
W.H. Coghill

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The Analysis of Layered Systems
Part V: Developer Notes and User Manual

by

W.H. Cogill

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Authorised for issue by
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The analysis of layered systems.
Part V: Developer Notes and User Manual

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June 2002

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Laboratory manual Part I

Part 1 : Developer notes
Chapter 1

Part 1: Hard over soft

1.1 Summary

This report contains notes for developers and a user manual.

Chapters 1 and 2 contain notes which were used in order to develop the programs described. The purpose of the notes is to obtain an approximate expression for the thickness of the surface layer of a single-layered system. The data required are the measurements of the reciprocal wavelength over a range of frequencies. The range of frequencies should encompass wavelengths of approximately a quarter to four times the thickness of the surface layer.¹

Chapters 3 and 4 constitute a manual intended for users of the programs described. They present attempts to make theoretical matches of experimental results. The aim is to utilize theory to estimate the properties of pavement structures. We attempt to estimate both the thicknesses and the elastic moduli of the component layers of the structures.²

1.2 Statement of the problem

1.2.1 Introduction

The problem is to determine the structure of a pavement using as input the results of measurements of the wavelength of waves of the Rayleigh type and the corresponding frequencies.

¹The notation employed throughout is that used by Ewing, Jardetzky and Press². 
²For simplicity, we denote by
1. “hard” the medium having the higher stiffness, “soft” the medium having the lower stiffness
2. “Direct” and “Inverse” the systems in which the stiffnesses increase and decrease with the depth respectively
3. “Forward” and “Reverse” the calculations which lead to observed surface behaviour and the system properties - the stiffnesses and the thicknesses of the layers - respectively.
1.2. **STATEMENT OF THE PROBLEM**

CHAPTER I. **PART I: HARD OVER SOFT**

Experimental results are obtained in the form shown in Figure (1.1). This figure shows the reciprocal wavelength plotted against the frequency. The reciprocal slope of this plot yields the group velocities corresponding with the component materials.

The velocity of propagation of shear waves in the surface layer $\beta_1$ can be determined from the reciprocal of the slope of the figure at high frequencies. The quantity $\beta_2$ is the velocity of shear waves in the underlying semi-infinite medium. $\beta_2$ can be determined from the reciprocal of the slope of the figure at low frequencies. The point of intersection of the two sloping lines can be used to estimate the thickness of the surface layer.

The problem is to determine this thickness.

The problem is approached in two stages. In the first, we attempt to approximate the frequency response of a single-layered system, by expanding the transcendental functions in the frequency equation. The expansion is carried out to the first power of the argument of the transcendental functions. The work is performed with the aid of Mathematica notebooks **APP.EQH.NB** and **APP.MH.NB**.

This stage is partly successful. It can only be used if the measured phase velocity exceeds the $\alpha$-velocity in the underlying medium. Phase velocities lower than this value lead to negative square roots in the approximation to the frequency equation.

To overcome this limitation we attempt an approximation of a higher order.

### 1.2.2 The forward problem

We start with a pavement having a simplified cross section. The pavement is of the "hard-over-soft" type, i.e. a layer of stiff material overlying a subgrade composed of a material having a lower stiffness. We start with a calculated response of a layered structure, and not with the results of measurements. The result is shown in Figure (1.1). This figure shows the reciprocal wavelength of waves of the Rayleigh type plotted against the frequency.

The interpretation of the results is based on A.W.Lee's paper, a condensed version of which is given in **Rayleigh waves in a single-layered system**, [A.W.Lee.DOC](#3). This version uses modern notation. The calculations for Figure (1.1) are based on imaginary arguments for the transcendental functions. The results shown in the figure were obtained with the aid of the program **SIN13.FOR**, which is based on A.W.Lee's paper.

### 1.2.3 The reverse problem

The output of the forward program yields the theoretically expected wave velocities as a function of the frequency.

The forward program **SIN13.FOR** appears to yield a valid solution, representing the zero of the determinant of the system.\(^3\)

We hypothesize that an inverse must exist which connects the physical properties of

---

\(^3\)The residual error oscillates, positive to negative. The amplitude of the oscillation is small.
the system to the measurements performed upon it. An attempt is made to find this inverse.

Mathematica notebooks have been written in order to provide numerical analysis of layered earthen structures. The object of this series of notebooks is to generate a Fortran program with which to interpret the field measurements made on waves of the Rayleigh type. This series of notebooks is based on APP.MH.NB. The notebook APP.MH.NB attempts to inverses a layered structure consisting of a hard layer overlying a soft semi-infinite medium.

We attempt to interpret the results shown in Figure (1.1). We attempt to find the thickness of the surface layer. We use as data the measurements of reciprocal wave velocity and the corresponding frequency.

The steps toward doing this are as follows. The Mathematica notebook APP.MH.NB defines the frequency function in terms of $\eta_1$, $\eta_2$, $\xi_1$, $\xi_2$ as first-order expansions [2].

1. LEE.EQH.NB uses the previous definitions of $\eta_1$, $\eta_2$, $\xi_1$, $\xi_2$; it generates the frequency equation; it solves the frequency equation for the wave number and writes the result in Fortran form. The wave number is expressed as a fraction of the thickness of the surface layer.

2. The output from APP.MH.NB is used as input to LEE.EQH.NB which writes its output in Fortran form. The corresponding Fortran program is APP.H.FOR.

3. The Fortran program APP.H.FOR is used in order to determine the depth of the interface. The data required are the reciprocal wave velocity and the corresponding frequency.

The logical sequence of these notebooks is: APP.MH.NB, LEE.EQH.NB, LEE11.NB, LEE2.NB, LEE21.NB, REVERSE.NB. The final output is REVERSE.RES, which is in Fortran form.

1.3 Notebook APP.MH.NB

The following definition is developed from A.W. Lee, "The effect of geological structure upon microseismic disturbance," Monthly Notices of the Royal Astronomical Society: Geophysical Supplement, Vol. 3, 1932, pp. 83-105, equation (20). See A.W.LEE.DOC for a condensed version in modern notation[3]. It gives an approximation to the determinant for the case $\lambda_1 = \mu_1$ and $\lambda_2 = \mu_2$, with the transcendental functions expanded to the first power of their arguments. The result is used to estimate the thickness of a surface layer from pairs of readings of reciprocal wavelength and the corresponding frequency. The output is indicative of equations (22) to (25) in A.W.LEE.DOC[3]. The output is used as input to LEE.EQH.NB which writes output in Fortran form. The resulting Fortran program is APP.H.FOR.

\*The suffixes .MA and .NB are interchangeable: both are capable of being executed by Mathematica version 4 and higher.
\[ In[1]:= \text{xil} = \left( 2 - \frac{\text{kbi}^2}{k^2} \right) (X + z2 Y h) - \frac{2 S}{k} \left( \frac{z2 W h + k z}{k} \right); \]

\[ \text{xil2} = \left( 2 - \frac{\text{kbi}^2}{k^2} \right) \left( \frac{s2 W + k z}{k} \right) h - \frac{2 S}{k} \left( \frac{x s h + k z^2}{k} \right); \]

\[ \text{ets1} = \left( 2 - \frac{\text{kbi}^2}{k^2} \right) \left( \frac{x2 W + k z}{k} \right) h - \frac{2 R}{k} \left( \frac{x h + k z^2 X}{k} \right); \]

\[ \text{ets2} = \left( 2 - \frac{\text{kbi}^2}{k^2} \right) (X + s2 Y h) - \frac{2 R}{k} \left( \frac{s2 W h + k z}{k} \right); \]

\[ In[2]:= \text{res} = \text{Numerator}\left[ \text{Factor}\left[ \text{Simplify}\left[ \text{xil} - \text{xil2} - \text{ets1} \right] \right] \right]; \]

\[ \text{res} = \text{res} / k \rightarrow k^2(1/2) \]

\[ In[3]:= \text{xil1} = \text{Simplify}[\text{xil}] \]
\[ \text{Out}[3]= -\frac{2 h r2 S^2 X}{k} + \frac{\left( 2 - \frac{\text{kbi}^2}{k^2} \right) (X + h r2 Y) - 2 Z}{k} \]

\[ In[4]:= \text{xil2} = \text{Together}[\text{xil1}]; \text{xil3} = \text{Apart}[\text{xil2}]; \]

The result for xil follows

\[ In[5]:= \text{xil4} = \text{Simplify}[\text{xil3} / S \rightarrow \sqrt{k^2 - \text{kbi}^2}] \]
\[ \text{Out}[5]= -2 h r2 W - \frac{2 h \text{kbi}^2 r2 W}{k^2} \left( \frac{2 - \frac{\text{kbi}^2}{k^2}}{k^2} \right) + 2 X - \frac{\text{kbi}^2 X}{k^2} + h \frac{\text{kbi}^2 r2 Y}{k^2} - 2 Z \]

\[ In[6]:= \text{xil21} = \text{Simplify}[\text{xil2}] \]
\[ \text{Out}[6]= -\frac{2 (h S^2 X + s2 Y)}{k} + \left( 2 - \frac{\text{kbi}^2}{k^2} \right) \left( \frac{s2 W + h k z}{k} \right) \]

\[ In[7]:= \text{xil22} = \text{Together}[\text{xil21}]; \text{xil23} = \text{Apart}[\text{xil22}]; \]

\[ In[8]:= \text{xil24} = \text{Simplify}[\text{xil23}]; \]

\[ In[9]:= \text{Expand}[\text{xil24} / S \rightarrow \sqrt{k^2 - \text{kbi}^2}] \]
\[ \text{Out}[9]= -\frac{2 s2 W}{k} \left( \frac{\text{kbi}^2 s2 w}{k^2} - 2 h k X + \frac{2 h \text{kbi}^2 X}{k} + 2 s2 Y - h \frac{\text{kbi}^2 Z}{k} \right) \]

\[ In[10]:= \text{ets1} = \text{Simplify}[\text{ets1} / S \rightarrow \sqrt{\text{kbi}^2 - k^2}] \]
\[ \text{Out}[10]= -\frac{2 (h R^2 X + r2 Y)}{k} + \left( 2 - \frac{\text{kbi}^2}{k^2} \right) \left( \frac{r2 X + h k z}{k} \right) \]

