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Computationally Efficient Inversion of Mixed Phase Plants with IIR Filters

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UNIVERSITY OF SOUTHAMPTON

INSTITUTE OF SOUND AND VIBRATION RESEARCH

FLUID DYNAMICS AND ACOUSTICS GROUP

Computationally Efficient Inversion of Mixed Phase Plants with IIR Filters

by

T Papadopoulos and P A Nelson

ISVR Technical Report No. 297

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Authorized for issue by
Professor C L Morfey, Group Chairman

Contents

Abstract

Inverse filtering in a single or in multiple channels arises as a problem in a number of applications including telecommunications, active control of sound and vibration, sound reproduction and virtual acoustic imaging. In such cases one is trying to deconvolve the effect of the transmission path from the received signal, control a physical plant, accurately reproduce a given set of audio signals or implement a cross-talk cancellation network.

In the single-channel case, when the plant $C(z^{-1})$ sought to be inverted has zeros outside the unit circle in the z-plane, an approximation to the inverse $1/C(z^{-1})$ can be realised with an FIR filter if an appropriate amount of modelling delay is introduced to the system. But the closer the zeros of $C(z^{-1})$ are to the unit circle in the z-plane the longer -in terms of number of coefficients- the FIR inverse has to be. This type of FIR inverse filters can reach several tens of times the number of coefficients in the plant.

In the work presented here a simplified single-channel version of a virtual acoustic imaging system is considered and an off-line implementation of the required inverse filtering is presented that utilises a variant of the backward-in-time filtering technique usually associated with zero-phase FIR filtering. On this basis a single-channel mixed phase plant can be inverted with an IIR filter of order roughly equal to that of $C(z^{-1})$, thus decimating the processing time required for the inverse filtering computation.

1. *Introduction*

The need to design a digital filter (or matrix of digital filters) that approximates as closely as possible the inverse frequency response of a given single-channel system (or the inverse of the frequency-response-matrix of a given multi-channel system) appears in a number of applications including transmission channel equalisation [see 1], active control [see 2 and 3], sound reproduction [see 4, 5 and 6] and virtual acoustic imaging [see 7]. In what follows, the specific problem of inverse filter design for a Virtual Acoustic Imaging System is addressed, an IIR digital filtering structure being proposed and simulated in MATLAB. A simplified single-channel version of the problem is considered but a description of the inherently multi-channel structure of such a system is first given in order to illustrate the context of the problem at hand. The IIR filtering structure is then described and finally the results of the simulations are presented and discussed.

1.1. *General model of a Virtual Acoustic Imaging System*

A Virtual Acoustic Imaging System is defined in [8] as a sound reproduction system that gives "... a listener the impression that there is a sound source [henceforward termed the *virtual source*] at a given position in space where no real source exists". As is also described in [8], "one way to achieve this is to ensure that the sound pressures that are reproduced at the listener's ears are the same as the sound pressures that would have been produced there by a real source at the same position as the virtual source". This situation is depicted in Figure 1–1 in which a practical way to achieve what was just described with a standard stereophonic sound reproduction system comprising two loudspeakers is proposed. Specifically one can place an actual sound source at the desired position of the virtual source, record the sound field generated at the ears of the listener when a given input signal is fed to this source and then try to reproduce exactly the same sound field at the listener's ears using the sources comprising the Virtual Acoustic Imaging System. Evidently, for this to be possible, the given input signal would have to be appropriately pre-filtered before been fed to the real sources.

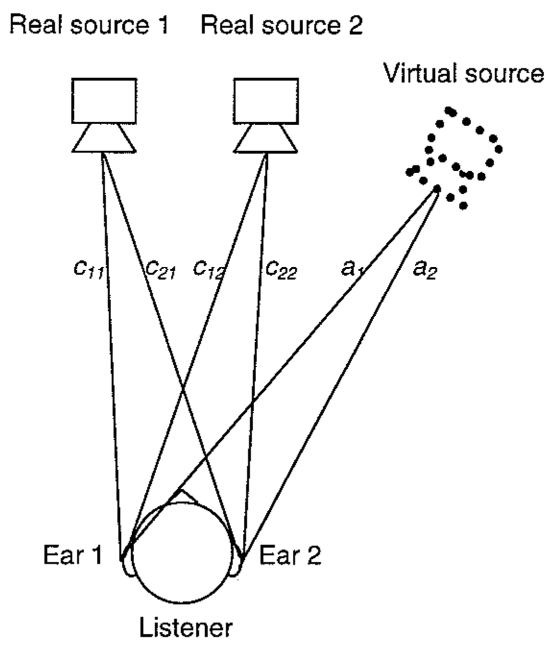

Figure 1–1 The objective of a Virtual Acoustic Imaging System

In Figure 1–1, each one of the symbols c_{ij} denotes the *Head Related Transfer Function* (HRTF) relating the sound pressure at ear number i to the j-th source's (electrical) input[1]. Similarly, each of the symbols a_i denotes the HRTF relating the sound pressure that would have been produced at ear number i should source was reproducing sound from the location of the *virtual source* to the input signal to that source. With this arrangement, if one does not have the possibility of physically recording the sound pressures at the listener's ears, they can simulate them by filtering the input signal through the pair of HRTFs denoted by a_i thus obtaining an approximation of the sound pressures that would have been produced by the virtual source at the listener's ears if this source was actually reproducing the input signal.

The schematic of Figure 1–1 translates to the block diagram shown in Figure 1–2 where the input signal $x(n)$ is, as was mentioned above, pre-filtered through a matrix of inverse filters **H** before it is fed to the loudspeakers comprising the system. In the block diagram, **A** denotes the *target matrix* i.e. a matrix containing an adequate amount of modelling delay[2] and, in the case when we are simulating the desired sound pressures as explained above, the HRTFs labelled a_i in Figure 1–1. With this arrangement the obvious goal is to design an optimal matrix of digital filters **H** so that

[1] In the relevant literature the term *transfer function* is strictly associated with the frequency domain description of a system but in what follows we use the acronym HRTF interchangeably to refer either to a system's z-transform representation or to the corresponding time sequence.
[2] The feature of modelling delay is discussed below.

the vector of reproduced signals labelled in Figure 1–2 as $\hat{\mathbf{d}}$ is as close as possible to the desired signals labelled as **d**.

