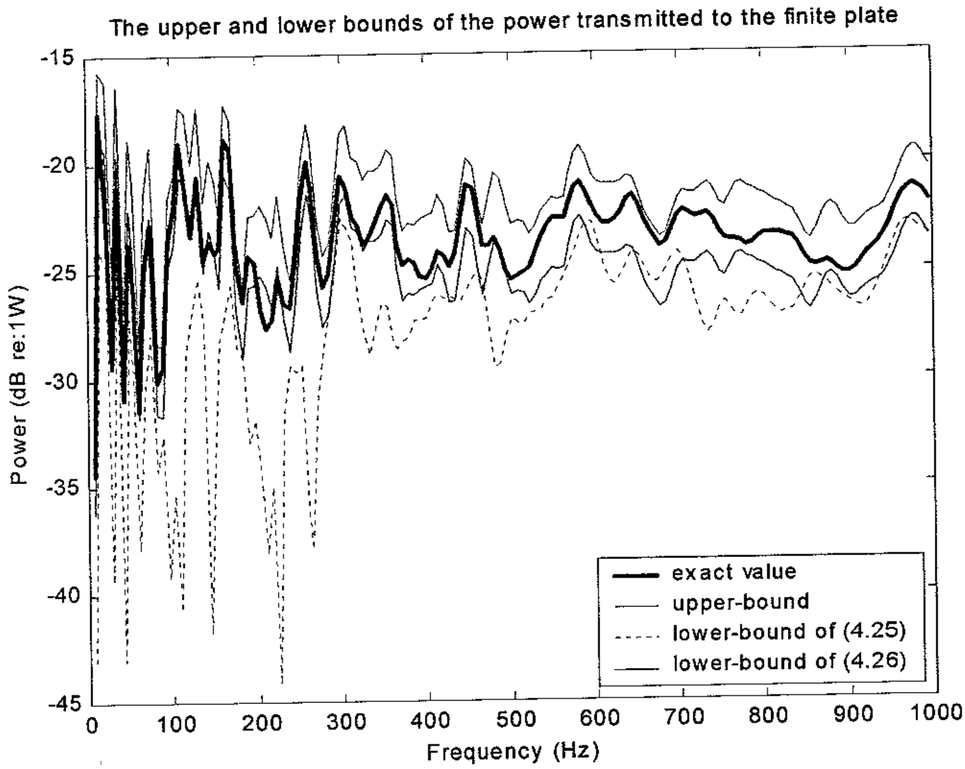


Figure 4.7

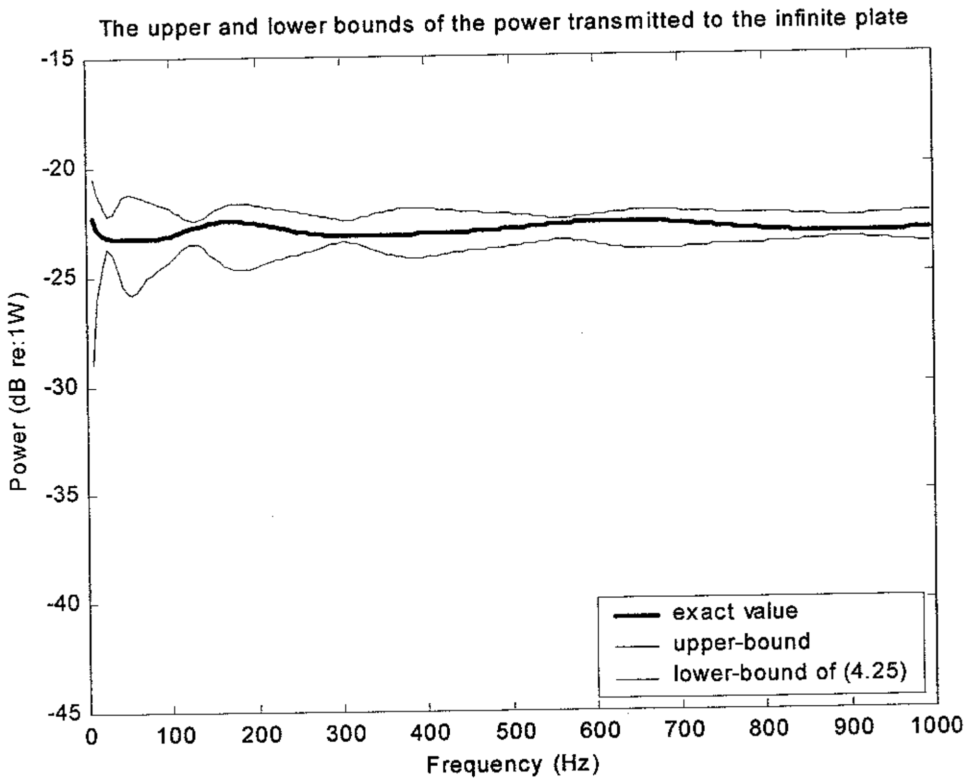


Figure 4.8

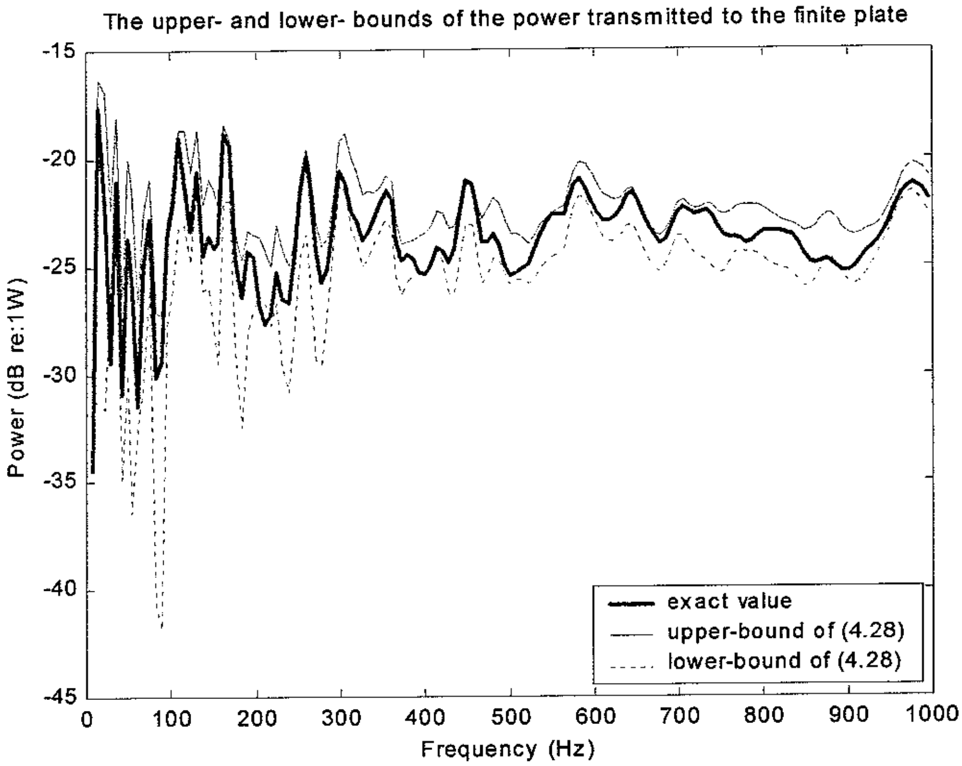


Figure 4.9

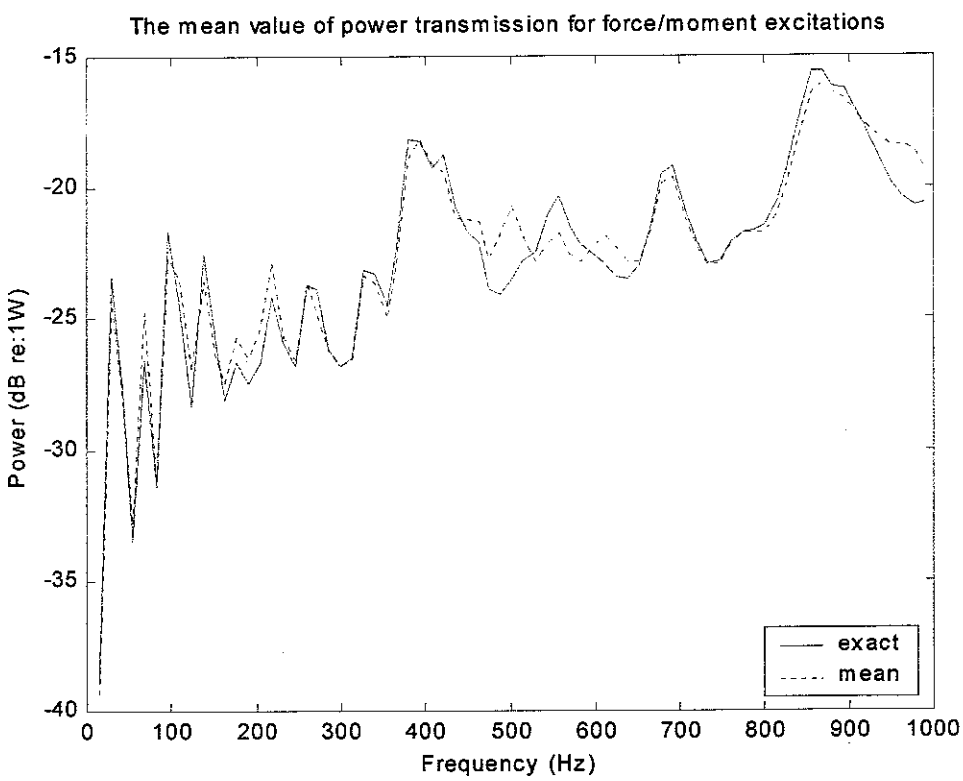
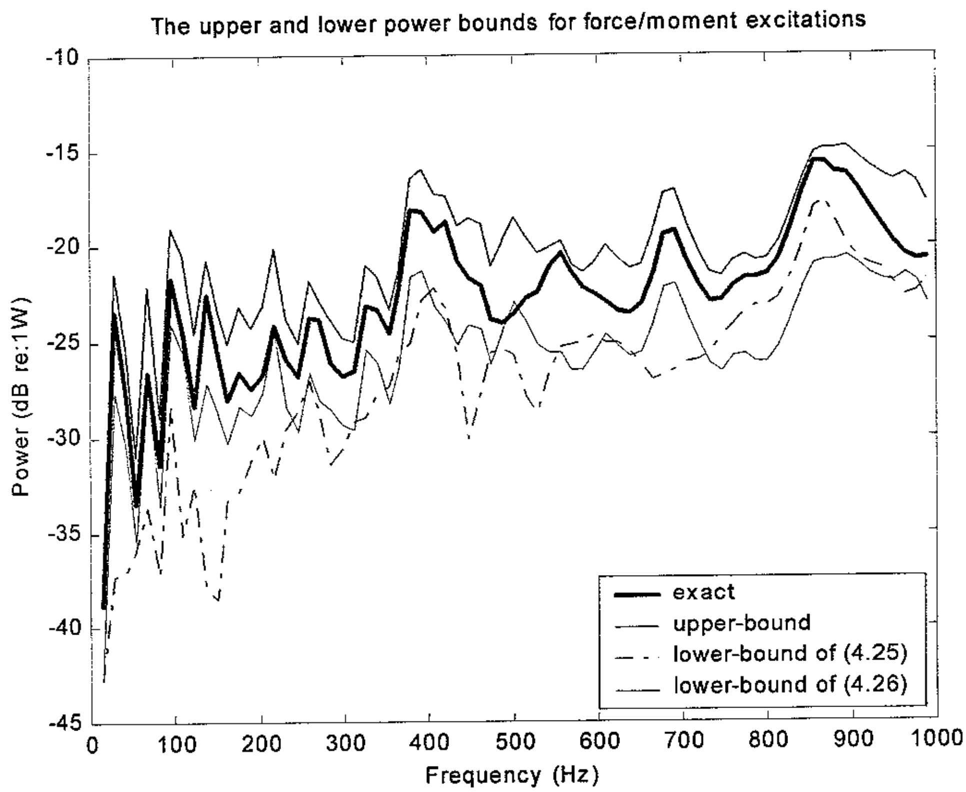


Figure 4.10



The upper and lower power bounds for force/moment excitations
Power (dB re:1W)
-10
-15
-20
-25
-30
-35
-40
-45
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)
exact
upper-bound
lower-bound of (4.25)
lower-bound of (4.26)

Figure 4.11

5 Conclusions

This work described in this memorandum is an attempt to develop simple and accurate approximations for the power transmitted to flexible receivers by force sources (either discrete or line-distributed). Three general approaches, FRF method, FE method and Green function method, are reviewed and some possibilities of power prediction simplifications using existing techniques are discussed. These include the multipole method, the mean emission method, the modal analysis method and the Fourier Transform method. Then a new technique – the "power mode" method – is described. By this theory the vibration power transmitted to a flexible receiver by N discrete point forces can be regarded as the power input by N independent power modes. For low mode-count structures, the first order power mode dominates the total power transmission, whereas for high mode-count or non-resonant structures, more power modes become significant. To predict the transmitted power using power modes directly usually has no practical advantages over the mobility matrix method. However, by the power mode theory one may approximate the power by its mean value as well as upper and lower bounds in a simple manner. It also has been shown that this approach can be used for the cases where both force and moment excitations are involved, provided the mobility matrix of the receiver structure is scaled appropriately.

Although in this memorandum only the force sources have been considered, similar results can be obtained for velocity source excitation cases by analogy, if the mobility matrix of the receiver is replaced by the corresponding impedance matrix.

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