\[ In[11]:= \text{ets12} = \text{Simplify}[\text{ets11}] \]
\[ \text{Out}[11]= -\frac{2 (h R^2 X + r2 Y)}{k} + \left( 2 - \frac{\text{kbi}^2}{k^2} \right) \left( \frac{r2 X + h k z}{k} \right) \]

\[ In[12]:= \text{ets13} = \text{Expand}[\text{ets12} / R^2 \rightarrow k^2 - \text{kbi}^2] \]
\[ \text{Out}[12]= -\frac{2 r2 W}{k} \left( \frac{\text{kbi}^2 r2 W}{k^2} - 2 h k X + \frac{2 h \text{kbi}^2 X}{k} + 2 r2 Y - 2 h k z - h \frac{\text{kbi}^2 Z}{k} \right) \]

\[ In[13]:= \text{ets21} = \text{Simplify}[\text{ets21} / S \rightarrow \sqrt{\text{kbi}^2 - k^2}] \]
\[ \text{Out}[13]= -\frac{2 (h S^2 X + s2 Y)}{k} - \frac{h \text{kbi}^2}{k^2} - 2 s2 Y - h \frac{\text{kbi}^2 s2 Y}{k^2} \]

\[ In[14]:= \text{ets22} = \text{Simplify}[\text{ets21}]; \]

\[ In[15]:= \text{ets23} = \text{Expand}[\text{ets22} / R^2 \rightarrow k^2 - \text{kbi}^2] \]
\[ \text{Out}[15]= -\frac{2 s2 W}{k} \left( \frac{2 h \text{kbi}^2 s2 W}{3 k^2} + 2 X - \frac{\text{kbi}^2 X}{k^2}^2 + 2 h s2 Y - h \frac{\text{kbi}^2 s2 Y}{k^2} \right) \]
The Mathematica notebook APP.MH.NB yields an approximate expression for the determinant governing a single-layered system. The approximation is made by expanding the transcendental functions to the first power of their arguments.

1.4 The frequency equation: LEE.EQH.NB

The notebook which follows is LEE.EQH.NB. It yields a result in Fortran form, and the output is used as input to write the program LEE.H.FOR.

1.4.1 The notebook LEE.EQH.NB

A.W.Lee

See A.W. Lee "The effect of geological structure upon microseismic disturbance," Monthly Notices of the Royal Astronomical Society: Geophysical Supplement, Vol.3, 1932, pp. 83-105, eqn (20)[3]. The following notebook is an approximation to A.W.Lee's equation (20), the 'hard over soft' case. It generates a quadratic in the layer thickness h and is used to solve for h in terms of pairs of measurements of k and omega[3]. The following leads to a polynomial in h^2, h^-1, h^0. Its output is used as input to write the program LEE.H.FOR. The logical sequence of these files is: LEE.EQH.NB, LEE1.NB, LEE2.NB, LEE21.NB, REVERSE.NB. The output of REVERSE.NB is REVERSE.RES, which is in Fortran form.

\[
\begin{align*}
  x_{i1} &= 2 h r z (1 + W) + (2 + X); \\
  x_{i2} &= h k (2 X - 2 z) - \frac{x_2 (2 + W)}{k}; \\
  \eta_{s1} &= h k \left(\frac{2 X}{3} - 2 z\right) - \frac{x_2 (2 + W)}{k}; \\
  \eta_{s2} &= h s_2 \left(\frac{2 W}{3} - 2 X\right) + (2 - X); \\
\end{align*}
\]

\text{In}[16] := eqn = x_{i1} \cdot x_{i2} - x_{i1} \cdot \eta_{s1};
\text{In}[17] := eqn2 = Expand[eqn];
\text{In}[18] := aa = Coefficient[eqn2, h, 2];
\text{In}[19] := bb = Coefficient[eqn2, h, 1];
\text{In}[20] := Simplify[bb]
\text{In}[21] := cc = Coefficient[eqn2, h, 0];
\text{In}[22] := tmp = OpenWrite[a : tmp, FormatType \rightarrow FortranForm, PageWidth \rightarrow 70]
\text{In}[24] := aaa = FortranForm[aa];
\text{bbb} = FortranForm[bb];
\text{ccc} = FortranForm[cc];
1.5. THE PROGRAM APP.H.FOR

We solve next for the layer thickness, h, which makes the determinant $x_1^2\eta_2 - x_1^2\eta_1 = 0$

In[25] := Write[tmp, aaa = , aaa, bbb = , bbb, ccc = , ccc ];

Write[tmp, Solve[aaa h^2 + bbb h + ccc == 0 , h]];  

In[26] := Close[tmp]  

APP.H.FOR

1.5 The program APP.H.FOR

The output of LEE.EQN.NB is the file artmp. This file can be used to write a Fortran program APP.H.FOR to perform the inverse computations. The data to the Fortran program APP.H.FOR consists of the values of the reciprocal wavelength and the corresponding frequency. Pairs of these data can be read from the plotted results of experimental measurements. In the case discussed here, the data are read from the results of calculations made with the aid of SIN13.FOR, and shown in Figure (1.1).

The output obtained from the program APP.H.FOR is shown in Figure (1.2). This figure shows the calculated thickness of the surface layer, plotted using successive data pairs obtained from Figure (1.1). The surface layer was one unit in thickness. The result obtained from the program APP.H.FOR is accurate to an order of magnitude only.

1.6 The program SIN13.FOR

Figure (1.1) shows the output from SIN13.FOR. The figure shows the dimensionless form of the results of measurements for a system composed of a layer of a stiff material overlying a subgrade composed of a less stiff material. The ratio of the shear moduli of the materials is $\frac{\mu_2}{\mu_1} = 0.01$. The dimensionless wavenumber is $\frac{\pi h}{\lambda}$, and the dimensionless frequency is $\frac{c}{\beta_1} \cdot \frac{\pi h}{\lambda}$. The ratios $\frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2} = 0.4$

In order to achieve a plot which is comparable with field measurements, the axes must be scaled. The frequency axis must be multiplied by $\beta_1$, and divided by $\pi h$. The axis of the reciprocal wavelength must be divided by $\pi h$.

1.6.1 Operating SIN13.FOR

On execution, SIN13.FOR echoes "Enter a Starting value of V". It expects an initial wavelength, expressed as $V = \frac{\lambda}{\pi}$. It uses the value of V supplied by the user to start a search for a solution. If the velocity in the surface material is unity, a value of 0.4 appears to be a suitable starting point. This value of V corresponds with a wavelength of approximately the thickness of the surface layer.
However the termination condition, at which the solution is accepted for output, is not rigorous (see lines 126-130, SIN13.FOR). It does not necessitate a crossing of the zero of the determinant of the system. It requires only that the change in the value of the determinant at the final step should be small (about one hundredth or less of the initial trial value). This requirement leads to a more ready acceptance of the result at low velocities than at high velocities. At high velocities, the requirement is relatively stringent.

The effect is to produce different outputs depending on the starting value of V.
Figure 1.1: Diagram showing the dimensionless form of the results of measurements for hard over soft; Output from SIN13.FOR. The wavenumber is \( \frac{\pi h}{\lambda} \), and the frequency is \( \frac{c}{\beta_1} + \frac{\pi h}{\lambda} \). The ratios \( \frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2} = 0.4 \).

Figure 1.2: Diagram showing the reverse of the results of measurements for hard over soft; Output from APP_H.EXE. The thickness \( h \) is unity, and the frequency is \( \frac{c}{\beta_1} = \frac{\pi h}{\lambda} \).
Chapter 2

Part 1: Soft over hard

2.1 Natural earthen systems

The following notebooks attempt to solve the problem (Section 1.2) for normal surface earthen systems. In these systems, the soft layer is at the surface. The underlying medium is a hard material. This is the normal progression in earthen systems.

2.1.1 The case of “soft over hard” - the normal geophysical progression

The case of “soft over hard” is the structure which is normally encountered in the field. Waves having a length shorter than the thickness of the surface layer yield information concerning the surface layer. Longer waves are generated by low frequencies. The results obtained indicate the properties of the underlying medium.

Two notebooks are utilized: LEE1.NB and LEE2.NB. The notebook LEE1.NB writes the series representing the determinant of the system in \( h^p \), \( p = 1..6 \), where \( h \) is the thickness of the surface layer.\(^1\) The output of LEE1.NB is atlee1tmp, which is read as input by LEE2.NB. LEE2.NB writes the coefficients of the series for the determinant. The output from LEE2.NB is LEE2.RES. This file contains the coefficients of the series representing the determinant.

The coefficients are in a form which is suitable for reversion. The constant term is zero, and the coefficient of the term in \( h^1 \) is unity.

The Mathematica program REVERSE.NB yields an output REVERSE.RES in Fortran form. This output in turn yields a Fortran program REW202.FOR\(^2\) and REV9x.F90. The reversion was carried out to terms in \( h \). The series obtained is either divergent or so slowly convergent as to be unusable.

---

\(^1\)Each of the expanded series of \( \xi_1, \xi_2, \eta_1, \eta_2 \) contains \( h^2 \); the product \( \xi_1 \eta_2 - \xi_2 \eta_1 \) contains \( h^6 \).

2.1.2 The program EW202.FOR

The example shown in Figure (2.2) is a forward calculation performed with the aid of EW202.FOR.

In Figure (2.2), the data used as input to the program EW202.FOR are as follows.

