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Active Control of Sound Transmission through an Opening Connecting Two Rooms

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ISVR Technical Memorandum No 889

July 2002

University of Southampton

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UNIVERSITY OF SOUTHAMPTON

INSTITUTE OF SOUND AND VIBRATION RESEARCH

SIGNAL PROCESSING & CONTROL GROUP

Active Control of Sound Transmission through an Opening Connecting Two Rooms

by

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ISVR Technical Memorandum N° 889

July 2002

Authorised for issue by
Prof S J Elliott
Group Chairman

ABSTRACT

This report presents the results of a theoretical study of active control of sound transmission through an opening connecting two rooms. The system studied consists of two lightly damped enclosures connected by a small window. A sound source is placed in the first cavity and a set of secondary loudspeaker sources are located along the perimeter of the window. The goal is to reduce the global acoustic potential energy in the receiver room by driving the control loudspeakers to cancel the measured acoustic pressure by an equal number of error microphones placed close to the loudspeakers.

This report introduces the theory of an impedance matrix model used to predict the sound transmission between the two cavities that are coupled via the window. The sound transmission between the two cavities has been modelled by considering the window to be virtually divided into rectangular elements, each of which behaves like a baffled piston. In this way a matrix formulation has been derived which gives the sound pressure in the two rooms and at the error sensors as a function of the primary and secondary sources strengths.

Simulations have been carried out for both passive and active sound transmission with different numbers and configurations of control units. The aim of this study is to assess whether or not it would be worth studying and developing a "window active noise control system".

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1. INTRODUCTION

During the past three decades, a lot of research about the active control of sound has been carried. In this area the first subject of study and modelling was the control of sound in ducts. At low frequency, the problem is in one dimension and the method generally used to cancel sound in ducts is the reflection of the primary wave disturbance by means of one or more secondary sources. This is usually achieved by cancelling the acoustic pressure at the secondary sources positions[1]. At low frequencies the sound cancellation requires a single input single output control system which is composed by a loudspeaker and a microphone that play respectively the role of actuator or secondary source and error sensor.

Most of the concrete problems are in three dimensions. For example, the problem analysed in this report is about the transmission of noise through an opening connecting two rooms. Everybody has already experienced this common problem. It can be at home when you hear the sound of radio from a near room or in transport vehicles with an opened window. The purpose of this study is to cancel the pressure at the middle of the opening[2]. The aim of the project is to reduce this kind of transmission using a local active sound control technique.

Here two enclosures are considered with a primary source placed in the first of them. An active control system at the opening is theoretically studied. As Romeu et al. (2002), the strategy is to cancel the pressure at the window but here, we use several secondary sources placed just close to the error microphones. The purpose is to obtain an appreciable diminution of the global acoustic potential energy in the second cavity using this control system. The collocated positioning of a microphone error sensor to each control loudspeaker should enable the implementation of feedback control. Indeed, it is hoped the response functions of each microphone sensor-loudspeaker actuator are minimum phase with real part positive so that decentralised feedback control could be implemented[3]. If feedback control can be implemented then both tonal, deterministic, and random disturbances could be controlled.

The report is organised in three parts. In the first part (section2), the geometric and physical constants of the system studied are given. In the second part (section 3), an impedance matrix model is formulated in order to derive the acoustic potential energy in both cavities. The opening, which transmits sound from one cavity to the other, acts as a sound source. Therefore, it has been approximated with a grid of rectangular elementary pistons moving at different velocities. The third part (section 4) describes the passive sound transmission through the opening for the model chosen. Some techniques to validate it are presented and several simulations results are discussed. The last part (section 5) is dedicated to active control simulations assuming ideal feed-forward control. The strengths of the secondary sources are driven in order to cancel the sound pressure measured at the error microphones. Finally, the potential energy has been plotted to evaluate the global benefits of such a control system on both cavities. The simulation results shown correspond to four different configurations, according to whether the active control system has one, two, four or eight control units.

2. THE SYSTEM STUDIED

As shown in Figure 2.1, two rectangular lightly damped enclosures connected by a window are considered.

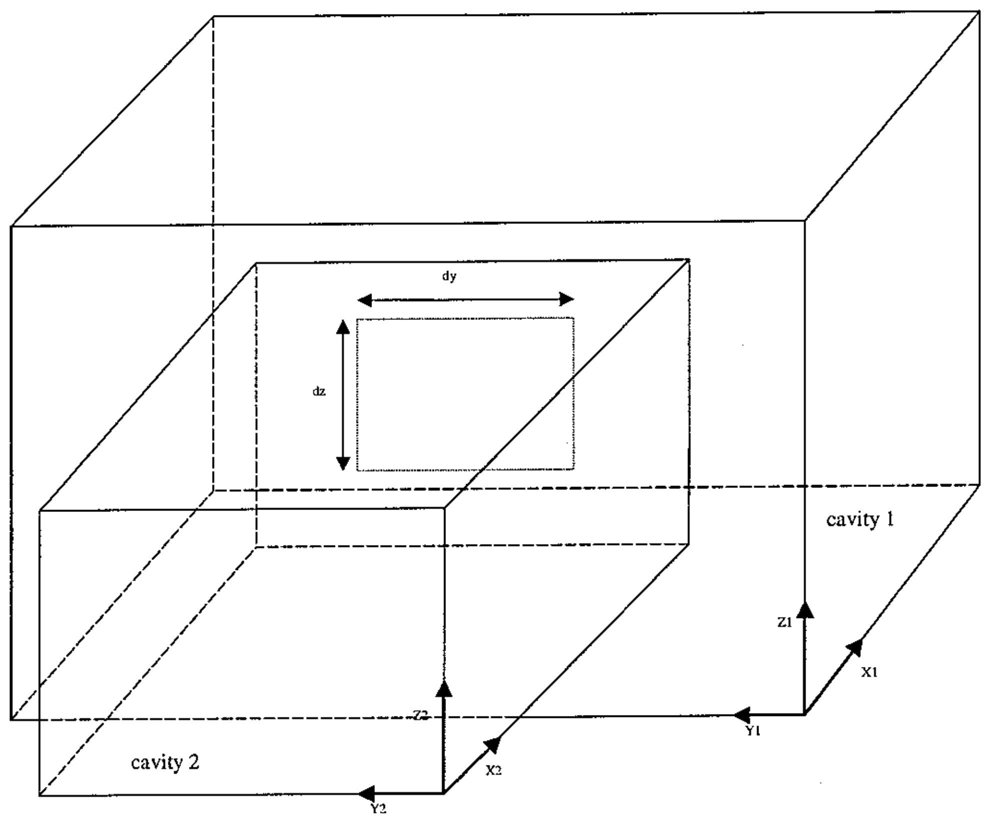

Figure 2.1: *View of the cavities and the opening in between*

Figure (2.2) presents a top view of the system: a sound source is placed in one of the corners of the first cavity. This position has been chosen in such a way as that all the acoustic modes are excited.