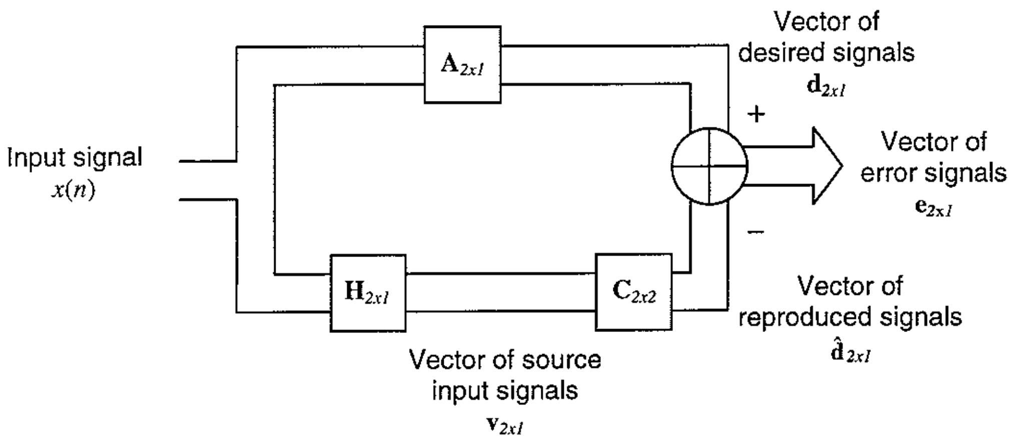

Figure 1–2 Block diagram of a Virtual Acoustic Imaging System

Using the z-transforms of the signals and systems involved in Figure 1–2 we derive the following equations for the error signal **e**:

$$\begin{aligned}\mathbf{e}(z^{-1}) &= \mathbf{d}(z^{-1}) - \hat{\mathbf{d}}(z^{-1}) \\ &= \mathbf{A}(z^{-1}) \cdot X(z^{-1}) - \mathbf{C}(z^{-1}) \cdot \mathbf{H}(z^{-1}) X(z^{-1})\end{aligned} \tag{1.1}$$

It is now obvious how the design of the system under discussion reduces to an inverse filter design problem as from equation (1.1) it is evident that the optimal value for **H** will be given by:

$$\mathbf{H}_{opt}(z^{-1}) = \mathbf{C}^{-1}(z^{-1}) \mathbf{A}(z^{-1}) \tag{1.2}$$

1.2. Single-channel version of the problem

The single-channel equivalent of Figure 1–2 is shown in Figure 1–3 where all the signals and systems are taken in their z-domain representation[3]. As can be seen in that figure, in one channel our problem essentially coincides with the typical equalisation problem in which we aim to design a filter h that inverts the plant[4] C so that the actually reproduced signal $\hat{d}(n)$ is as close as possible to a delayed version of the input signal $x(n)$. In other words, one seeks to design h so that the following z-domain equations holds

[3] For notation-simplicity reasons we did not include a single-channel equivalent of the target matrix **A** in the version of the problem presented here but it is easy to see that this does not really restrict the generality of the analysis that follows.

[4] In what follows, using control theory terminology, we call the transfer function c (or the matrix of transfer functions **C**) "*the plant*".

$$H(z^{-1}) \cdot C(z^{-1}) = z^{-\Delta} \tag{1.3}$$

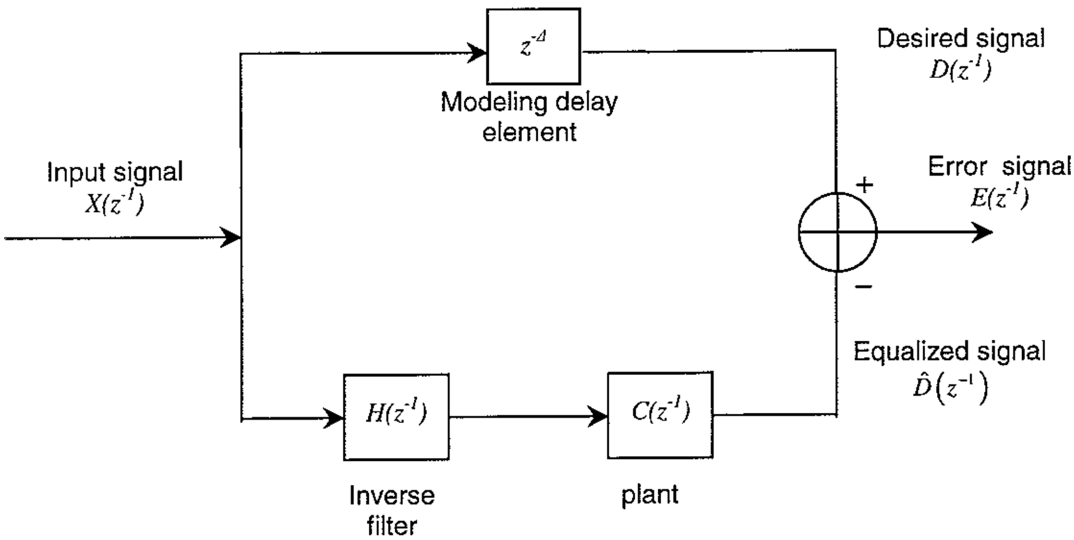

Figure 1–3 Single-channel block diagram of a Virtual Acoustic Imaging System

Now assuming that the plant is a causal all-zero system containing no pure delay as is described in (1.4), and further assuming that it is minimum phase, i.e. has all its zeros inside the unit-circle in the *z*-plane, no modelling delay would be necessary and the optimal inverse filter would be given by equation (1.5)