The values of $\beta_2/\alpha_3$, $\alpha_1/\alpha_2$, $\beta_2/\beta_1$, and $\mu_2/\mu_1$, which are coded as B1A1, A1A2, B2A2, XMU2M1, are the first line of the data input. The subsequent lines, eight values per line, are the values of $c/\alpha_1$ for which calculations are required. The figures "999" in the final field specify a continuation line; a zero "0" specifies the final line of input.

\[
\begin{array}{cccccc}
0.4 & 0.1 & 0.4 & 100 \\
1.1 & 1.3 & 1.5 & 1.7 & 1.9 & 2.1 & 2.3 & 999 \\
2.6 & 2.9 & 3.1 & 3.3 & 3.5 & 3.53 & 3.57 & 999 \\
3.6 & 3.63 & 3.66 & 3.69 & 3.72 & 3.74 & 3.77 & 0 \\
\end{array}
\]

The program EW202.FOR reads the velocity as a fraction of $c/\alpha_1$. $c$ must exceed $\alpha_1$ and must be less than $\beta_2$. The corresponding wavelength is calculated within the program EW202.FOR as $V = \lambda/\pi$. The output of the reciprocal wavelength is $\lambda/\pi$, which is $\pi$ times the actual reciprocal wavelength in the system, measured in reciprocal meters.

The frequency output is $c/\beta_1 + \pi h/\lambda$. It is thus $\pi h/\beta_1$ times the actual frequency in the system measured in Herz. The thickness $h$ of the surface layer is unity.

2.1.3 Expansion of the determinant in $h$

The determinant, given by the notebook LEE1.NB, is expanded in powers of $h$. The coefficients are $[a(i), i = 1..power]$. The independent variable of the reversion is $z = -a[0] / a[1]$. The remaining coefficients $[a(i), i = 2..power]$ are all divided by $a[1]$.

A generalized series is then produced and reversed in notebook REVERSE.NB. The output is in REVERSE.RES. The output contains the coefficients of the reversion and the reversion itself, in terms of its coefficients.

The output of LEE1.NB is LEE1TMPDAT. LEE1TMPDAT was used as input to LEE2.NB. The output of LEE2.NB is LEE2.RES and is in Fortran form.

The program was tested using the data shown in Figure (2.2), a system composed of a layer having a thickness of unity overlying a semi-infinite medium.

2.1.3.1 Expansion to the power of three in $h$

Expansion was carried out to the power three in $h$. The result was that $a[7], a[8], a[9]$ are all equal to zero. The terms in the reversed series are large but defined. The program is EW183.F90.\footnote{The source language of this program EW183.F90 and of the programs REV96.F90 and REV99.F90 are not included as they are of small significance. They are generated from LEE2.RES which is in Fortran form.}

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The series converges, at least for some values of frequency and reciprocal wavelength (the values 0.13 0.00073 lead to a value of 0.9 for $< h >$)

2.1.3.2 Expansion to the power of six in $h$

Expansion was carried out to the power of six in $h$. The program is REV96.F90. The result was that, the independent variable of the reversion, was defined. All other terms, up to an including the sixth power of $h$, were defined. The series converges, at least for some values of frequency and reciprocal wavelength (the values 0.2 0.00238 lead to a value of 0.91 for $< h >$).

2.1.3.3 Expansion to the power of nine in $h$

Expansion was carried out to the power of nine in $h$. The program is REV99.F90. The series converges, at least for some values of frequency and reciprocal wavelength (the values 0.2 0.00238 lead to a value of 0.89 for $< h >$).

2.1.4 Mathematica notebook LEE1.NB

In[27]:=

(* LEE1.NB : The following is a definition of the period equation (4-202) in Ewing, Jardetzky and Press, page 193. It leads to a polynomial 'f' in h to the power of six. Use Coefficient[f, h, n], to extract the coefficients, see page 791 of the Mathematica manual, Version 4*)

\[
x_1 = ((-k_1^2/2) (xx \cos[w r_1 h] + x_2/x_1 yy \sin[w r_1 h])
+ 2 s_1/k (z_2/k w w \sin[w s_1 h] - k/s_1 z z \cos[w s_1 h]));
\]

\[
x_2 = ((-k_1^2/2) (s_2/k w w \cos[w r_1 h] + k/s_1 z z \sin[w r_1 h])
+ 2 s_1/k (xx \sin[w s_1 h] - s_2/s_1 y y \cos[w s_1 h]));
\]

In[28]:=

\[
e_1 = ((-k_1^2/2) (z_2/k w w \cos[w s_1 h] + k/s_1 z z \sin[w s_1 h])
+ 2 r_1/k (xx \sin[w r_1 h] - r_2/r_1 y y \cos[w r_1 h]));
\]

\[
e_2 = ((-k_1^2/2) (xx \cos[w s_1 h] + s_2/s_1 y y \sin[w s_1 h])
+ 2 r_1/k (s_2/k w w \sin[w r_1 h] - k/s_1 z z \cos[w r_1 h]));
\]
2.1. NATURAL EARTHEN SYSTEMS

\text{CHAPTER 2. PART I: SOFT OVER HARD}

\texttt{In[29]} :=
\texttt{\{k = w / c; kbl = w / b1; kb2 = w / b2; \}}
\texttt{r1 = \{ - 1/c^2 + 1/a1^2 \} ^ (1/2); s1 = \{ - 1/c^2 + 1/b1^2 \} ^ (1/2); r2 = \{ - 1/c^2 + 1/a2^2 \} ^ (1/2); s2 = \{ - 1/c^2 + 1/b2^2 \} ^ (1/2); \}}
\texttt{(* Expand each of the elements in a series, and simplify *)}
\texttt{xil = Factor[Normal[Series[x1, {h, 0, 3}]]]; xil2 = Factor[Normal[Series[x2, {h, 0, 3}]]];}
\texttt{In[30]} :=
\texttt{et1 = Factor[Normal[Series[e1, {h, 0, 3}]]]; eta2 = Factor[Normal[Series[e2, {h, 0, 3}]]];}
\texttt{In[31]} :=
\texttt{(* Now write the determinant, to be solved for the thickness, h *)}
\texttt{f = xil eta2 - xil2 eta1; \{ EJP's equation 4-202 \}}
\texttt{y = Simplify[Factor\{ f \}]; \{ Prepare the polynomial to extract \}}
\texttt{(* the coefficients; 'y' is now a polynomial, and the coefficients \}}
\texttt{of h^n can be extracted by a Coefficient\{y,h,n\} command *)}
\texttt{stmp = OpenWrite\{ "a:lee1tmp\" \}; \{ Open a file to receive \}}
\texttt{intermediate output\} \texttt{Write\{ stmp, y \}; \{ Write 'y' to A:LEE1TMP as input to LEE2.M \}}
\texttt{Close\{ stmp \}; \{ Close the file \}}
\texttt{(* Definition of the ancilliary variables follows *)}
\texttt{ww = 2 \{ nu2/nu1 - 1 \}; xx = nu2/nu1 kb2^2/k^2 - ww; yy = kb1^2/k^2 + ww; zz = nu2/nu1 kb2^2/k^2 - kb1^2 /k^2 - ww; \}}
2.1.5 Generation of the coefficients for the reversion: Mathematica notebook LEE2.NB

\( In[32] := \)

(* File LEE2.NB: Continuation of LEE1.NB the expansion
of Ewing Jarzetsky and Press eqn (4-202) *)

\( y = <<a:lee1tmp; (* Read in the file LEE1TMP, y1 from LEE1.NB *) \)
\( z = -\text{Coefficient}[y,h,0]/\text{Coefficient}[y,h,1]; (* Independent variable *) \)

\( y1 = y - \text{Coefficient}[y,h,0]; (* Subtract constant term from \)
series, and *)

\( y2 = y1/\text{Coefficient}[y2,h,1]; (* arrange that h'1 has a coefficient \)
of unity.*)

\( a[1] = \text{Simplify}[\text{Coefficient}[y2,h,1]]; (* Check that h'1 has a \)
coefficient of unity.*)

\( a[0] = \text{Coefficient}[y2,h,0]; (* Check that the constant term is \)
zero.*)

\( a[2] = \text{Simplify}[\text{Coefficient}[y2,h,2]]; (* Prepare coefficients *) \)
\( a[3] = \text{Simplify}[\text{Coefficient}[y2,h,3]]; (* for *) \)
\( a[4] = \text{Simplify}[\text{Coefficient}[y2,h,4]]; (* reversion *) \)
\( a[5] = \text{Simplify}[\text{Coefficient}[y2,h,5]]; (* of *) \)
\( a[6] = \text{Simplify}[\text{Coefficient}[y2,h,6]]; (* series *) \)

\( \text{stmp} = \text{OpenWrite}["a:lee2.res"]; \)

\( \text{Write}[\text{stmp}, \text{ " z = ", FortranForm[z],} \)
\( \text{ "a[0] =", FortranForm[a[0]],} \)
\( \text{ "a[1] =", FortranForm[a[1]],} \)
\( \text{ "a[2] =", FortranForm[a[2]],} \)
\( \text{ "a[3] =", FortranForm[a[3]],} \)
\( \text{ "a[4] =", FortranForm[a[4]],} \)
\( \text{ "a[5] =", FortranForm[a[5]],} \)
\( \text{ "a[6] =", FortranForm[a[6]] ;} \)

\( \text{Close[\text{stmp]}] \)

\( \text{Out[32]} = 1 \)
\( \text{Out[32]} = 0 \)
\( \text{Out[32]} = \text{a : lee2.res} \)

\( LEE21.NB \)

2.1.6 Notebook LEE21.NB

The output of the coefficients of the series to be reversed can be obtained with the aid of Notebook LEE21.NB.
2.2 LEE21.NB

Output of the coefficients of the reversed series: Mathematica notebook LEE21.NB

```mathematica
In[33]:=

    tmp = OpenWrite["a:lee.dat"];
    Write[tmp, " z = ", x,
            "a[0] =", a[0],
            "a[1] =", a[1],
            "a[2] =", a[2], (* Write results *)
            "a[3] =", a[3], (* results *)
            "a[4] =", a[4],
            "a[5] =", a[5],
    Close[tmp]
```

2.3 Series for the thickness \( h \) in terms of \( c \) and \( \omega \).

REVERSE.NB

The notebook REVERSE.NB generates a series, then finds the reverse of the series. The \( a(i), i = 1..6 \) are the coefficients of the forward series. This file includes an independent variable of the reversion, \( p \). Then the coefficients of the reversed series \( b(i), i = 1..6 \) are written in terms of \( a(i), i = 1..6 \). The results are written to REVERSE.RES.