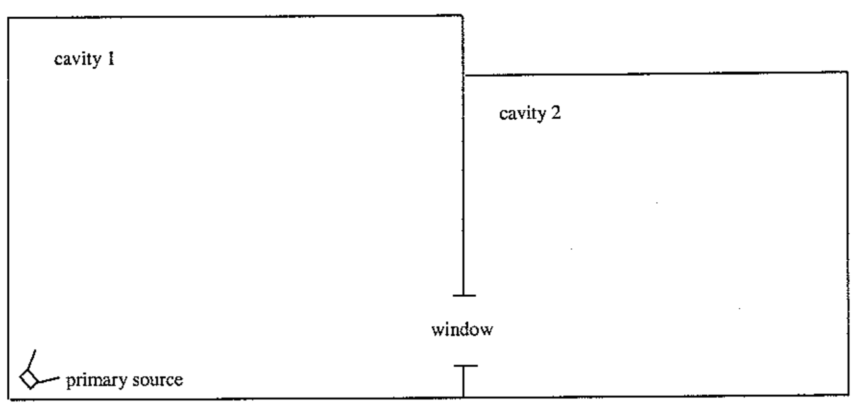

Figure 2.2: *2D view of the system studied*

The window can be equipped with a variable number of loudspeakers and microphones, playing the role of secondary sources and error sensors respectively. In order to simplify the calculus of the optimal secondary sources, we have assumed that the number of microphones L is equal to the number of secondary sources. Moreover, the microphones are assumed to be placed close to the window, but on the side of cavity 2.
Table 2.1 below summarize the geometrical and physical properties of the system.

Speed of sound	$c_0 = 343ms^{-1}$
Air density	$\rho_0 = 1.21kgm^{-3}$
Dimensions of cavity 1	$L_{x1} = 0.8m$ $L_{y1} = 0.6m$ $L_{z1} = 0.7m$
Dimensions of cavity 2	$L_{x2} = 0.6m$ $L_{y2} = 0.5m$ $L_{z2} = 0.4m$
Position of the primary source	$x_p = 0.01m$ $y_p = 0.01m$ $z_p = 0.01m$
Dimensions of the window	$dy = 0.1m$ $dz = 0.2m$
Position of the centre of the window	$x_g^{C1} = Lx_1$ $y_g = 0.1m$ $z_g = 0.15m$

Table 2.1 *Geometry and physical constants for the system*

3. MATRIX MODEL FOR THE SOUND TRANSMISSION

In this section, it is described the impedance matrix model used to derive the acoustic response of both cavities for a given primary harmonic acoustic excitation in the first cavity.
The responses of both cavities in a frequency range 0 to 1 kHz have been calculated assuming the primary source to be harmonic with time dependence of the form $\exp(j\omega t)$. The velocity-type and pressure-type parameters used in the model have been taken to be real part of a counter clock while rotating complex vectors, e.g. phasors. The velocity or pressure parameters are therefore given by $v(t) = \mathrm{Re}\{v(\omega)e^{j\omega t}\}$ or $p(t) = \mathrm{Re}\{p(\omega)e^{j\omega t}\}$ where $v(\omega)$ and $p(\omega)$ are the velocity and pressure phasors at the instant $t = 0$.
In order to study the influence of the window on the sound field, we have virtually divided it into rectangular elements, each of which behaves like a piston. Therefore the mathematical model built in for the simulations considers the window divided into a grid of rectangular elements whose dimensions have been taken to be $l_{yw} = dy / n_e$ and $l_{ze} = dz / n_e$, where $(n_e)^2$ is the total number of elements.

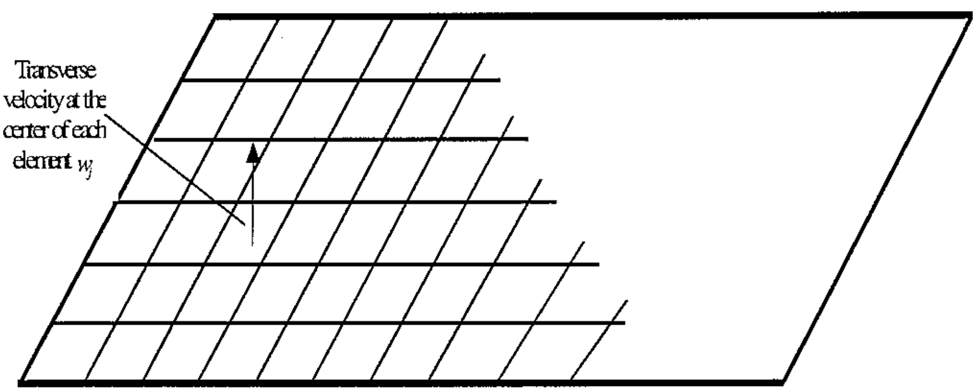

Figure 3.1: *Velocity notation used in the mathematical model*

The influence of the window on each cavity has been considered separately. Hence the phasors of the transverse velocities $v_{wj}^{C1}(\omega)$ and $v_{wj}^{C2}(\omega)$ at the centre of the elements of the window due to the sound in cavity 1 and 2 respectively have been grouped into the following vectors:

$$\mathbf{v}_w^{C1}(\omega) \equiv \begin{Bmatrix} v_{w1}^{C1}(\omega) \\ v_{w2}^{C1}(\omega) \\ \vdots \\ v_{wP}^{C1}(\omega) \end{Bmatrix}, \qquad \mathbf{v}_w^{C2}(\omega) \equiv \begin{Bmatrix} v_{w1}^{C2}(\omega) \\ v_{w2}^{C2}(\omega) \\ \vdots \\ v_{wP}^{C2}(\omega) \end{Bmatrix}, \qquad (3.1, 3.2)$$

where $P = n_e^2$ is the total number of elements. Similarly, $\mathbf{p}_w^{C1}(\omega)$ and $\mathbf{p}_w^{C1}(\omega)$ are the vectors with the sound pressure phasors in front of the window at the centre positions of the grid of elements, on both sides of the window.

$$\mathbf{p}_w^{C1}(\omega) \equiv \begin{Bmatrix} p_{w1}^{C1}(\omega) \\ p_{w2}^{C1}(\omega) \\ \vdots \\ p_{wP}^{C1}(\omega) \end{Bmatrix} \tag{3.3}$$

$$\mathbf{p}_w^{C2}(\omega) \equiv \begin{Bmatrix} p_{w1}^{C2}(\omega) \\ p_{w2}^{C2}(\omega) \\ \vdots \\ p_{wP}^{C2}(\omega) \end{Bmatrix} \tag{3.4}$$

The primary excitation is given by the complex strength q_p of the primary source in cavity 1.
The complex strengths of the L secondary sources introduced for the active control are grouped into the following vector:

$$\mathbf{q}_s(\omega) \equiv \begin{Bmatrix} q_{s1}(\omega) \\ q_{s2}(\omega) \\ \vdots \\ q_{sL}(\omega) \end{Bmatrix} \tag{3.5}$$