$$C(z^{-1}) = c_0 + c_1 z^{-1} + \ldots + c_n z^{-n} \tag{1.4}$$

$$H(z^{-1}) = \frac{1}{C(z^{-1})} \tag{1.5}$$

However, even though the assumption that the plant is modelled as a causal all-zero system is quite realistic, hardly ever does one come across an experimentally measured plant that has all its zeros inside the unit circle in the *z*-plane. Hence, for every z_i for which $\left|C\left(z_i^{-1}\right)\right| > 1$ we will have a pole outside the unit circle in the inverse filter $H(z^{-1})$ of (1.5). As is explained in [9 pp. 356-359], each pole inside the unit circle appears in the impulse response corresponding to the inverse *z*-transform of $H(z^{-1})$ as a geometric series that decays forward in time while each pole outside the unit circle as a geometric series which either increases exponentially in forward time or decays in backwards time. So evidently, in the usual case where the plant $C(z^{-1})$ to be inverted is mixed-phase, the filter of (1.5) is either unstable or non-causal and, thus, in both cases non-realisable.

The typical technique to address such a problem when one is to invert a mixed phase plant [see e.g. 10 pp. 236-244] is to introduce an adequate number of modelling delay samples Δ so that $H(z^{-1})$ is a delayed and truncated version of the -both in backwards and in forward time- infinitely long ideal inverse. In order to show analytically that such a realisable approximation to the ideal but non-realisable inverse is possible, we take a factorisation of $C(z^{-1})$ into a factor $C_{min}(z^{-1})$ with all its roots strictly inside the unit circle and a $C_{max}(z^{-1})$ with all its roots outside or on the unit circle. In this view equation (1.3) translates to (1.7) where we have used the fact that there will always be two polynomials $R_{min}(z^{-1})$ and $R_{max}(z^{-1})$ in negative powers of z such that equation (1.6) holds[5]. Thus

$$\frac{1}{C_{min}(z^{-1})\cdot C_{max}(z^{-1})}=\frac{R_{min}(z^{-1})}{C_{min}(z^{-1})}+\frac{R_{max}(z^{-1})}{C_{max}(z^{-1})} \tag{1.6}$$

and the expression for the inverse filter becomes

$$\begin{aligned} H(z^{-1}) &= \frac{z^{-\Delta}}{C_{min}(z^{-1})\cdot C_{max}(z^{-1})} \\ &= \frac{R_{min}(z^{-1})z^{-\Delta}}{C_{min}(z^{-1})}+\frac{R_{max}(z^{-1})z^{-\Delta}}{C_{max}(z^{-1})} \end{aligned} \tag{1.7}$$

Now the right-sided time sequence[6] corresponding to the first term in (1.7) will be stable since all the roots of $C_{min}(z^{-1})$ are inside the unit circle but for the second term one will have to choose the left-sided time sequence among the two possible time representations since it is the only one which is stable. But, even though the elements of such a sequence appear in negative time, the delay introduced by the term $z^{-\Delta}$ shifts these elements by Δ positions forward in time. This way one can truncate this double-sided infinitely long inverse time sequence keeping only a finite number of samples appearing in positive time and this is the FIR inverse filter that is used when modelling delay is used for the inversion of a mixed phase plant.

[5] The polynomials $R_{min}(z^{-1})$ and $R_{max}(z^{-1})$ are guaranteed to exist as a partial-fraction expansion of the left-hand side of (1.6) [see 9 pp. 188-197] as a proper collection of terms can readily show.
[6] The terms *right-sided* and *left-sided sequence* are used as in [11 pp. 45-52]

2. *The IIR inverse filtering structure*

2.1. Problems faced with conventional IIR filter design

The technique presented above is guaranteed to give an FIR filter that approximates the inverse quite satisfyingly but usually leads to very long filters. Specifically, as is explained in [12], the closer one or more of the zeros of $C(z^{-1})$ are to the unit circle, the slower the inverse time sequence will decay, the zeros close but inside the unit circle slowing the decay of the forward-in-time part and those close but outside the unit circle that of the backwards-in-time part. As will be made apparent below, it is quite common for measured HRTFs to have several tens or even hundreds of zeros clustered virtually on the unit circle in the z-plane thus demanding an FIR filter of several thousands of coefficients and a quite large amount of modelling delay in order to approximate closely the ideal non-causal inverse time-sequence. An undesirable implication of this fact is that, since the output of filters of this length requires a quite substantial amount of processing time to be computed, the inversion of a typical measured HRTF network of the type shown in Figure 1–1 can be implemented in real-time only by use of dedicated signal processing hardware and by no means in an ordinary personal computer.

A possible way out of this restriction would be to try and implement the inverse filtering by use of IIR filters hoping that equivalent performance could be achieved with a much lower number of coefficients and thus less computation demand. However, the problem with this approach is that a number of established IIR filter design techniques that would seem well suited for this purpose fail mainly due to numerical problems.

Namely, the whole family of algorithms associated with the design of fixed parameter IIR filters based on the minimisation of some sort of mean-square-error cost function in the time domain (e.g. *Prony's* or *Shank's* methods [see 9 pp. 706-710]) require the computation and usually the inversion of auto-correlation and cross-correlation matrices of the same order as the number of samples of the inverse time sequence to be modelled. Since the inverse time sequences we are dealing with here can be several thousands of samples long, the use of this family of design methods is not feasible for obvious numerical reasons.

Similar considerations apply to the algorithms that iterate towards the optimal IIR inverse of a given order by minimising a cost function defined on the basis of either the output or the equation error signal [see 13]. As argued in [14], under certain conditions -related to the initial conditions of the output error technique and the absence of measurement noise for the equation error technique- these algorithms do indeed succeed in converging to the optimal recursive inverse filter but only when the plant's order is kept to no more than a few tens of coefficients.