The coefficients \( a(i), i = 1..6 \) can be read from LEE2.RES. REVERSE.NB performs the same operation on its input series as that performed on the output series in LEE2.NB.
2.4 Series reversion: Notebook REVERSE.NB

```mathematica
(* REVERSE.NB Mathematica program to generate the reversed series of a polynomial having defined coefficients. The program reverts a series having coefficients a[i]. The coefficients of the reversion are b[i]. The coefficients of the reversion and the reversion itself are written to a file A:REVERSE.RES *)

y3 = Sum[{a[i] x^-i}, {i, 0, 7}]; (* Write a series with defined coeffs *)

y2 = Expand[Simplify[(y3[[1]] - a[0])/a[1]]]; (* Set zero and unity coeffs *)

z = - a[0]/a[1]; (* Independent variable of the reversion *)

yl = Series[y2, {x, 0, 7}]; (*Set the series form for the reversion *)

y = Normal[InverseSeries[yl, x]]; (* Carry out the reversion *)

b[1] = Simplify[Coefficient[y, p, 1]]; (* Extract the coefficients *)

b[2] = Simplify[Coefficient[y, p, 2]]; (* *)

b[3] = Simplify[Coefficient[y, p, 3]]; (* of the *)

b[4] = Simplify[Coefficient[y, p, 4]]; (* reversed *)

b[5] = Simplify[Coefficient[y, p, 5]]; (* *)

b[6] = Simplify[Coefficient[y, p, 6]]; (* series *)

w = Sum[b[i] p^i, {i, 1, 6}]; (* Form the output series *)

*) tmp = OpenWrite["a:reverse.rep"];

Write[tmp, "p = ", x , (* Define p *)

"b[1] =", ForForm[b[1]], (* Write *)

"b[2] =", ForForm[b[2]], (* the *)

"b[3] =", ForForm[b[3]], (* results *)

"b[4] =", ForForm[b[4]], (* to a Fortran file *)

"b[5] =", ForForm[b[5]], (* and *)

"b[6] =", ForForm[b[6]] ]; (* close the file *)

Write[tmp, " w = ", ForForm[w ]]; (* Write the output to a file *)

Close[tmp]
```

figures (1.1) and (1.2) indicate the results obtained from a single-layered system, of the "hard-over-soft" type. Figure (1.1) is the output from SIN13.FOR. It shows the reciprocal of the wavelength plotted against the frequency, for a system consisting of a hard layer of unit thickness overlying a soft semi-infinite medium.

Figure (1.2) shows the results obtained with the aid of the Fortran program APP_H.FOR. The input to APP_H.FOR consists of the data pairs read from Figure (1.1). The result is an approximation to the thickness of the surface layer.
2.4. SERIES REVERSION: NOTEBOOK REVERSE MRT I: SOFT OVER HARD

Figure 2.1: Diagram showing the dimensionless form of the results of measurements for hard over soft; Output from SIN13.FOR. The wavenumber is \( \frac{\pi h}{\lambda} \), and the frequency is \( \frac{c}{\beta_1} = \frac{\pi h}{\lambda} \). The ratios \( \frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2} = 0.4 \)

Figure 2.2: Diagram showing the dimensionless form of the results of measurements for soft over hard; Output from EW202.FOR. The wavenumber is \( \frac{\pi h}{\lambda} \), and the frequency is \( \frac{c}{\beta_1} = \frac{\pi h}{\lambda} \). The ratios \( \frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2} = 0.4 \)
Laboratory manual Part II

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Chapter 3

Part 2: Theoretical Match with Experimental Results

3.1 Maple worksheets hardk.mws and softk.mws

The following is a series of outputs obtained with the aim of matching experimental results to theoretical predictions.

![Plot of reciprocal wavelength k vs frequency omega for hard over soft](image)

Figure 3.1: Implicit Plot from hardk.mws: a stiff layer overlying a less stiff semi-infinite medium.

The two figures (3.1) and (3.2) were plotted with the aid of Maple version 7. The figures show the reciprocal wavelength $k$ plotted against the angular frequency $\omega$. The programs were hardk.mws (see section 5.8 on page 83) and softk.mws (see section 5.9 on page 85). The programs hardk.mws and softk.mws were developed from A.W.Lee, and are based on Ewing, Jardetzky and Press equation (4-202)[2].

The system consists of a single layer overlying a semi-infinite medium. In Figure (3.1), the surface layer has the higher modulus. In Figure (3.2), the surface layer has the lower modulus.
Figure 3.2: Implicit Plot from softk.mws:a less stiff layer overlying a more stiff semi-infinite medium.

The scale of the reciprocal wavelength in figures (3.1) and (3.2) is \( \frac{2\pi}{\lambda} \) where \( \lambda \) is in units of the thickness of the layer. The scale of frequency \( \omega = 2\pi f \) where \( f \) is the frequency in Herz.

The shear-moduli \( \mu_1 \) and \( \mu_2 \) refer to the materials in the surface layer and the underlying medium respectively. The values of the moduli are 1.0 and 100.0, and Poisson’s ratio is equal to 0.4. The thickness of the surface layer is 0.1 units in both cases.

3.2 Hard over soft: reverse calculation

3.2.1 Scaling the output from SIN13.FOR

The results shown in Figure (3.3) were obtained with the aid of SIN13.FOR (see section 5.1 on page 53).

The output variables are the reciprocal wavelength \( V = \text{LAM} \text{BDA/PI} \) and the frequency, in the form \( CB1/V \).

The velocity of shear waves in the material composing the surface layer \( \beta_1 \) is 1300 m/s. The ratio of the shear modulus in the underlying medium to that in the surface layer \( \frac{\mu_2}{\mu_1} \) is 0.01. The thickness of the surface layer is 0.05 m. The ratio of velocities of transverse waves, \( \beta \), to the velocities of dilatational waves, \( \alpha \), is \( \frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2} = 0.4 \).

---

\(^1\)In the output from SIN13.FOR: The wavenumber is \( \frac{\pi h}{\lambda} \), and the frequency is \( \frac{c}{\beta_1} \times \frac{\pi h}{\lambda} \). The ratio
\( \frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2} = 0.4 \)
3.2.2 Routine for finding the theoretical match

The routine for finding a theoretical match with the experimental results shown in Figure (3.3) was as follows.

1. The ratio of the elastic modulus in the surface course to that in the underlying course was estimated from the plot of reciprocal wavelength against the frequency. This ratio is taken as the square of the ratio of the group velocities.\footnote{In a system composed of two media, the ratio of the elastic moduli $\frac{\mu_1}{\mu_2}$ is greater than the ratio of the group velocities.}

2. Two sets of theoretical points are shown. The two trial sets of points shown have first-order discontinuities at (1000, 2.5) assuming a thickness of 0.045m, and (1600, 3.9) assuming a thickness of 0.065m. The first-order discontinuity in the plot of theoretical results was used to scale the theoretical results to correspond with the experimental results.

3. The program SIN13.FOR was used to obtain a table of values of reciprocal wavelength against the corresponding frequency. The velocity in the surface material was assumed to be unity.

   The output from SIN13.FOR is shown in Figure (2.1). The discontinuity in the curve of the theoretical results is at (0.19, 0.55). This curve is calculated for a thickness of unity for the surface layer.

4. In the output from SIN13.FOR: The dimensionless reciprocal wavelength is $\frac{\pi h}{\lambda}$, and the dimensionless frequency is $\frac{c}{\beta_1} = \frac{\pi h}{\lambda}$. The ratios $\frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2} = 0.4$
3.2. Hard Over Soft: Reverse Calculation

The match shown is obtained as follows.

1. The **Wavenumber** output from **SIN13.FOR** is multiplied by \( \frac{1}{0.045 \pi} \) and by \( \frac{1}{0.065 \pi} \) to obtain matches for thicknesses of 0.045 meters and of 0.065 meters respectively.

2. The **Frequency** output from **SIN13.FOR** is multiplied by \( \frac{1300}{0.045 \pi} \) and \( \frac{1300}{0.065 \pi} \) to obtain matches for a structure having a surface-layer thickness of 0.045 meters and of 0.065 meters respectively.

### 3.2.3 Problems in attaining a theoretical match - Hard over soft: inverse system

#### 3.2.3.1 Hurn Airport hurn547.tex

We attempt to improve on the present match between the measurements and the expected theoretical results.

The experimental results labelled **hurn547** were obtained on a parking bay at Hurn Airport at a spacing of 547 millimeters. The results are shown in Figure (3.4). They indicate that three media are present in the system. The velocities in the media are 1300 m/s, 450 m/s and 160 m/s. We attempt first to match the results with those expected from a system consisting of two layers.

The theoretical match was reached as follows.

1. Read the velocity in the surface layer from the experimental results. It is the reciprocal of the slope of the plot of reciprocal wavelength versus frequency, measured at high frequencies.

2. Choose a value of \( h \) which most nearly matches the remainder of the experimental results.

3. Increase the contrast in the matching data, between the upper and lower layers. This is done by increasing the ratio \( \frac{\mu_1}{\mu_2} \) and the corresponding \( \frac{\alpha_1}{\alpha_2} \) in the data file. See **SIN13.FOR** (Section 5.1 page 53) for the format of the data file.