We have assumed that the primary and each secondary source are point monopoles sources. The phasors of sound pressure at the L error microphones, placed exactly at the window, have been grouped as follows:

$$\mathbf{p}_e^{W}(\omega) \equiv \begin{Bmatrix} p_{e1}^{W}(\omega) \\ p_{e2}^{W}(\omega) \\ \vdots \\ p_{eL}^{W}(\omega) \end{Bmatrix} \tag{3.6}$$

3.1. Impedance functions

The pressure at the centres of the window elements generated by the two cavities is given by the following impedance equations:

$$\mathbf{p}_w^{C1}(\omega) = \mathbf{z}_{wp}^{C1}(\omega)q_p(\omega) + \mathbf{Z}_{ww}^{C1}(\omega)\mathbf{v}_w^{C1}(\omega) + \mathbf{Z}_{ws}^{C1}(\omega)\mathbf{q}_s(\omega) \tag{3.7}$$

$$\mathbf{p}_w^{C2}(\omega) = \mathbf{Z}_{ww}^{C2}(\omega)\mathbf{v}_w^{C2}(\omega) + \mathbf{Z}_{ws}^{C2}\mathbf{q}_s(\omega) \tag{3.8}$$

Also the acoustic pressure at the error microphones is given by:

$$\mathbf{p}_e^{W}(\omega) = \mathbf{z}_{ep}^{C1}(\omega)q_p(\omega) + [\mathbf{Z}_{ew}^{C1}(\omega) + \mathbf{Z}_{ew}^{C2}(\omega)]\mathbf{v}_w^{C2}(\omega) + [\mathbf{Z}_{es}^{C1}(\omega) + \mathbf{Z}_{es}^{C2}(\omega)]\mathbf{q}_s(\omega) \tag{3.9}$$

where the elements of the eight impedance matrices and the two impedance vectors are defined as follows (Nelson and Elliott, 1992):

$$z_{wp}^{C1^{i,1}}(\omega) = \omega\rho_o c_o{}^2 \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{wi}, y_{wi}, z_{wi})\psi_{n1}(x_p, y_p, z_p)}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} \tag{3.10}$$

$$Z_{ww}^{C1^{i,j}}(\omega) = -S\omega\rho_o c_o{}^2 \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{wi}, y_{wi}, z_{wi})\psi_{n1}(x_{wj}, y_{wj}, z_{wj})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)].} \tag{3.11}$$

$$Z_{ws}^{C1^{i,l}}(\omega) = \omega\rho_o c_o{}^2 \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{wi}, y_{wi}, z_{wi})\psi_{n1}(x_{sl}, y_{sl}, z_{sl})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} \tag{3.12}$$

$$Z_{ww}^{C2^{i,j}}(\omega) = S\omega\rho_o c_o{}^2 \sum_{n2=0}^{N2} \frac{\phi_{n2}(x_{wi}, y_{wi}, z_{wi})\phi_{n2}(x_{wj}, y_{wj}, z_{wj})}{V_2[2\zeta_{n2}\omega_{n2}\omega + j(\omega^2 - \omega_{n2}^2)]} \tag{3.13}$$

$$Z_{ws}^{C2^{i,l}}(\omega) = \omega\rho_o c_o{}^2 \sum_{n2=0}^{N2} \frac{\phi_{n2}(x_{wi}, y_{wi}, z_{wi})\phi_{n2}(x_{sl}, y_{sl}, z_{sl})}{V_2[2\zeta_{n2}\omega_{n2}\omega + j(\omega^2 - \omega_{n2}^2)]} \tag{3.14}$$

$$z_{ep}^{C1^{k,1}}(\omega) = \omega\rho_o c_o{}^2 \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{ek}, y_{ek}, z_{ek})\psi_{n1}(x_p, y_p, z_p)}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} \tag{3.15}$$

$$Z_{ew}^{C1^{k,j}}(\omega) = -S\omega\rho_o c_o{}^2 \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{ek}, y_{ek}, z_{ek})\psi_{n1}(x_{wj}, y_{wj}, z_{wj})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} \tag{3.16}$$

$$Z_{ew}^{C2^{k,j}}(\omega) = S\omega\rho_o c_o^2 \sum_{n2=0}^{N2} \frac{\phi_{n2}(x_{ek}, y_{ek}, z_{ek})\phi_{n2}(x_{wj}, y_{wj}, z_{wj})}{V_2[2\zeta_{n2}\omega_{n2}\omega + j(\omega^2 - \omega_{n2}^2)]} \tag{3.17}$$

$$Z_{es}^{C1^{k,l}}(\omega) = \omega\rho_o c_o^2 \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{ek}, y_{ek}, z_{ek})\psi_{n1}(x_{sl}, y_{sl}, z_{sl})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} \tag{3.18}$$

$$Z_{es}^{C2^{k,l}}(\omega) = \omega\rho_o c_o^2 \sum_{n2=0}^{N2} \frac{\phi_{n2}(x_{ek}, y_{ek}, z_{ek})\phi_{n2}(x_{sl}, y_{sl}, z_{sl})}{V_2[2\zeta_{n2}\omega_{n2}\omega + j(\omega^2 - \omega_{n2}^2)]} \tag{3.19}$$

where

ω	is the angular frequency of excitation,
j	is the complex operator,
n_1, n_2	are modal indices,
ω_{n1}	is the angular frequency of the n_1-th mode of cavity 1,
ω_{n2}	is the angular frequency of the n_2-th mode of cavity 2,
$\psi_{n1}(x, y, z)$	is the n_1-th eigenfunction of cavity 1 at position (x, y, z),
$\phi_{n2}(x, y, z)$	is the n_2-th eigenfunction of cavity 2 at position (x, y, z),
ζ_{n1}	is the damping ratio of the of the n_1-th mode of cavity 1,
ζ_{n2}	is the damping ratio of the of the n_2-th mode of cavity 2,
ρ_0	is the air density,
c_0	is the speed of sound,
V_1	is the volume of cavity 1,
V_2	is the volume of cavity 2,
S	is the surface of the window.

NB: As shown in Figure (3.2), considering $\omega \to 0$, then for a force exerted on the window, if the pressure increases in the second cavity, the pressure should decrease in the first cavity, hence, $Z_{ww}^{C1^{i,j}}$ has minus sign in accordance with physics.