2.2. *The proposed IIR inverse filtering algorithm*

It was shown in section 1.2 that the reciprocal of the *z*-transform of the plant $1/C(z^{-1})$ can be written as the sum of two rational terms one of which has all its poles inside and the other either on or outside the unit circle [see equation (1.6)]. We also described how a realisable approximation to the ideal can be implemented by means of an FIR filter with the introduction of modelling delay. Here, in order to explain how the proposed IIR inverse filtering technique works, we start again from equation (1.6) and shifting the notation from polynomials in the *z*-variable to polynomials in the *delay operator* q^{-1} [see 21 pp. 48-51] we write the stable but non-causal inverse impulse response as

$$\frac{1}{C(q^{-1})} = \frac{R_{min}(q^{-1})}{C_{min}(q^{-1})} + \frac{R_{max}(q)}{C_{max}(q)} \tag{2.1}$$

Again, in (2.1) the first term of the right-hand side is a rational expression in the delay operator q^{-1} thus corresponding to an infinite time series advancing in forward time while the second term is a rational expression in the forward shift operator q thus corresponding to an infinite time series advancing backwards in time. But, unlike what was done before, this time in order to realise the filtering through the anti-causal part of the inverse impulse response we utilise the fact that given a finite-length impulse response $H(q^{-1})$ and an input signal $x(n)$, the same output $y(n)$ will be obtained if one filters $x(n)$ through $H(q^{-1})$ or if, alternatively, filters a time-reversed version of $x(n)$ through a time-reversed version of $H(q^{-1})$ and then reverses the outcome in time [see 15 pp. 431-433]. For the purposes of the problem discussed here the above translates to the fact that the (non-realisable) output that one would get by filtering $x(n)$ through the second term of the right-hand side of (2.1) can be obtained by simply reversing $x(n)$, filtering it through the mirror-image of this anti-causal infinite impulse response obtained by replacing q with q^{-1}, and finally reversing the output. Using the "^" symbol to denote reversion in time, what was said above is formally written in the next formula the filter appearing in the right-part of which being trivially realisable. Thus

$$y(n)=\frac{R_{max}(q)}{C_{max}(q)}x(n)\Leftrightarrow \hat{y}(n)=\frac{R_{max}(q^{-1})}{C_{max}(q^{-1})}\hat{x}(n) \qquad (2.2)$$

So to summarise, given a signal $s(n)$ that it the output of a plant $C(q^{-1})$ been fed with an input signal $x(n)$, in order to recover $x(n)$ one has to:

i) Compute the decomposition of equation (2.1),

ii) Filter $s(n)$ through the minimum phase term,

iii) Filter a time-reversed version of $s(n)$ through the mirror image of the maximum phase term as shown in the right-part of (2.2) and reverse the output in time and finally,

iv) Add the two outputs to obtain $x(n)$

2.3. Practical issues regarding the implementation of the proposed algorithm

Before we present the simulation results obtained in MATLAB using the inverse filtering technique described above, there are a number of points that have to be made in relation to the algorithm listed in the last part of section 2.2. First one has to say that even though the algorithm presented above is rather straightforward to set down analytically it is more complicated to realise numerically, the critical point in its implementation being the decomposition described in equation (2.1) or equivalently equation (1.6). In the simulation results presented in section 3 the decomposition was computed by use of the *residuez* MATLAB command [see 16] which introduced numerical error to the whole chain of computations[7]. This fact reduces the effectiveness of the algorithm to a considerable degree since (as will be demonstrated in the next section) the higher the order of the plant to be inverted the less accurate the inversion.

[7] It is easy to see that when a rational expression with real coefficients is expanded to partial fractions, the coefficients of these fractions have to be either real or to appear in conjugate pairs. Still, in the expansion computed by means of the *residuez* command some of the roots in the fractional expansion were only approximately conjugate and thus the (very small) imaginary parts that appeared in the final coefficients of the two terms in the right-hand side of (1.6) had to be set to zero.

On the other hand, more sophisticated numerical techniques can be used to compute the decomposition of equation (1.6) as it is easy to see that this equation can be reduced to the Diophantine equation (2.3) in the unknown polynomials $R_{min}(z^{-1})$ and $R_{max}(z^{-1})$ which, given that the greatest common divisor of $C_{min}(z^{-1})$ and $C_{max}(z^{-1})$ is 1, will always have a solution [see 18 pp. 41-59].

$$R_{min}(z^{-1}) \cdot C_{max}(z^{-1}) + R_{max}(z^{-1}) \cdot C_{min}(z^{-1}) = 1 \qquad (2.3)$$

Dedicated algorithms exist for the numerical solution of this kind of equation formulated in the time-domain [see 19 and 22] and hopefully better results can be obtained when these numerical techniques are incorporated into the proposed algorithm. Alternatively, homomorphic signal processing techniques can be used for the decomposition of the plant's reciprocal z-transform to minimum and maximum phase components [see 20 and 11 pp. 500-527] but this direction has not yet been studied in the course of the present research project.

A second shortcoming of the method as presented up-to-now is that it is strictly off-line as for the implementation of the 3rd step of the algorithm listed in section 2.2 one has to have knowledge of the whole input signal before the inverse filtering procedure can start. Furthermore this step increases the amount of memory required for the filtering procedure since the whole input signal has be stored in memory before the recursive filtering is computed unlike the typical FIR case where only a number of input samples equal to the filter's length have to be kept in memory for the computation of each new output sample.

A way around these restrictions, however, would be to segment the input signal and process one segment at a time. Effectively, this is equivalent to introducing an amount of modelling delay equal to the segment's length and truncating the maximum phase component's infinitely long impulse response to the same length[8]. This way, superior performance to that of an FIR inverse can be achieved with far fewer coefficients and with no restrictions other than the introduction of an amount of modelling delay equal to the one required by the FIR inversion.