The match was obtained using **SIN13.FOR** and the following data file.

```
0.4 100.0 0.4 1.0E-4 \ \\
0.64 0.53 0.62 0.61 0.60 0.59 0.585 999. \ \\
0.58 0.37 0.56 0.555 0.55 0.546 0.54 999. \ \\
0.524 0.523 0.52 0.51 0.50 0.49 0.48 999. \ \\
0.47 0.46 0.45 0.42 0.40 0.39 0.38 999. \ \\
0.36 0.35 0.34 0.33 0.32 0.31 0.30 999. \ \\
0.290 0.285 0.280 0.275 0.270 0.265 0.250 999. \ \\
0.283 0.282 0.281 0.280 0.279 0.278 0.277 999. \ 
```
Figure 3.4: Experimental results and theoretical match assuming that the system consists of two media. Hurn airport Hurn547 \( \beta_1 = 1300 \text{ms}^{-1}, h = 0.060 \text{m} \). hurn547.dat is result of measurements; hurn547l.dat is the theoretical match.

\[
\begin{align*}
0.276 & \ 0.275 & \ 0.274 & \ 0.273 & \ 0.272 & \ 0.271 & \ 0.270 & \ 999. & \ \backslash \backslash \\
0.269 & \ 0.268 & \ 0.266 & \ 0.264 & \ 0.262 & \ 0.260 & \ 0.258 & \ 999. & \ \backslash \backslash \\
0.257 & \ 0.256 & \ 0.255 & \ 0.254 & \ 0.252 & \ 0.251 & \ 0.250 & \ 999. & \ \backslash \backslash \\
0.23 & \ 0.21 & \ 0.19 & \ 0.17 & \ 0.13 & \ 0.11 & \ 0.09 & \ \backslash \backslash \\
0.0 & \ 0.0 & \ 0.0 & \ 0.0 & \ 0.0 & \ 0.0 & \ 0.0 &
\end{align*}
\]

The thickness is calculated from the height of the first-order discontinuity in the curve, in both the experimental results and in the results of the calculation. The match was achieved by

1. multiplying the theoretical values of the wavenumber by \( \frac{1}{\pi \ 0.060} \), indicating that the thickness is 0.060 m.

2. multiplying the theoretical values of the frequency by \( 1300 \frac{1}{\pi \ 0.060} \), indicating that the velocity of Rayleigh type waves in the surface medium is 1300 m/s.

The best match achieved is shown in Figure (3.4). The match achieved for the underlying layers and for the semi-infinite medium is poor.
3.3 Soft over hard: reverse calculation

The results shown in Figure (3.5) were obtained on a parking bay at Hurn Airport. The figure shows also a theoretical match with the experimental results. The match indicates that the surface layer is approximately 64 millimeters in thickness.

The calculations for the theoretical match were performed with the aid of the program SON.FOR. The datafile SON.DAT was as follows.

```
0.4 0.101 0.4 100. \$
2.51 2.52 2.53 2.54 2.55 2.56 2.57 999. \$
2.58 2.6 2.75 2.8 2.9 3.1 3.3 999.
3.45 3.8 3.9 3.95 4.0 4.05 4.08 999. \$
4.2 4.4 5.1 5.5 6.7 7.5 7.9 999. \$
8.0 8.1 8.2 8.3 8.4 8.5 8.6 999. \$
8.7 8.8 8.9 9.0 9.1 9.15 9.18 999. \$
0.0 0.0 0.0 0.0 0.0
```

The match was achieved by placing $\beta_1 = 670 \text{ms}^{-1}$ and $h = 0.64/\pi$, $h = 0.32/\pi$, and $h = 1.28/\pi$.r
3.3.1 Problems in attaining a theoretical match - Soft over hard: Direct system

The steps taken were as follows.

1. Read the ratio of the highest to the lowest velocity from the plot of the reciprocal wavelength against the frequency.

2. This ratio, squared, gives the ratio of the shear moduli $\frac{\mu_2}{\mu_1}$.

3. Enter the value of the ratio of the shear moduli in SON.DAT Section 5.2 on page 56 or in EW202.DAT Section 5.4.1 on page 58.

4. Enter the values of the Poisson’s ratios in the surface layer and in the underlying medium in SON.DAT or in EW202.DAT.

5. Enter the value of $\frac{\alpha_1}{\alpha_2}$. This value can be arbitrarily small: it needs only to ensure that $\alpha_1 < c < \beta_2$.

6. Enter the values of the velocity ratios to be calculated, as fractions $\frac{c}{\alpha_1}$. The velocity must exceed $\alpha_1$ and be less than $\beta_2$.

There are infinitely many solutions to Ewing’s equation (4-202). The solution produced by SON.FOR or by EW202.FOR depends upon the initial value of $V = \lambda$. The numerical solution is probably easiest for $\frac{\mu_2}{\mu_2}$ large, 60 or more. For $\frac{\mu_2}{\mu_2}$ small, equal to 4 or less, the response depends upon the initial value of $V$. For $\frac{\alpha_1}{\alpha_2} = 0.1$, the first-order discontinuity in the curve is sharp for $V=1.0$, and is gradual for $V=4.0$.

3.3.2 Scaling the output from SON.FOR or EW202.FOR

The output from SON.FOR and from EW202.FOR is in dimensionless form. The reciprocal of the wavelength is in dimensionless units of $\frac{\pi h}{\lambda}$. Divide the output of dimensionless reciprocal wavelength by $\pi h$ in order to make it dimensional.

The frequency output from SON.FOR and from EW202.FOR is in dimensionless units of $\frac{c}{\beta_1} = \frac{4 h}{\lambda}$. The output from SON.FOR and from EW202.FOR must be divided by $\frac{\pi h}{\beta_1}$, in order to make it dimensional.

3.3.3 Routine to provide a theoretical match with experimental data

3.3.3.1 Burn Airport burn12.epsp

The output obtained theoretically requires scaling in order to match it with experimental results. The following procedure was followed in order to produce Figure (3.6).
CHAPTER 3. PART 2: THEORETICAL MATCH WITH EXPERIMENTAL

3.3. SOFT OVER HARD: REVERSE CALCULATION RESULTS

1. Using the first-order discontinuity of the curve of the reciprocal wavelength against the frequency, adjust the calculated reciprocal wavelength so that it matches with the experimental measurements.

2. Calculate and supply the value of \( \beta_1 \) which yields the measured frequency at this point.

3. The supplied numerical values then represent the field quantities.

Figure (3.5) shows the experimental results obtained for Hurn airport, site 1. The theoretical match shown was obtained with the aid of EW202.FOR. In the surface layer, \( \beta_1 = 800 \text{ms}^{-1} \), and the layer thickness was \( h = 0.4 \text{m} \). The value of Poisson's ratio was equal to 0.4 in both the layer and in the underlying semi-infinite medium.

![Graph showing experimental results and theoretical match.]

**Figure 3.6. Experimental results and theoretical match. Hurn Airport hurn12p \( \beta_1 = 800 \text{ms}^{-1}, h = 0.4 \text{m} \)**

The datafile `EW202.DAT` used in the program `EW202.FOR` was as follows.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>0.4</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.005</td>
<td>1.01</td>
<td>1.013</td>
<td>1.014</td>
<td>1.015</td>
<td>1.017</td>
<td>1.019</td>
<td>999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.02</td>
<td>1.04</td>
<td>1.05</td>
<td>1.06</td>
<td>1.07</td>
<td>1.08</td>
<td>1.09</td>
<td>999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
<td>1.6</td>
<td>1.7</td>
<td>999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td>1.82</td>
<td>1.84</td>
<td>1.86</td>
<td>1.88</td>
<td>1.93</td>
<td>2.0</td>
<td>999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>2.1</td>
<td>2.15</td>
<td>2.2</td>
<td>2.28</td>
<td>2.3</td>
<td>2.32</td>
<td>999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.33</td>
<td>2.34</td>
<td>2.35</td>
<td>2.36</td>
<td>2.37</td>
<td>2.38</td>
<td>2.40</td>
<td>999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.6</td>
<td>2.8</td>
<td>3.0</td>
<td>3.4</td>
<td>3.6</td>
<td>3.7</td>
<td>3.8</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The starting point for the iteration was \( V = 1 \). The ratio of the moduli was obtained as the square of the ratio of the extreme velocities, \( \left( \frac{2010}{670} \right)^2 = 9 \). Note that the datafile contains the unrealistic value for \( \frac{\alpha_1}{\alpha_2} = 0.1 \). A realistic value would be \( \frac{1}{3} \).

This indicates an error in the assumption concerning the ratio of the velocities \( \frac{\alpha_1}{\alpha_2} \): this ratio is not equal to the ratio of the group velocities.

---

3The first-order discontinuity is not sharp in this case. It is sharper as the ratio \( \frac{\mu_2}{\mu_1} \) is decreased to about 0.1. See the next example.
3.3.3.2 Hurn Airport hurn13.txt

The attempt to match the experimental results with theoretical calculations was continued. Figure (3.7) shows the result obtained using the program EW202.FOR with the following data file

\[
\begin{array}{cccccccc}
0.4 & 0.1 & 0.4 & 100 \\
1.1 & 1.3 & 1.5 & 1.7 & 1.9 & 2.1 & 2.3 & 999 \\
2.6 & 2.9 & 3.1 & 3.3 & 3.5 & 3.53 & 3.57 & 999 \\
3.6 & 3.63 & 3.66 & 3.69 & 3.72 & 3.74 & 3.77 & 0 \\
\end{array}
\]

The theoretical match of the experimental results shown in Figure (3.7) was obtained by scaling the theoretical results shown in Figure (2.2).

3.3.3.3 Details of the scaling process

The velocity in metres per second at high frequencies is 570 m/s. It was obtained from the plot of the experimental results, showing the wavenumber plotted against the frequency, Figure (3.7). It is the reciprocal of the slope of this plot, at high frequencies.

The starting point for the iteration in the program EW202.FOR was \( V = 1 \). The output from EW202.FOR was scaled as follows. The frequency was multiplied by \( \frac{570 \times 0.15}{0.08} \) and the wavenumber was multiplied by \( \frac{0.15}{0.08} \). The values 0.15 and 0.08 are the wavenumber ordinates of the first-order discontinuity in the curve, for the experimental results and for the theoretical match obtained from EW202.FOR shown in Figure (2.2) respectively.

3.4 Conclusion

The aim of this chapter is to provide details of the process of scaling theoretical results. The operations are

1. Estimate the group velocities in the surface and the underlying media, with the aid of a plot of the experimental measurements of the reciprocal wavelength against the frequency.

2. Use the co-ordinates of the first-order discontinuity in the plot of the experimental measurements to estimate the depth of the interface between the surface layer and the underlying semi-infinite medium.
Figure 3.7: Experimental results and theoretical match. Hurn airport Hurn13p
\( \beta_1 = 570\text{ms}^{-1}, h = 0.6\text{m} \)
Chapter 4

Part 2: Field Results

4.1 Pavement structure

The general pavement structure is more complex than those considered so far.