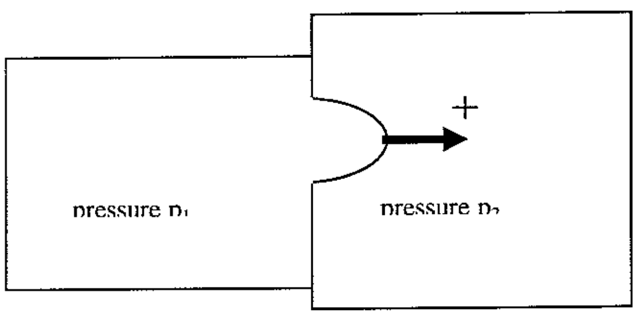

Figure 3.2: *diagram for the sign convention of pressure.*

3.2. Approximations

In the above formulation, the following approximations were made:

1. The damping ratio is the same for all the modes: $\zeta_{n1} = \zeta_{n2} = 0.01 \;\; \forall n_1 \;\; \forall n_2$.

2. We consider the sound field to be represented by a *finite* number of modals contributions by truncating the series representation after N_1 (resp. N_2) terms (Nelson and Elliot, 1992). To avoid numerical problems with MatLab, we had to take into account more than 5000 modes. This approximation is valid for frequencies up 1000 Hz.

The eigenfunctions of the first cavity can be written as:

$$\psi_{n1}(x, y, z) = \sqrt{\varepsilon_{n1,1}\varepsilon_{n1,2}\varepsilon_{n1,3}} \cos\left(\frac{n_{1,1}\pi}{L_{x1}}x\right)\cos\left(\frac{n_{1,2}\pi}{L_{y1}}y\right)\cos\left(\frac{n_{1,3}\pi}{L_{z1}}z\right), \tag{3.20}$$

where L_{x1} L_{y1} L_{z1} are the dimensions of the cavity and n_1 denotes the trio of modal integers $n_{1,1}$, $n_{1,2}$ and $n_{1,3}$. The normalisation factors are built as follows: $\varepsilon_{n1,k} = 1$ if $n_{1,k} = 0$ and $\varepsilon_{n1,k} = 2$ if $n_{1,k} > 0$.

The expression of ϕ_{n2} is similar except that it involves the dimensions and the modes of the second cavity.

For the first cavity, the natural frequencies are given by:

$$\omega_{n1} = c_0 \sqrt{\left[\frac{n_{1,1}\pi}{L_{x1}}\right]^2 + \left[\frac{n_{1,2}\pi}{L_{y1}}\right]^2 + \left[\frac{n_{1,3}\pi}{L_{z1}}\right]^2} \tag{3.21}$$

and for the second cavity:

$$\omega_{n2} = c_0 \sqrt{\left[\frac{n_{2,1}\pi}{L_{x2}}\right]^2 + \left[\frac{n_{2,2}\pi}{L_{y2}}\right]^2 + \left[\frac{n_{2,3}\pi}{L_{z2}}\right]^2} \tag{3.22}$$

3.3. Acoustic Sound Pressure at monitor sensors

The complex pressure at a set of monitor sensors in the both cavities is given by the following relations:

$$\mathbf{p}_m^{C1}(\omega) = \mathbf{Z}_{mp}^{C1}(\omega)q_p(\omega) + \mathbf{Z}_{mw}^{C1}(\omega)\mathbf{v}_w^{C1}(\omega) + \mathbf{Z}_{ms}^{C1}(\omega)\mathbf{q}_s(\omega) \tag{3.23}$$

$$\mathbf{p}_m^{C2}(\omega) = \mathbf{Z}_{mw}^{C2}(\omega)\mathbf{v}_w^{C2}(\omega) + \mathbf{Z}_{ms}^{C2}(\omega)\mathbf{q}_s(\omega) \tag{3.24}$$

where the elements of transfer coefficients are defined as follows:

$$Z_{mp}^{C1^{r,l}}(\omega) = \omega\rho_o c_o^{\;2} \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{mr}, y_{mr}, z_{mr})\psi_{n1}(x_p, y_p, z_p)}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]}, \tag{3.25}$$

$$Z_{mw}^{C1^{r,j}}(\omega) = -S\omega\rho_o c_o^{\;2} \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{mr}, y_{mr}, z_{mr})\psi_{n1}(x_{wj}, y_{wj}, z_{wj})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]}, \tag{3.26}$$

$$Z_{ms}^{C1^{r,l}}(\omega) = \omega\rho_o c_o^{\;2} \sum_{n1=0}^{N1} \frac{\psi_{n1}(x_{mr}, y_{mr}, z_{mr})\psi_{n1}(x_{sl}, y_{sl}, z_{sl})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]}, \tag{3.27}$$

$$Z_{mw}^{C2^{r,j}}(\omega) = -S\omega\rho_o c_o^{\;2} \sum_{n2=0}^{N2} \frac{\phi_{n2}(x_{mr}, y_{mr}, z_{mr})\psi_{n1}(x_{wi}, y_{wi}, z_{wi})}{V_2[2\zeta_{n2}\omega_{n2}\omega + j(\omega^2 - \omega_{n2}^2)]}, \tag{3.28}$$

$$Z_{ms}^{C2^{r,l}}(\omega) = \omega\rho_o c_o^{\;2} \sum_{n2=0}^{N2} \frac{\phi_{n2}(x_{mr}, y_{mr}, z_{mr})\phi_{n2}(x_{sl}, y_{sl}, z_{sl})}{V_2[2\zeta_{n2}\omega_{n2}\omega + j(\omega^2 - \omega_{n2}^2)]}. \tag{3.29}$$

Knowing the sound pressure at a given point in one of the cavities gives some indications about the benefits of the active control of sound transmission. However, it is much helpful to determine the global acoustic potential energy in each cavity.

3.4. Total time-averaged potential energy

In this subsection, the method to calculate acoustic potential energy is only given for the first cavity, but the results can be easily applied to the second cavity by adapting the notations.

The total time-averaged acoustic potential energy in the first cavity is:

$$E_p^{C1} = \frac{1}{4\rho_0 c_0^2} \int_{V1} | p_{tot}(x, y, z)|^2 \; dx.dy.dz \;, \tag{3.30}$$

where p_{tot} is the superposition of the sound pressures due to the different sound sources.

As done above, the approximate solution for the complex acoustic pressure in the harmonically excited lightly damped sound field in cavity 1 can be written using the following truncated series expansion:

$$p^{C1}(x, y, z) = \sum_{n1=0}^{N1} a_{n1}\psi_{n1}(x, y, z) \tag{3.31}$$

Then, using eq. (3.30-31) and the modal orthogonality property, the total acoustic potential energy can be evaluated from the sum of the squares of the complex mode amplitudes:

$$E_p^{C1} = \frac{V_1}{4\rho_0 c_0^2} \sum_{n1=0}^{N1} | a_{n1} |^2 . \tag{3.32}$$