As a last point we have to mention that, even though the proposed technique was formulated in a single-channel context, a multi-channel generalisation is feasible

[8] The segmentation of the input signal does not have to affect to affect the filtering through the minimum phase part as an appropriate number of the filter's outputs can be stored in memory and used as the initial conditions of the filtering of the next segment to arrive.

that could easily fit as a solution to the inverse filtering demands of the generic Virtual Acoustic Imaging System described in section 1.1. This is because, as is described in detail in [4], the inverse of the filter-matrix **C** corresponding to the transfer function of a multi-channel plant can be realised by a matrix **H** containing rational expressions in the z-variable having numerators of order equal to the plant's order (and thus typically very low) and all sharing a common denominator. It is exactly this denominator that requires very long FIR filters to be realised in a non-recursive fashion but could, however, be implemented with proper use of the technique described above, and, even more important, only once for all the elements of **H**.

3. Simulation results

The inverse filtering algorithm presented above was simulated in Matlab and in this section we present the results we obtained and compare them with the results we get when we use conventional FIR inverse filtering to equalise the same plant. As a typical case of electroacoustic plant we chose to invert an HRTF measured by Gardner and Martin at the MIT Media Lab [see 17] and specifically the one measured with the source placed exactly in front of the KEMAR dummy head[9]. This HRTF (henceforward denoted as c) is 128-samples long and is depicted in Figure 3–1 where the distribution of its roots in the z-plane is also shown. As it can be seen in this figure, the plant is mixed-phase and as was mentioned above most of its roots are clustered very close to the unit circle. A 1500-coefficients FIR inverse of this plant (henceforward denoted as h) was computed in the frequency domain as the inverse DFT of the reciprocal of the plant's DFT after proper zero padding was applied in order to make use of the FFT's computational advantage and to avoid circular convolution effects [see 11 pp. 105-109]. The modelling delay was chosen to 700 samples. This FIR inverse is shown in Figure 3–2 (left part) along with the result we get when we convolve this inverse with the plant (middle part) and the difference between this convolution result and the ideal delta function δ(n-700) (right part). As can be seen in this figure the inversion should be quite successful when h is used but a

[9] Thus the chosen HRTF is the one contained in the left and right channels of H0e000a.wav (both channels contain the same time sequence as is explained in [17])

residual mismatch error should still be present between the delayed input and the equalised signal as part (**c**) of the figure suggests.

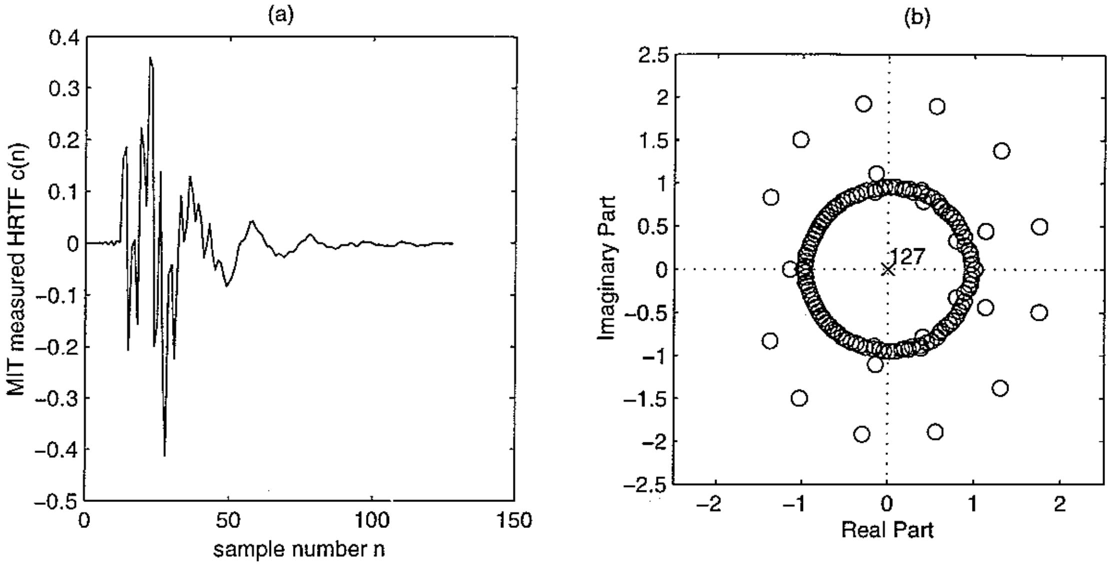

Figure 3–1 (**a**) Measured HRTF's time history and (**b**) its zeros in the z-plane

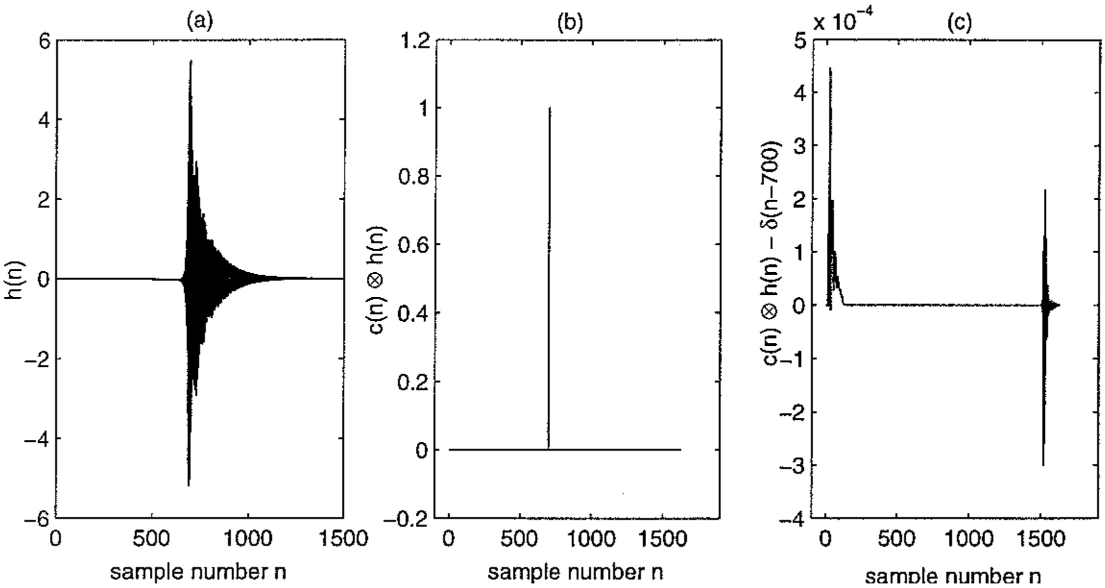

Figure 3–2 (**a**) 1500-samples-long FIR inverse $h(n)$ with 700 samples of modelling delay, (**b**) convolution of $c(n)$ with $h(n)$ and (**c**) mismatch between the ideal $\delta(n\text{-}700)$ and $c(n)\otimes h(n)$