The stiffnesses of the materials within the layers can be in any order. The material having the highest elastic modulus may not be that in the top layer.

The results of the field measurements are matched with those expected theoretically from waves of the Rayleigh type. The solution is performed with the aid of the theory of Rayleigh waves [5]. The solution is not unique, as the component matrices contain trigonometric functions. In finding a theoretical match for experimental results, it is attempted to obtain the solution corresponding with the first mode. This is the mode having the lowest frequencies, and is probably the mode which is generated in the field.

In conclusion, suggestions are made on how to avoid mode-skipping during the calculations, and to concentrate on output relating to the first or any other selected mode.

4.2 Results of measurements

The results of experimental measurements made on two structures follow. The structures were of the general type, having stiffnesses in any order of depth. We make attempts to find matching theoretical structures.

4.2.1 LCPC results

Measurements were performed on a major road by the Laboratoire Central des Ponts et Chausées (LCPC) [1]. The results of the measurements are shown in Table 4.1.

The measurements of the phase velocity made over a range of frequencies were plotted. An attempt was made to match the results of the measurements with theoretical predictions. See Figure (4.2.1). This figure shows the phase velocity plotted against the wavelength. It shows also the theoretical prediction obtained with the aid of the
4.2. RESULTS OF MEASUREMENTS  
CHAPTER 4. PART 2: FIELD RESULTS

<table>
<thead>
<tr>
<th>Velocity(m/s)</th>
<th>Wavelength(m)</th>
<th>Frequency(Hz)</th>
<th>Recip w'length(1/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1320</td>
<td>0.2</td>
<td>6600</td>
<td>5</td>
</tr>
<tr>
<td>1350</td>
<td>0.25</td>
<td>5400.00</td>
<td>4.00</td>
</tr>
<tr>
<td>1370</td>
<td>0.4</td>
<td>3423.00</td>
<td>2.50</td>
</tr>
<tr>
<td>1300</td>
<td>0.65</td>
<td>2000.00</td>
<td>1.54</td>
</tr>
<tr>
<td>1085</td>
<td>0.75</td>
<td>1446.67</td>
<td>1.33</td>
</tr>
<tr>
<td>1120</td>
<td>1.1</td>
<td>1018.18</td>
<td>0.91</td>
</tr>
<tr>
<td>1040</td>
<td>1.25</td>
<td>832.00</td>
<td>0.80</td>
</tr>
<tr>
<td>900</td>
<td>1.8</td>
<td>500.00</td>
<td>0.56</td>
</tr>
<tr>
<td>850</td>
<td>2.15</td>
<td>395.35</td>
<td>0.47</td>
</tr>
<tr>
<td>840</td>
<td>2.8</td>
<td>300.00</td>
<td>0.36</td>
</tr>
<tr>
<td>790</td>
<td>2.6</td>
<td>303.85</td>
<td>0.38</td>
</tr>
<tr>
<td>700</td>
<td>3.05</td>
<td>229.51</td>
<td>0.33</td>
</tr>
<tr>
<td>560</td>
<td>2.8</td>
<td>200.00</td>
<td>0.36</td>
</tr>
<tr>
<td>500</td>
<td>2.95</td>
<td>169.49</td>
<td>0.34</td>
</tr>
<tr>
<td>460</td>
<td>3.25</td>
<td>141.54</td>
<td>0.31</td>
</tr>
<tr>
<td>410</td>
<td>4.6</td>
<td>89.13</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 4.1: Table giving the results of measurements from a pavement structure on Route A4 Auxerre-Lyon

programs PREPFOR and RAYFOR.

4.2.1.1 Theoretical prediction

The theoretical prediction was made using the data obtained concerning the properties of the structure. The elastic moduli used were those measured by static means. The values of the moduli may not be correct in a situation which involves wave propagation.

The data file used in the program PREPFOR was as follows.

```
LCPC DATA
E1=11200, E2=28500, E3=200; H1=0.1, H2=0.4;
NU1=NU3=0.25, NU2=0.4 3
11200 .25 28500 .4 200 .3 0.0
  2.  2.  2.
  0.1 0.4
(5.0.0) (0.05.0.00) (0.1.0.01) 0.05 0.1
```

The parameters used differ from those given by Bonitzer and Leger[1]. The values used by Bonitzer and Leger are shown in Table 4.2. Table 4.2 shows also the values used in Figure (4.2.1) and Figure (4.1).

The scatter of the experimental points in Figure (4.2.1) may be explained by the results shown in Figure (3.3). The results shown in Figure (3.3) were obtained automatically, by means of a programmed routine. The results shown in Figure (4.2.1) were obtained manually and depend upon the judgement of the operator.

The results showing the reciprocal of the wavelength plotted against the frequency are
Table 4.2: Table showing velocities and thicknesses used in order to match the experimental results with theoretical calculations.

<table>
<thead>
<tr>
<th>Bonitzer and Leger</th>
<th>Figure 4.2.1 and Figure 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1 = 1320ms$</td>
<td>$\beta_1 = 1020ms$</td>
</tr>
<tr>
<td>$\beta_2 = 1415ms$</td>
<td>$\beta_2 = 1300ms$</td>
</tr>
<tr>
<td>$\beta_3 = 400ms$</td>
<td>$\beta_3 = 400ms$</td>
</tr>
<tr>
<td>$h_1 = 0.2m$</td>
<td>$h_1 = 0.2m$</td>
</tr>
<tr>
<td>$h_2 = 0.15m$</td>
<td>$h_2 = 0.4m$</td>
</tr>
</tbody>
</table>

Figure 4.1: Results of measurements and theoretical curve: reciprocal wavelength plotted against the frequency: LCPC structure Route A4 Auxerre-Lyon.
4.2. RESULTS OF MEASUREMENTS  

![Graph showing reciprocal wavelength vs frequency with data points labeled 'mart.dat' using 3:4.]

Figure 4.2: Figure giving results of measurements from Martinzec[4] Figure 1.55

<table>
<thead>
<tr>
<th>Composition</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt cement</td>
<td>5cm</td>
</tr>
<tr>
<td>Coated sand/gravel</td>
<td>8cm</td>
</tr>
<tr>
<td>Cement stabilized gravel</td>
<td>25cm</td>
</tr>
<tr>
<td>Lime stabilized gravel</td>
<td>10cm</td>
</tr>
</tbody>
</table>

Table 4.3: Table giving the composition of the pavement from Martinzec[4] Figure 55

shown in Figure (4.1). This figure shows both the results of measurements and the calculated output from RAY.FOR.

4.2.2 Martinzec, experimental results

Measurements were made by Martinzec [4]. The results of some of the measurements are shown in Figure (4.2) and in Table 4.4. The velocities of shear waves read from Figure (4.2) are $\beta_1 = 1520\text{ms}^{-1}$, $\beta_2 = 1300\text{ms}^{-1}$, $\beta_3 = 240\text{ms}^{-1}$. Assuming that the density is 2000$\text{kg/m}^3$, the values of the shear moduli are 4.5, 3.38 and 0.11 GPa. The corresponding values of $E = (1 + \sigma) \beta^2$ for $\sigma = 0.25$ are 11.1, 8.4 and 0.28 GPa.

The composition of the pavement is shown in Table 4.3.

Figure (4.2) shows the velocity of waves of the Rayleigh type plotted against the wavelength. The figure shows experimental measurements, and also the expected theoretical values. The pavement is treated as a structure composed of three media. All media extend laterally to an infinite extent. The subgrade extends to an infinite depth.

The theoretical curve shown in Figure (4.3) was obtained with the aid of the programs PREP.FOR and RAY.FOR.
### Table 4.4: Table giving results of measurements from Martinzec[4] Figure 1.55

<table>
<thead>
<tr>
<th>Velocity(m/s)</th>
<th>Wavelength(m)</th>
<th>Frequency(Hz)</th>
<th>Recip w’length(1/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1496</td>
<td>0.05</td>
<td>29920</td>
<td>20</td>
</tr>
<tr>
<td>1400</td>
<td>0.15</td>
<td>9333</td>
<td>6.67</td>
</tr>
<tr>
<td>1250</td>
<td>0.3</td>
<td>4167</td>
<td>3.33</td>
</tr>
<tr>
<td>1000</td>
<td>0.55</td>
<td>1818</td>
<td>1.82</td>
</tr>
<tr>
<td>800</td>
<td>0.7</td>
<td>1143</td>
<td>1.43</td>
</tr>
<tr>
<td>700</td>
<td>1</td>
<td>700</td>
<td>1</td>
</tr>
<tr>
<td>500</td>
<td>1.55</td>
<td>323</td>
<td>0.65</td>
</tr>
<tr>
<td>400</td>
<td>2</td>
<td>200</td>
<td>0.5</td>
</tr>
<tr>
<td>350</td>
<td>2.5</td>
<td>140</td>
<td>0.4</td>
</tr>
<tr>
<td>300</td>
<td>3</td>
<td>100</td>
<td>0.33</td>
</tr>
<tr>
<td>250</td>
<td>4</td>
<td>63</td>
<td>0.25</td>
</tr>
</tbody>
</table>

### Figure 4.3: Results of measurements of phase velocity and corresponding wavelength, showing theoretical curve: Martinzec Figure 55 [4]
4.2. RESULTS OF MEASUREMENTS  CHAPTER 4. PART 2: FIELD RESULTS

The following data were used for the program PREP.FOR.

\[
\text{MARTINZEC55E1=11100,E2=8400,E3=280;H1=0.13,H2=0.25,NU1=NU2=NU3=0.25} \\
3 11100 .25 8400 .25 280 .25 0.0 \\
2. 2. 2. \\
0.13 0.25 \\
(5, 0.0) (0.2, 0.00) (0.01, 0.01) 0.02 0.1
\]

The figure shows a discrepancy between the measured and predicted values of wave velocity at the shorter wavelengths. This indicates that the properties of the surface layers, their thicknesses and elastic moduli, are not accurately defined. We attempt to improve the theoretical match by treating the system as composed of five media. The corresponding data file for PREP.FOR follows.

\[
\text{MARTINZEC55E1=11100,E2=84000,E5=280;H1=0.13,H2=0.25,NU1=NU2=NU3=0.25} \\
5 11100 .25 84000 .25 20000 .25 40000 .25 280 .25 0.0 \\
2. 2. 2. 2. \\
0.05 0.28 0.25 0.10 \\
(5, 0.0) (0.4, 0.00) (0.05, 0.01) 0.0001 0.1
\]

This data file leads to a continuous dispersion curve of wave velocity against the wavelength, as shown in Figure (4.3). However the measured velocities at wavelengths less than 1.5 meters are higher than the predicted values.