The total time-averaged acoustical potential energy of the first cavity per unit primary excitation can be deduced from eq. (3.32) adopting the vector notation (3.33) for the sum of the squared mode amplitudes:

$$\mathbf{a}^{C1} \equiv \begin{Bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{N1} \end{Bmatrix}, \qquad E_p^{C1}(\omega) = \frac{V_1}{4\rho_0 c_0^2} \mathbf{a}^{C1^{\mathrm{H}}} \mathbf{a}^{C1} , \tag{3.33-34}$$

where $^{\mathrm{H}}$ is the hermitian transpose operator and:

$$\mathbf{a}^{C1}(\omega) = \mathbf{T}_p^{C1}(\omega) q_p(\omega) + \mathbf{T}_w^{C1}(\omega) \mathbf{v}_w^{C1}(\omega) + \mathbf{T}_s^{C1}(\omega) \mathbf{q}_s(\omega) , \tag{3.35}$$

and:

$$T_p^{C1^{n1,1}}(\omega) = \omega \rho_o c_o{}^2 \frac{\psi_{n1}(x_p, y_p, z_p)}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} , \tag{3.36}$$

$$T_w^{C1^{n1,j}}(\omega) = -S\omega \rho_o c_o{}^2 \frac{\psi_{n1}(x_{wj}, y_{wj}, z_{wj})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} , \tag{3.37}$$

$$T_s^{C1^{1,l}}(\omega) = \omega \rho_o c_o{}^2 \frac{\psi_{n1}(x_{sl}, y_{sl}, z_{sl})}{V_1[2\zeta_{n1}\omega_{n1}\omega + j(\omega^2 - \omega_{n1}^2)]} . \tag{3.38}$$

4. PASSIVE SOUND TRANSMISSION

The passive sound transmission between the coupled cavities has been studied first by plotting the acoustical potential energy in both cavities as a function of the frequency of excitation.

4.1. Boundary conditions

The pressure and the transverse velocity are continuous crossing the window, hence:

$$\mathbf{v}_w^{C1}(\omega) = \mathbf{v}_w^{C2}(\omega) = \mathbf{v}_w(\omega) \ , \qquad \mathbf{p}_w^{C1}(\omega) = \mathbf{p}_w^{C2}(\omega) . \tag{4.1-2}$$

4.2. Acoustic potential energy

Without control, there are not any secondary sources. Then from eq. (3.7) and (3.8) using boundary conditions (4.1)-(4.2) above, the transverse velocity at the centres of the elements of the window can be expressed as:

$$\mathbf{v}_w(\omega) = (\mathbf{Z}_{ww}^{C2}(\omega) - \mathbf{Z}_{ww}^{C1}(\omega))^{-1}\mathbf{Z}_{wp}^{C1}(\omega) q_p(\omega) \tag{4.3}$$

Then, the acoustic potential energy of the first cavity can be calculated using eq. (4.3) and (3.34-38):

$$\mathbf{a}^{C1}(\omega) = \left\{\mathbf{T}_p^{C1}(\omega) + \mathbf{T}_w^{C1}(\omega)\left[(\mathbf{Z}_{ww}^{C2}(\omega) - \mathbf{Z}_{ww}^{C1}(\omega))^{-1}\mathbf{Z}_{wp}^{C1}(\omega)\right]\right\} q_p(\omega) \tag{4.4}$$

The acoustic potential energy of the second cavity can be obtained as well:

$$\mathbf{a}^{C2}(\omega) = \left\{\mathbf{T}_w^{C2}(\omega)\left[(\mathbf{Z}_{ww}^{C2}(\omega) - \mathbf{Z}_{ww}^{C1}(\omega))^{-1}\mathbf{Z}_{wp}^{C1}(\omega)\right]\right\} q_p(\omega) \tag{4.5}$$

4.3. Simulations results

Having implemented the expressions above in a *MatLab* computer code, the total acoustic potential energy in both cavities has been plotted with reference to a larger and larger number of window elements for a modelling of the window in a more and more accurate way. For instance, Figures (4.1) and (4.2) show the results obtained when the window is virtually divided into 4 or 64 rectangular elements.
The simulations carried out have shown that the estimate of the acoustic potential energy of the second cavity was a good approximation with a (8,8) grid. From now on the results presented will be those obtained for this grid.

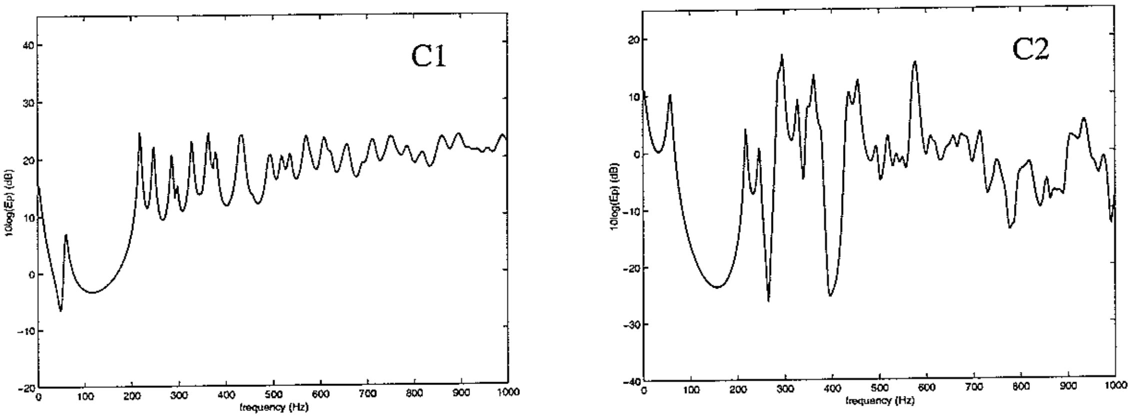

Figure 4.1: *Time averaged acoustic potential energy in cavity 1 (left) and cavity 2 (right) without control, modelling the window with (2 ,2) grid.*

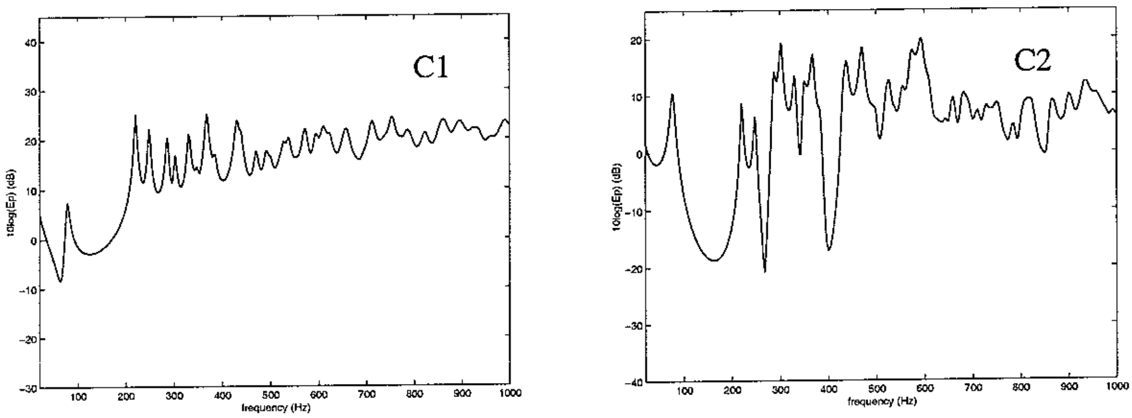

Figure 4.2: *Time averaged acoustic potential energy in cavity 1 (left) and cavity 2 (right) without control, modelling the window with (8 ,8) grid.*

Looking globally to both Figures 4.1 and 4.2, some general remarks can be made:

1. In the first cavity, for excitation frequencies under 500 Hz, a relatively large number of modes of the cavity are excited and resonance peaks of amplitude from 30 dB to15 dB are found.