Next, in order to test the performance of the IIR algorithm presented above and to compare it with the corresponding performance of the FIR filter of Figure 3–2, we created a random signal of $2{\cdot}10^4$ samples (henceforward denoted as x) shown in part (**a**) of Figure 3–3 and filtered it through the cascade arrangement of $c(n)$ with $h(n)$. The output $y_{FIR}(n)$ of this filtering procedure should be a delayed version of the input signal and this is shown in part (**b**) of Figure 3–3 where the first 10^4 samples of the difference $y_{FIR}(n)$-$x(n$-$700)$ are displayed. The same input signal $x(n)$ was then processed through the IIR filtering procedure described above to acquire the output $y_{IIR}(n)$. The difference between the equalised output $y_{IIR}(n)$ and the original input $x(n)$ is shown in part (**c**) of Figure 3–3.

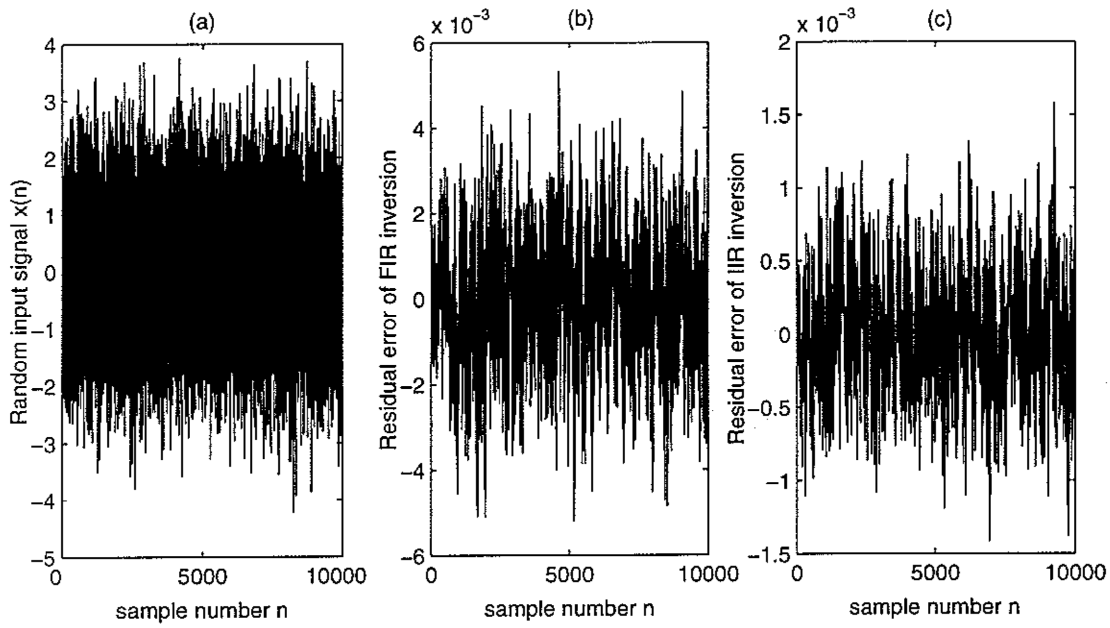

Figure 3–3 (**a**) Random input signal $x(n)$, (**b**) Performance of the 1500-coefficients FIR inverse and (**c**) Performance of IIR inversion utilising approximately 250 coefficients

It is evident from the comparison of the middle and the rightmost parts of Figure 3–3 that, using as many coefficients as twice the plant's order[10] i.e. less than one fifth of the number used for the conventional FIR equalisation, the IIR filtering procedure introduced here achieves better results.

In section 2.3 we mentioned that the critical point in the implementation of our IIR filtering procedure is the decomposition of the rational expression $1/C(z^{-1})$ corresponding to the reciprocal of the plant's z-transform as it is this stage of the procedure that is extremely sensitive to numerical error. From the results presented above it is evident that the introduced algorithm can indeed invert the 128-samples-long measured HRTF but as it can be seen in the rightmost part of Figure 3–3 the residual error has a rather substantial value. This is exactly due to the fact that the decomposition into minimum and maximum phase parts is not as accurate as one would desire. In order to emphasise the proposed technique's potential advantages we turn now to a more favourable situation for the algorithm where we try to invert a shorter plant thus reducing the numerical strain imposed in the decomposition of

[10] This is can be readily made evident by simple inspection of equation (2.1)

equation (2.2). To this aim we have run the algorithm using a truncated version of $c(n)$ comprising only its first 92 samples and shown in Figure 3–4 along with the distribution of its zeros in the z-plane