In order to correct the discrepancy, a five-medium system was analyzed. The wavelength output from the program RAY.FOR was multiplied by 0.3 and the velocity of waves of the Rayleigh type was multiplied by 1020. This suggests that the actual layer thickness are 0.3 times those assumed in the data used for the program RAY.FOR, shown in Table 4.3.

4.2.2.1 Theoretical prediction

The program RAY.FOR operates using trigonometric functions. Therefore infinitely many solutions exist, providing phase velocity of waves of the Rayleigh type at specified wavelengths. The experimental results match with only one of these solutions. This solution may correspond with the mode having the lowest possible frequency.

Mode selection is carried out using the wave phase velocity as the criterion: the solution is sought for a given phase velocity. A better criterion may be to use the frequency. Use of the frequency may possibly decrease the tendency to skip modes.

The theoretical curve shown in Figure (4.3) shows adequate agreement with measurements at large wavelengths. At short wavelengths there is a discrepancy between the measurements and the theoretical curve.

This discrepancy is corrected in Figures (4.4) and (4.5) as follows. The wavelengths output from RAY.FOR have been multiplied by 0.3; the velocity at the surface has
Figure 4.4: Results of measurements and theoretical curve: the reciprocal of the wavelength plotted against the frequency of waves of the Rayleigh type, Martinzec Figure 55. Theoretical values of the wavelengths have been multiplied by 0.3; $\beta_1 = 1020 \text{ms}^{-1}$

Figure 4.5: Results of measurements and theoretical curve: phase velocity of waves of the Rayleigh type plotted against the wavelength, Martinzec Figure 55. Wavelengths have been multiplied by 0.3; $\beta_1 = 1020 \text{ms}^{-1}$
been made equal to $1020\text{ms}^{-1}$. The match between measured and theoretical values at short wavelengths is improved, as shown in Figures (4.4) and (4.5).

It may be deduced that the system is 0.3 times as thick as the data indicates in Table 4.3.

4.3 Routine for obtaining a theoretical match of the general structure

The results shown suggest a routine to follow in order to obtain a theoretical match with a series of experimental measurements.

1. Estimate the velocity of propagation of shear waves in the components of the structure. The estimate is obtained from a plot of the reciprocal wavelength against the frequency. The velocities are successively the reciprocals of the slopes of the plot. The largest values of the reciprocal wavelength correspond with the layers closest to the surface.

2. Estimate the thickness of the surface layer. The thickness is approximately the wavelength at which the plot of velocity against the wavelength passes through a maximum.

3. Estimate the thickness of the underlying layer. Increase the thickness of the underlying layer, if the drop in the phase velocities occurs at too small a wavelength.

4. Execute the program RAY.FOR. Plot the following.

   (a) Reciprocal wavelength against the frequency. The frequency output of RAY.FOR must be multiplied by the selected value of $\beta_1$, in order to yield actual frequencies in Hz.

   (b) The velocity of propagation of waves of the Rayleigh type against the wavelength. The output of RAY.FOR gives the velocity as a fraction $\frac{c}{\beta_1}$.

   (c) Return to (1).

5. If the maximum occurs at too large a wavelength, decrease the thickness of the surface layer.

6. Continue until a satisfactory match is reached.

4.4 Conclusion

The object of the present work is to determine the properties of a pavement structure. The velocities of waves of the Rayleigh type are measured at the surface of a layered structure, such as a road surface. The results of the measurements are used to estimate the thicknesses and the elastic moduli of the media of which the structure is composed.
1. Measurements of wave velocities on pavements are able to lead to information concerning the properties of the pavement.

2. A simplified interpretive method exists. It is applicable to a structure consisting of a single layer overlying a semi-infinite medium. It involves the use of the programs SIN13.FOR and SON.FOR

3. The general structure can be analysed with the aid of the program RAY.FOR. The program RAY.FOR is able to analyse the results obtained from the surface of a multi-layered structure. The underlying medium is treated as being semi-infinite.

4. The use of Maple worksheets provides an indication of the valid solutions of the determinant given by equation (4-202) in Ewing, Jardetzky and Press[2].
Bibliography


Chapter 5

Appendix

5.1 Listing of $SIN13.FOR$

The following is a listing of the program $SIN13.FOR$. It leads to a table of reciprocal wavelength and the corresponding frequency, for a system consisting of a hard layer overlying a soft semi-infinite medium.

The quantities $r_1, r_2, s_1, s_2$ are imaginary.

The reciprocal wavelength is calculated as $\frac{\pi h}{\lambda}$. The output must be divided by $\pi$ to obtain reciprocal wavelength, with the thickness $h$ of the layer as unity.

The frequency is output as $CB1/V$. That is $\frac{C \cdot \pi h}{\lambda \cdot \beta_1}$, or $\frac{\pi h}{\beta_1}$ times the frequency $\frac{C}{\lambda}$.

5.1.1 Program listing

```
C  003G4  001  025 TTI BASEMENT OET BUILDING WHC33F
C THERE ARE SEVEN TEN COLUMN DATA FIELDS ON THE VELOCITY CARD.
C ANY FIELD MAY CONTAIN 999, WHICH CAUSES
C THE PROGRAM TO READ A FURTHER VELOCITY CARD.
C Version for hard over soft see Lee,A.R. "The effect of structure
C upon microseismic disturbance" page 91 eqn (20)
C*************************************************************************
C*************************************************************************
C*************************************************************************
C RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM. JANUARY 1968 WHC33F
C*************************************************************************
C*************************************************************************
C*************************************************************************
c Trial data follows, works on starting value of 0.4
C  0.4 10.0 0.4 0.01
C  0.58 0.53 0.529 0.528 0.527 0.526 0.525 999.
```
C       0.524 0.523 0.52 0.51 0.50 0.49 0.48 999.
C       0.47 0.46 0.45 0.42 0.40 0.39 0.38 999.
C       0.36 0.35 0.34 0.33 0.32 0.31 0.30 999.
C       0.290 0.285 0.280 0.275 0.270 0.265 0.250 999.
C       0.283 0.282 0.281 0.280 0.279 0.278 0.277 999.
C       0.276 0.275 0.274 0.273 0.272 0.271 0.270 999.
C       0.269 0.268 0.266 0.264 0.262 0.260 0.258 0.0
C       0.0 0.0 0.0 0.0
C
C
PROGRAM WHC33F
DIMENSION CB(8)
REAL*8 R2K,S1K,S2K,
   R1H,R2H,S1H,S2H,XKR1,XKS1,R2R1,S2S1,XII, XI2, ETA1, ETA2, XITEM1, 30
   2XITEM2
101 FORMAT (4F10.0)
102 FORMAT (13X,'RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM.'
   15X' EQUATING EQUATION (4 - 202)'//7X' POISSON' S RATIO IN LAYER = ',
   2F5.2,' IN HALF SPACE = ', F5.2,' MU2/MU1 = ', F7.2)///)
103 FORMAT (8F10.0)
104 FORMAT (6X,'VELOCITY    WAVELENGTH    FREQUENCY
   1 WAVE NUMBER    COUNTER REG.' ///)
105 FORMAT (5X,G12.4,5X,G12.4,7X,G12.4,4X,G12.4,4X,G12.4,7X,I3)
106 FORMAT (5X,G12.4,5X,G12.4,7X,G12.4,4X, G12.4,7X,I3,3X,
   1 'PARAMETERS EQUALS ', E18.6)
107 FORMAT (1H1)
  OPEN(1,FILE='SIN13.DAT',STATUS='OLD',FORM='FORMATTED')
  OPEN(3,FILE='SIN13.PRN',STATUS='UNKNOWN',FORM='FORMATTED')
201 READ(1,*) B1A1,A1A2,B2A2,XMU2M1
IF(B1A1.EQ.0.0) GO TO 999
WRITE(3,102) SIGMA1,SIGMA2,XMU2M1
WRITE(3,104) C "2/V" is 2*pi/lambda = k; so V = lambda / pi
WRITE(*,*), ' ENTER A STARTING VALUE OF V'
READ(*,*), V
202 READ(1,*) (CB(J),J=1,8)
C       REF=10.V
I=0
C
C******************************************
C**********STATEMENT 203 IS RE-ENTRY POINT FOR NEW VALUE OF CBJ**********
C******************************************
203 CONTINUE
C       V = VST ! RESET V TO THE STARTING VALUE
I=I+1
IF(I.GT.8) GO TO 201
KOUNT = 0
CB1=CB(I)
IF(CB1.EQ.0.0) GO TO 201
IF(CB1.EQ.999.0) GO TO 202
CA1=CB1*B1A1
CA2=CB1*B1A1*A1A2
CB2=CB1*B1B2

WRITE(*,*) CA1,CA1,CA2,CB2

R1K=SQRRT(1.0-CA1*CA1)
R2K=SQRRT(-1.0+CA2*CA2)
S1K=SQRRT(1.0-CB1*CB1)
S2K=SQRRT(-1.0+CB2*CB2)
W = 2.*(XMU2M1-1.)
X = XMU2M1*CB1*CB1*B1B2*B1B2-W
Y = CB1*CB1+W
Z = X-CB1*CB1