2. In the second cavity, for excitation frequencies under 500 Hz, a lower number of modes seems to be excited which however have relatively high amplitudes.

3. Most of the resonance frequencies in the two cavities differs from the natural frequencies of the cavities modes (see table below) because of the coupling effect of the window which generates a new set of coupled modes. Some of these modes are quite similar to those of the two cavities considered individually with a small perturbation in correspondence of the window. Other are instead quite different to those of the two cavities when considered individually and are indeed given by the coupled acoustic field in the two rooms. .

4. Above 600 Hz the phenomenon of modal overlap takes place.

These results were expected: indeed the primary source placed in cavity 1 is exciting most of the cavity modes. In contrast the sound in cavity 2 is determined by the window which is exciting only part of the cavity 2 modes. Also it should be noted that a relatively lower frequency resonance is found at about 100 Hz. This is due to the low frequency volumetric acoustic mode of the two cavities coupled together.

Mode 1	Frequency 1 (Hz)
(1,0,0)	214.4
(0,0,1)	245.0
(0,1,0)	285.8
(1,0,1)	325.5
(1,1,0)	357.3
(0,1,1)	376.5
(2,0,0)	428.8

Mode 2	Frequency 2 (Hz)
(1,0,0)	282.8
(0,1,0)	343.0
(0,0,1)	428.8
(1,1,0)	446.5
(1,0,1)	515.3
(0,1,1)	549.1
(2,0,0)	571.7

Table 4.1: First seven natural frequencies of cavity 1 (left) and cavity 2 (right).

4.4. Validation of the model

The analysis of the passive sound transmission was useful to validate the model.

Several points have been checked such as:

- Problems have been found for the inversion of the impedance matrices $\mathbf{Z}_{ww}$ whose elements are given by a summation of modal terms as described in Section 3.1. Thus, the values of each of $\psi_{n1}(x_{wi}, y_{wi}, z_{wi})$ and $\Phi_{n2}(x_{wj}, y_{wj}, z_{wj})$ for a window in the middle of the cavities were verified for some of their natural frequencies. The results were coherent. For instance, the coefficients were even and odd with an origin at the middle of the window according to the modes considered. Indeed, after many considerations, the problem proved to be of numerical origin. The contribution to the impedance functions of the low frequency modes has been found to differ by fractions at the various centre points of the window elements. As a consequence the impedance matrix was found to be singular. The solution to the problem was to increase the number of natural modes taken into account. A number of 5000 modes is generally required for the given dimensions of the cavity and for the frequency range (0 – 1 kHz) chosen.

- The calculus of the potential energy was validated comparing simulations results to those obtained for one room in paragraph 10.10 of *Active Control and Sound* (Nelson and Elliott, 1992)[1].

At the best knowledge of the authors, the literature does not offer a similar study to that presented in this reported. So there is no comparison possible to compare and validate the model developed and here presented. The method used for the coupling is the decomposition of the opening in some small elements. An integral method has been presented by Morse and Ingard (1987) but the special case of considering a set of control sensors-actuators makes it to be difficult to be implemented The technique of the decomposition in elements has the advantage of an easy manipulation and gives good results.

5. FEEDFORWARD CONTROL

5.1. Principle

The basic principle of Active Noise Control is to cancel the sound pressure at some determined error points using a set of loudspeakers that are driven by either a Single Input Single Output or a Multi Input Multi Output a feed-forward controller which uses a reference signal about the primary disturbance. Figure 5.1 give an example of this technique applied to sound control in a duct. There is a detection sensor with output signal $x(t)$, a secondary source, being driven by the signal $y(t)$, and an error sensor with output waveform $e(t)$ (Nelson and Elliott, 1992).

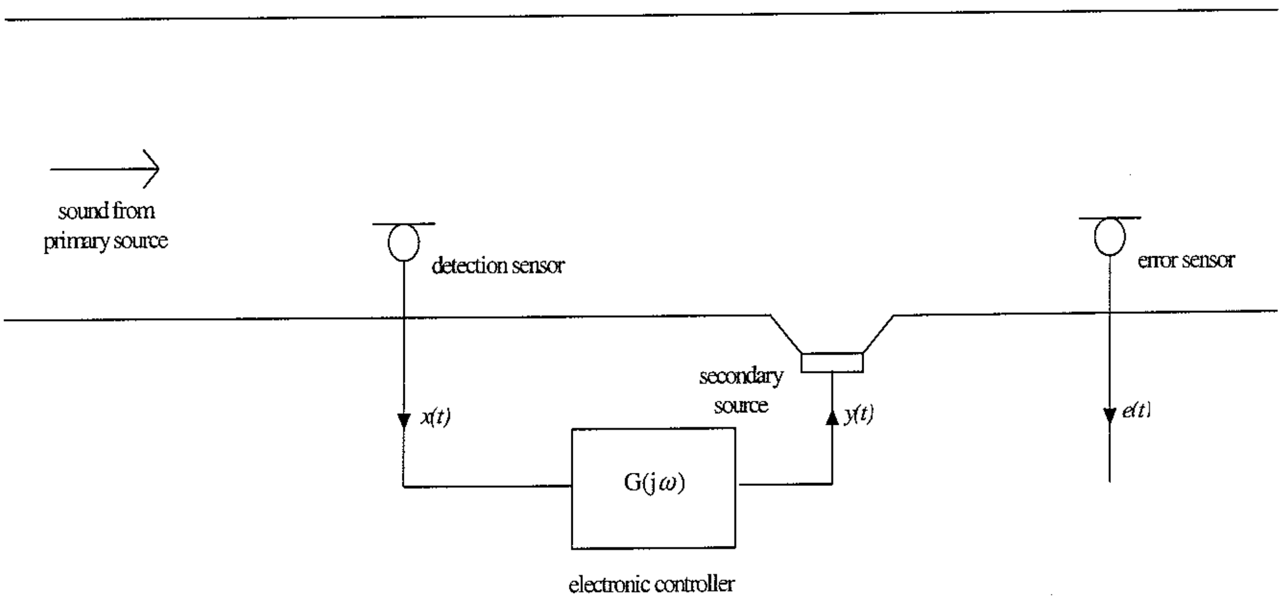

Figure 5.1: *General Physical Block Diagram of a single channel control system for the control of random noise, using an acoustic detection sensor (Nelson and Elliott, 1992).*

However for the problem under study the disturbance is generally of random type which poses several problems for the implementation of a feed forward control scheme. Random disturbances can be better controlled using feedback control schemes which however require more care at design level particularly for stability issues.