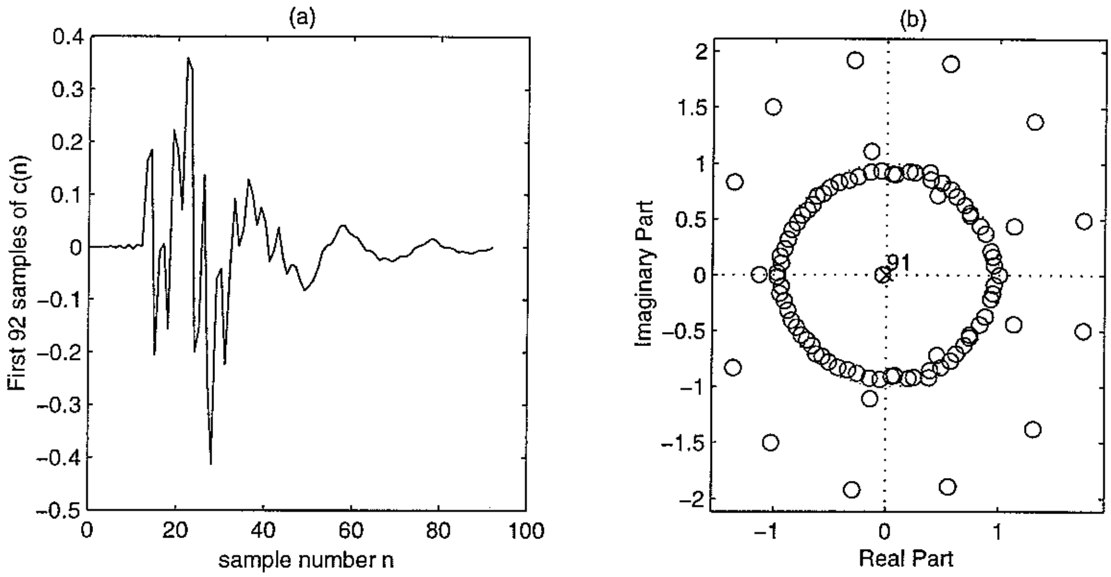

Figure 3–4 (**a**) Truncated version of MIT's HRTF and (**b**) its zeros in the z-plane

As can be seen in Figure 3–5 when set to invert this shorter -but again mixed-phase- plant our algorithm leaves a residual error of around 180dB with reference to the input signal $x(n)$ using 92 feedforward and 92 recursive coefficients. On the other hand the error for the conventional FIR inverse filter with 1000 coefficients is barely around 25dB, the performance of the FIR inversion remaining significantly inferior even when 4000 coefficients are used. It is thus evident that if a means to successfully compute the decomposition of (2.1) for longer plants is established, results superior to the ones obtained by the conventional FIR inversion technique with modelling delay can be achieved with far lower computational demands.

As a final set of results we compare in Figure 3–6 the residual error we obtain when we try to invert the truncated plant with the proposed IIR filtering technique versus the ones we get when an FIR filter using 8000 coefficients is used. It can be seen in this figure that only when more than 40 times more coefficients are used does the conventional FIR inverse give equivalent results with our algorithm and this only when the exactly optimum choice of modelling delay is made, as a comparison of the middle and the right parts of Figure 3–6 suggests.

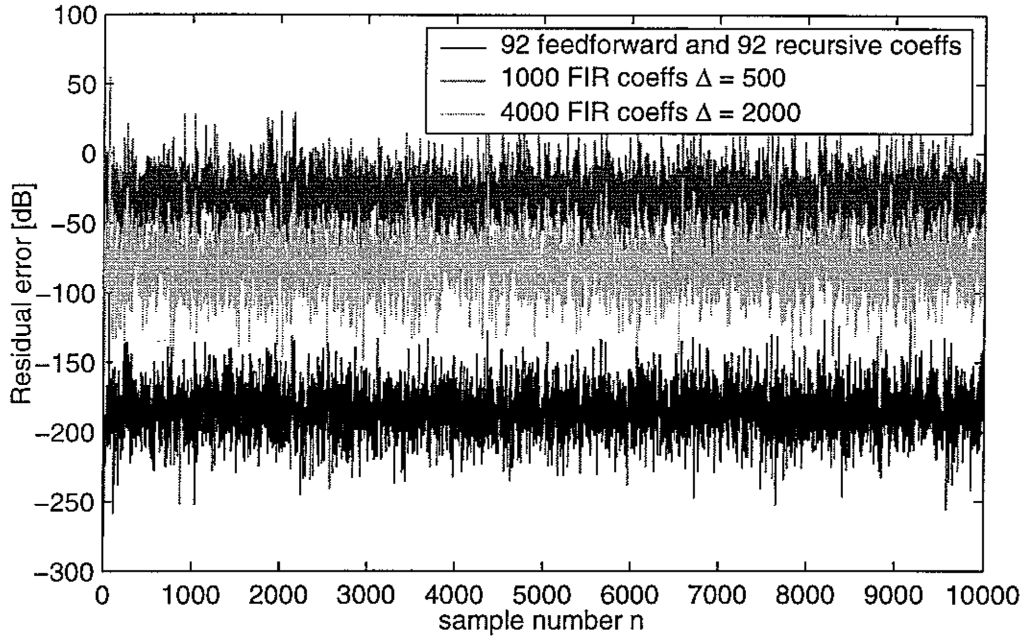

Figure 3–5 Comparison of the performance achieved in the inversion of the truncated HRTF by the proposed algorithm utilising approximately 200 coefficients, a 1000-coefficients FIR filter and a 4000-coefficients FIR filter