The specific case under study considers the excitation to be purely harmonic and ideal feed-forward control is implemented. The error microphones where the sound pressure is cancelled are localised on the window. But, what interest us is to diminish the global pressure in the cavity 2. Therefore the control effectiveness will be assessed by comparing the acoustic potential energy in the two rooms before and after control. The "collocation" of the sensors (microphones) and actuators (loudspeakers) should then enable the implementation of a feedback control system which would enable the control of both tonal and random disturbances.

5.2. Configurations

A limited number of configurations is studied in this report, as shown in Figure (5.2) below:

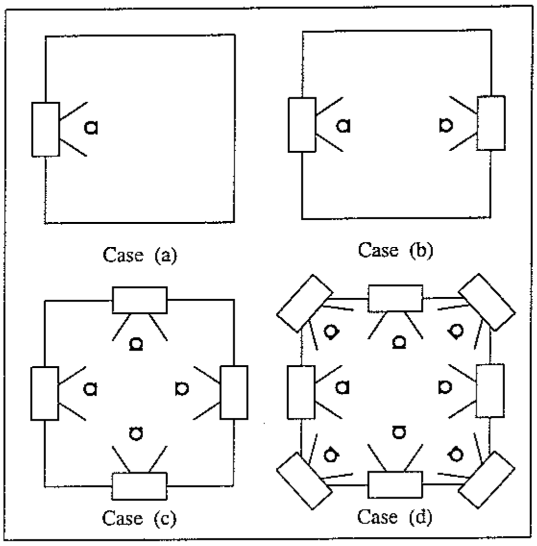

Figure 5.2: *Positions of loudspeakers and microphones at the window.*

To simplify the formulation for the calculus of the optimal secondary sources strengths, the number of secondary sources and error sensors will be assumed to be equal.

5.3. Mathematics

At the window, pressure is given by eq (2.7) and (2.8). Since, the pressure and velocity are continuous across the window, we can determine $\mathbf{v}_w$ in function of q_p and $\mathbf{q}_s$ as follows:

$$\mathbf{v}_w = \mathbf{v}_{wp}\, q_p + \mathbf{V}_{ws}\, \mathbf{q}_s \tag{5.1}$$

with

$$\mathbf{V}_{ws} = (\mathbf{Z}_{ww}^{C2} - \mathbf{Z}_{ww}^{C1})^{-1}(\mathbf{Z}_{ws}^{C1} - \mathbf{Z}_{ws}^{C2}) \tag{5.2}$$

$$\mathbf{v}_{wp} = (\mathbf{Z}_{ww}^{C2} - \mathbf{Z}_{ww}^{C1})^{-1}\mathbf{z}_{wp}^{C1} \tag{5.3}$$

The optimal control sources are then found by imposing the sound pressure at the error sensor positions to be zero so that:

$$\mathbf{q}_s = -((\mathbf{Z}_{ew}^{C2} + \mathbf{Z}_{ew}^{C1})\mathbf{V}_{ws} + \mathbf{Z}_{es}^{C2} + \mathbf{Z}_{es}^{C1})^{-1}(\mathbf{z}_{ep}^{C1} + (\mathbf{Z}_{ew}^{C2} + \mathbf{Z}_{ew}^{C1})\mathbf{v}_{wp})q_p . \qquad (5.4)$$

Finally, using eq. (5.1) and (5.5), it is possible to express $\mathbf{v}_w$ as a function of q_p.

We can calculate the value of the pressure at every point $(x_m^{C1}, y_m^{C1}, z_m^{C1})$ in the cavity 1 and $(x_m^{C2}, y_m^{C2}, z_m^{C2})$ in the cavity 2 with eq. (3.23) and (3.24).

We can also plot the total energy in both cavities and evaluate the influence of the number of secondary sources at the window. The expression of the acoustic energy in the first cavity is obtained replacing $\mathbf{q}_s$ and $\mathbf{v}_w$ by their expressions in eq. (3.35).

5.4. Simulations results

The following figures present the estimate of the total acoustic potential energy with and without control for 1, 2, 4 and 8 control units.

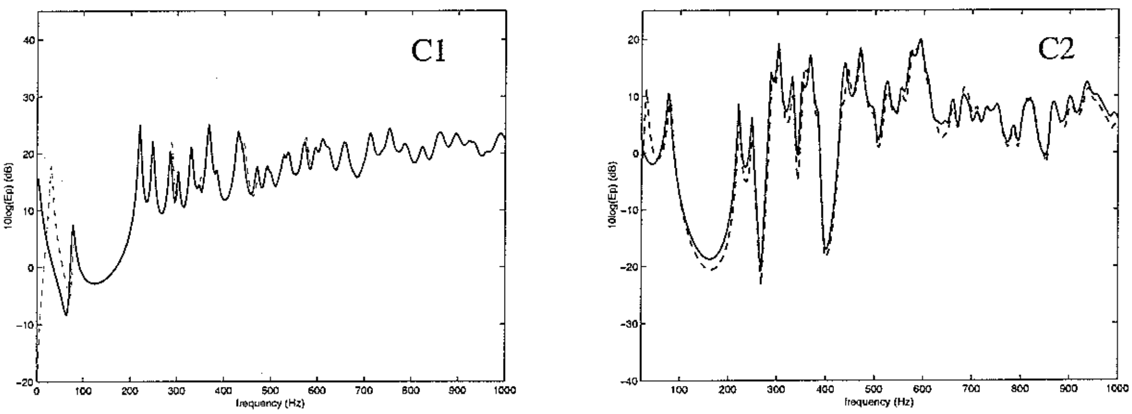

Figure 5.3: Estimate of the acoustic potential energy in cavity 1 (left) and cavity 2 (right) without control (solid line) and with control (dash line) for 1 loudspeaker (case (a)).

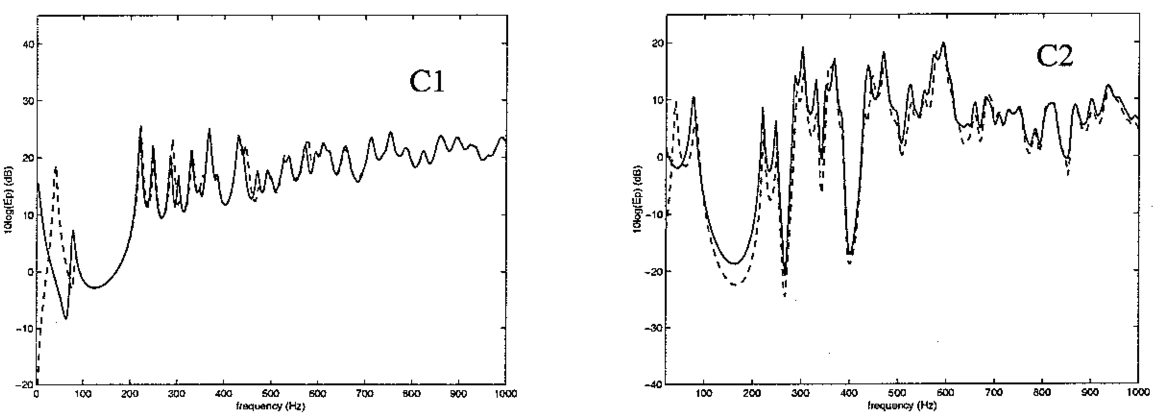

Figure 5.4: *Estimate of the acoustic potential energy in cavity 1 (left) and cavity 2 (right) without control (solid line) and with control (dash line) for 2 loudspeakers (case (b)).*