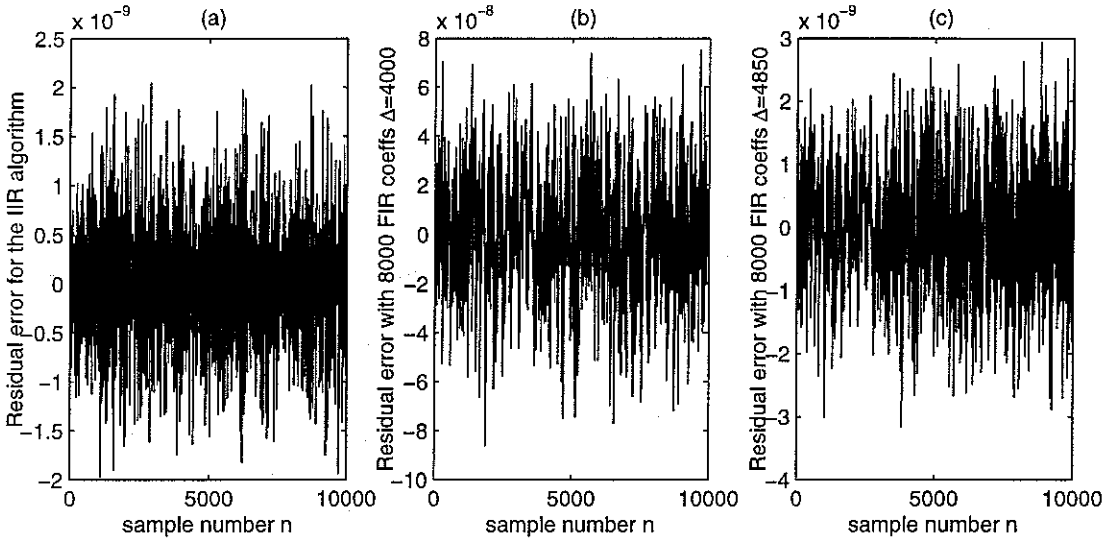

Figure 3–6 Performance comparison of the proposed algorithm and two 8000-coefficients FIR filters with different modelling delays

4. *Conclusions*

We presented above a technique based on IIR filtering that can be used for the equalisation of mixed phase single-channel plants and which was shown to achieve superior results compared to the conventional FIR-based technique albeit requiring far less processing power. The successful implementation of the proposed algorithm would mean that the inversion of longer, and consequently more accurate, measured

HRTFs could become practical (it could for example be used to design inverse filters for the 512-samples-long set of measured HRTFs included in the MIT-database or even better for the inversion of subject-specific measured HRTFs).

The weak point of the proposed implementation was pointed out and possible ways to improve it were suggested. It has to be said though, that even a trade-off between extremely high computational demand in the inverse filters' design for a significantly lower demand in the actual filtering process is indeed preferable since the inversion of the plant is a "one-off" operation that can be carried out by a very powerful machine unlike the filtering process which has to be computed time and time again in a possibly low-end-of-the-range machine every time the "user" wants to create a virtual image.

Furthermore, in spite of the fact that the presentation was restricted in a single-channel and formulated in a way that an on-line realisation is not possible, ways to overcome these restrictions were proposed. The actual implementation of the real-time, multi-channel generalisation of the algorithm and of course a set of subjective experiments for the evaluation of a Virtual Acoustic Imaging System based on the algorithm are going to be the next steps of the present research project.

References

1. J. G. PROAKIS *Digital communications*. McGraw-Hill, (1983)

2. P. A. NELSON and S. J. ELLIOTT *Active control of sound* Academic Press, (1992)

3. WIDROW, B. and E. WALACH *Adaptive Inverse Control*. Prentice Hall, (1996).

4. P. A. NELSON *Active control of acoustic fields and the reproduction of sound*. Journal of Sound and Vibration, **177**(4) pp. 447-77, (1994).

5. O. KIRKEBY, P. A. NELSON, F. ORDUNA BUSTAMANTE and H. HAMADA *Local sound field reproduction using digital signal processing*. Journal of the Acoustical Society of America, **100**(3) pp. 1584-93, (1996).

6. P. A. NELSON, H. HAMADA and S. J. ELLIOTT *Adaptive inverse filters for stereophonic sound reproduction*. IEEE Transactions on Signal Processing, **40**(7) pp. 1621-32, (1992).

7. P. A. NELSON, O. KIRKEBY, T. TAKEUCHI and H. HAMADA *Sound fields for the production of virtual acoustic images*. Journal of Sound and Vibration, **204**(2) pp. 386-396, (1997).

8. O. KIRKEBY, P. A. NELSON and H. HAMADA *The "Stereo Dipole" – A Virtual Source Imaging System using two closely spaced loudspeakers*. Journal of Audio Engineering Society, **46**(5) pp. 387-395, (1998).

9. J. G. PROAKIS and D. G. MANOLAKIS *Digital signal processing Principles, algorithms and applications*. Prentice-Hall, (1996).

10. B. WIDROW and S. D. STEARNS *Adaptive Signal Processing*. Prentice-Hall, (1985).

11. V. A. OPPENHEIM and R. W. SHAFER *Digital signal processing*. Prentice-Hall, (1975).

12. O. KIRKEBY, P. A. NELSON, H. HAMADA and F. ORDUNA BUSTAMANTE *Fast Deconvolution of Multi-Channel Systems Using Regularization*. Technical Report 255, ISVR, Southampton, U.K., (1996)

13. J. J. SHYNK *Adaptive IIR filtering*. IEEE ASSP Magazine, **6**(2) pp. 4-21, (1989).

14. T. PAPADOPOULOS *The use of IIR filtering in Virtual Acoustic Imaging Systems*. MSc Thesis Southampton, (2000).

15. A. V. OPPENHEIM, A. S. WILLSKY and I. T. YOUNG *Signals and systems*. Prentice-Hall, (1983).

16. *Signal Processing Toolbox User's Guide Version 5*. MathWorks, (2000).

17. B. GARDNER and K. MARTIN *HRTF Measurements of a KEMAR Dummy-Head Microphone*. Technical Report #280, MIT Media Lab, (1994).

18. V. KUCERA *Discrete linear control the polynomial equation approach*. Wiley, (1979).

19. *The Polynomial Toolbox 2 manual*. PolyX Ltd, (1999).

20. J. MOURJOPOULOS, P. M. CLARKSON and J. K. HAMMOND *A comparative study of least-squares and homomorphic techniques for the inversion of mixed phase signals*. IEEE International Conference on Acoustics Speech and Signal Processing, Paris, (1982).

21. K. J. ÅSTRÖM, AND B. WITTENMARK *Computer-controlled systems theory and design*. Prentice Hall, (1997).

22. V. KUCERA, J. JEZEK and M. KRUPICKA *Numerical-Analysis of Diophantine Equations*. Lecture Notes in Control and Information Sciences, **158** pp. 128-136, (1991).