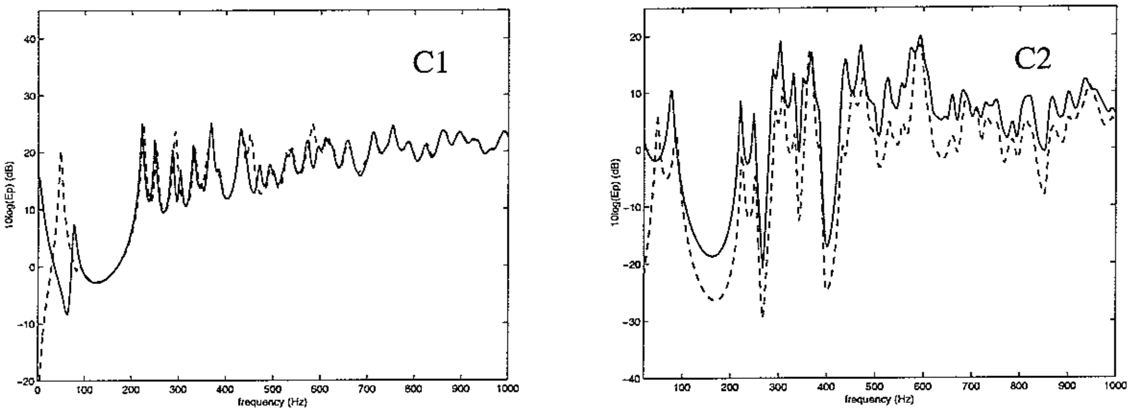

Figure 5.5: *Estimate of the acoustic potential energy in cavity 1 (left) and cavity 2 (right) without control (solid line) and with control (dash line) for 4 loudspeakers (case (c)).*

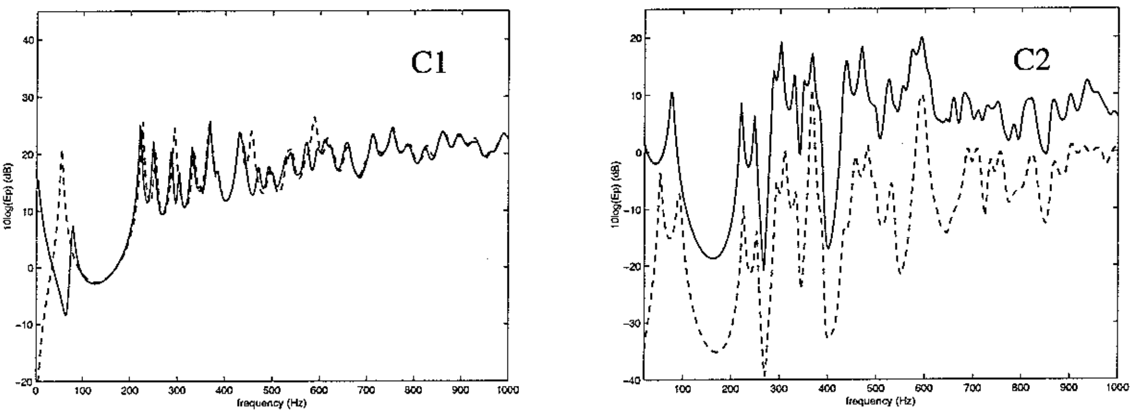

Figure 5.6: *Estimate of the acoustic potential energy in cavity 1 (left) and cavity 2 (right) without control (solid line) and with control (dash line) for 8 loudspeakers (case* (d)).

5.5. Analysis

1. For cavity 1, the potential acoustic energy without or with control is almost the same for any number of control units as it can be seen in Figure 5.3-6. The control does not affect the first room.

2. Figure 5.3, 5.4 and 5.5 show the acoustic potential energy before and after control with references to one, two and four loudspeakers.
For four loudspeakers an average reduction of 5 dB is achieved. More generally, the effects of active control are visible at low frequencies but the best reduction is 10dB less. To have an efficient control, a great number of loudspeakers is necessary.

3. Figure 5.6 shows that with eight loudspeakers, the average decrease obtained reaches 10-15dB.

4. After 600 Hz, the control is not efficient. This is not surprising. Active control techniques in enclosures are potentially most effective at low frequencies specifically for frequencies at which the modal overlap is small as asserted by Nelson and Elliott (1992).

6. CONCLUSIONS

The study presented is concerned with the transmission of sound through an opening connecting two rooms. In order to achieve the reduction of noise in the second cavity a control system with an equal number of loudspeakers and error microphones placed along the perimeter of the window has been used.

This report introduces the theory of an impedance matrix model used to predict the sound transmission between the two cavities. The cancellation of pressure at some error microphones using the control loudspeakers at the window has been investigated.

Simulations have been carried out for passive and active transmission with different numbers and configurations of control units.

The main conclusion can be summarised by the following points:

- A great number of natural modes range have to be taken into account compared to the frequency to avoid numerical problems (inversion of a singular impedance matrix),
- A decomposition of the window in an 8x8 grid of elementary pistons gives a good approximation for the control of sound,
- For an efficient control, a minimum of eight loudspeakers is required,
- The control is not affecting the first cavity.

Keeping in mind all those remarks an average diminution of 15 dB can be reached.

This work is only an introductory study. Main steps of the numerical modelling have been validated with simpler problems.

7. FUTURE WORK

The project described in the report is only the very beginning of a new project that will involve the Signal Processing and Active Control of ISVR for several years.

During this study, we have considered an ideal feed forward control system since we have assumed the exact cancellation of the pressure at the error microphones. Moreover, the dimensions of the window were much bigger than the wavelengths of excitation. Therefore, for future work, it would be necessary to adapt the theoretical study to a larger frequency range, with reference to a more realistic room and window. Eventually, it will be possible to get an idea of the true potential of such a decentralized control systems, paying attention to the modelling of sound transmission. Then, it would be interesting to extend the study to a feedback control architecture, taking into account the problems of stability of controllers.

8. REFERENCES

[1]Nelson P. and Elliott S., (1992), Active control of sound, chapters 1, 5 and 10, Academic Press.

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[3] Elliot S., Gardonio P., Sors T. and Brennan M., (2002), Active vibroacoustic control with multiple local feedback loops, *J.Acoust.Soc.Am.*, Vol.**111**, No. 2, 908-915.

[4]Morse P. M. and Ingard K. U., (1987), Theoretical Acoustics, chapter 10, Princeton University Press.