XKR1=1./R1K
XKS1=1./S1K
R2R1=R2K/R1K
S2S1=S2K/S1K

C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************

C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************

CONTINUE
R1H = (2./V)*R1K
KOUNT = KOUNT + 1
R2H=(2./V)*R2K
S1H=(2./V)*S1K
S2H=(2./V)*S2K

XI1:=(2.-CB1*CB1)*(X*DCOSH(R1H)+R2R1*Y*DSINH(R1H))
1 -2.*S1K*(R2K*W*DSINH(S1H)+XKS1*Z*DCOSH(S1H))
XI2:=(2.-CB1*CB1)*(S2K*W*DCOSH(R1H)+XKR1*Z*DSINH(R1H))
1 -2.*S1K*(X*DSINH(S1H)+S2S1*Y*DCOSH(S1H))

C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************

ETAI=-(2.-CB1*CB1)*(R2K*W*DCOSH(S1H)+XKS1*Z*DSINH(S1H))
1 -2.*R1K*(X*DSINH(R1H)+R2R1*Y*DCOSH(R1H))
ETAE2=(2.-CB1*CB1)*(X*DCOSH(S1H)+S2S1*Y*DSINH(S1H))
1 -2.*R1K*(S2K*W*DSINH(R1H)+XKR1*Z*DCOSH(R1H))

C********************************************************************
C********************************************************************
C********************************************************************
C********************************************************************

XITEM1=XI1*ETAE2
XITEM2=XI2*ETAI

DEL = REAL(XITEM1 - XITEM2)
IF (KOUNT = 1) 301, 301, 302
301 VVEN = V
V = 1.27 * V
VODD = V
DELEVEN = DEL
3011 GO TO 250
C*** \begin{verbatim}
302 IF (KOUNT - 2*(KOUNT/2)) 305, 305, 304
304 DELEVEN = DEL
    VEVEN = V
3041 GO TO 306
305 DELODD = DEL
    VODD = V
C*** \end{verbatim}

C*** \begin{verbatim}
306 IF(ABS(DELODD-DELEVEN),LE. 0.01) GO TO 599 ! Failed output
    DIFF = ABS(DELODD-DELEVEN)/(DELODD-DELEVEN)
    IF(DIFF,LE. .000001*DELODD) GO TO 600 ! Successful output
    V = VEVEN -0.5*DELEVEN*(VODD - VEVEN)/(DELODD- DELEVEN) \(\text{Convergence D}
3061 GO TO 250
C*** \end{verbatim}

C*** \begin{verbatim}
599 FREQ=CBI/V
    VI = 1/V
    WRITE(3,106) CBI, V, FREQ, VI, KOUNT,DELODD
    GO TO 203
600 FREQ=CBI/V
    VI = 1/V
    \begin{verbatim}
700 \end{verbatim}
    WRITE(3,105) CBI, V, FREQ, VI, KOUNT
    GO TO 203
999 WRITE(3,107)
STOP
END
\end{verbatim}
The result shown in Figure (1.1) was obtained using the following data for SIN13.FOR

0.4 10.0 0.4 0.01
0.66 0.65 0.64 0.63 0.62 0.61 0.59 999.
0.58 0.575 0.57 0.565 0.56 0.555 0.55 999.
0.58 0.53 0.529 0.528 0.527 0.526 0.525 999.
0.524 0.523 0.52 0.51 0.50 0.49 0.48 999.
0.47 0.46 0.45 0.42 0.4 0.39 0.38 999.
0.36 0.35 0.34 0.33 0.32 0.31 0.30 999.
0.290 0.285 0.280 0.275 0.270 0.265 0.250 999.
0.283 0.282 0.281 0.280 0.279 0.278 0.277 999.
0.276 0.275 0.274 0.273 0.272 0.271 0.270 999.
0.269 0.268 0.266 0.264 0.262 0.260 0.258 0.0
0.0 0.0 0.0 0.0

5.2 Listing of SON.FOR

The following is a listing of the program SON.FOR. It leads to a table of reciprocal wavelength and the corresponding frequency, for a system consisting of a soft layer
overlying a hard semi-infinite medium.

5.2.1 Program listing

C 003G4 001 025 TTI BASEMENT OET BUILDING WHC33F
C THERE ARE SEVEN TEN COLUMN DATA FIELDS ON THE VELOCITY CARD.
C ANY FIELD MAY CONTAIN 999,, WHICH CAUSES
C THE PROGRAM TO READ A FURTHER VELOCITY CARD.
C SON.FOR IS FOR SOFT OVER HARD
C TEST DATA FOLLOWS: (STARTING POINT V=3.0)
C 0.4 0.101 0.4 100.
C 2.51 2.52 2.53 2.54 2.55 2.56 2.57 999.
C 2.58 2.6 2.75 2.8 2.9 3.1 3.3 999.
C 3.43 3.8 3.9 3.95 4.0 4.05 4.08 999.
C 4.2 4.4 5.1 5.5 6.7 7.5 7.9 999.
C 8.0 8.1 8.2 8.3 8.4 8.5 8.6 999.
C 8.7 8.8 8.9 9.0 9.1 9.15 9.18 999.
C 0.0 0.0 0.0 0.0
C
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM. JANUARY 1968 WHC33F 20
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************

PROGRAM WHC33F
DIMENSION CB(8)
C COMPLEX*16 R1K,R2K,S1K,S2K,
C 1R1H,R2H,S1H,S2H,XKR1,XKS1,R2R1,S2S1,XI1,XI2,ETA1,ETA2,XITEM1,
C 2XITEM2
101 FORMAT (4F10.0)
102 FORMAT (1H0,26X,'RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM.',/)
103 FORMAT (2F5.2,), IN HALF SPACE = ', F5.2,', M2/MU1 = ',F7.2,/II)
104 FORMAT (8F10.0)
105 FORMAT (1H0.26X,'VELOCITY WAVELENGTH FREQUENCY WAVE
1 NUMBER COUNTER REG.','//I)
106 FORMAT (27X,G10.5,7X,G10.5,6X,G10.5, 9X,J3)
108 FORMAT (27X,G10.5,7X,G10.5,6X,G10.5, 9X, I3, 3X,
1 'Y EQUALS ', E18.6)
109 FORMAT (1H1)
OPEN(1,FILE='SON.DAT',STATUS='OLD',FORM='FORMATTED')
OPEN(3,FILE='SON.PRN',STATUS='UNKNOWN',FORM='FORMATTED')
201 READ(1, *) B1A1,A1A2,B2A2,XMU2M1
IF(B1A1.EQ. 0.0) GO TO 999
WRITE(3,102) SIGMA1,SIGMA2,XMU2M1  
WRITE(3,104)  
WRITE(*,*) ' ENTER A STARTING VALUE OF V'  
C "2/V" is 2*pi/lambda = k; so V = lambda / pi  
READ(*,*) V ! V IS LAMDA / PI  
202 READ(1, *) (CB(J),J=1,9)  
C  
REF=10.V  
I=0  
C********************************************************************  
C********************************************************************  
C********************************************************************  
C********************************************************************  
C********************************************************************  
C********************************************************************  

203 I=I+1  
IF(LGT.8) GO TO 201  
KOUNT = 0  
CB1=CB(I)  
IF(CB1.EQ.0.) GO TO 201  
IF(CB1.EQ.999.) GO TO 202  
C********************************************************************  
CA1=CB1*B1A1  
CA2=CB1*B1A1*A1A2  
CB2=CB1*B1B2  
WRITE(*,*)ca1,ca2,cb1,cb2  
C********************************************************************  
R1K=SQR(-1.0+CA1*CA1)  
R2K=SQR(1.0-CA2*CA2)  
S1K=SQR(-1.0+CB1*CB1)  
S2K=SQR(1.0-CB2*CB2)  
W = 2.*(XMU2M1-1.)  
X = XMU2M1*CB1*B1B2+B1B2-W  
Y = CB1*CB1+W  
Z = X-CB1*CB1  
C********************************************************************  
XKR1=1./R1K  
XKS1=1./S1K  
R2R1=R2K/R1K  
S2S1=S2K/S1K  
C********************************************************************  
C********************************************************************  
C********************************************************************  
C********************************************************************  
C********************************************************************  
C********************************************************************  

250 R1H = (2./V)*R1K  
KOUNT = KOUNT + 1  
R2H=(2./V)*R2K  
S1H=(2./V)*S1K  
S2H=(2./V)*S2K  
C********************************************************************  
XI1=(-2.*CB1*CB1)*(X*COS(R1H)+R2R1*Y*SIN(R1H))  
1 +2.*S1K*(R2K*W*SIN(S1H)-XKS1*Z*COS(S1H))  
XI2=(-2.*CB1*CB1)*(S2K*W*COS(R1H)+XKR1*Z*SIN(R1H))  
1 +2.*S1K*(X*SIN(S1H)-S2S1*Y*COS(S1H))
5.3 Listing of APP.H.FOR

The following is a listing of the program APP.H.FOR. The program yields an approximate value of the thickness of the surface layer. The data are pairs consisting of the
5.3.1 Program listing

c The following file is an approximation to Lee's determinant for the
c "hard over soft" case, using a first order expansion of the transcendental
c functions. It is based on equation (21) in the condensed version of
c A.W.Lee's paper, A.W.LEE.DOC. It solves for \( <N> \) in terms of the
c measured quantities, \( c \) the reciprocal wavelength and the corresponding
c frequency. The coefficients "aaa", "bbb", "ccc" are derived in the
c Mathematica notebook LEE.EQH.NB, where they are expressed in Fortran
c form.

c
```fortran
implicit real(-n)

201 format(3g16.3)
10 write(*,*) " Enter beta1 and beta2 "
read(*,*) beta1, beta2
alpha1=1.732*beta1
alpha2=1.732*beta2
20 write(*,*) "Enter reciprocal wavelength, frequency"
read(*,*) rlnghth,freq
if(rlnghth.eq.0) goto 10 !backtrack if required
write(*,201) rlnghth,freq
nu1=1.0
nu2=nu1*(beta2/beta1)**2
r=nu2/rlnghth
omega= 6.28*freq
k=6.28*rlnghth
ka1=omega/alpha1
kb1=omega/beta1
ka2=omega/alpha2
kb2=omega/beta2

write(*,*) " to here!"
write(*,*) "omega=" ,omega," c=" ,c
Rarg= -((omega**2/beta1**2) + omega**2/c**2)
if(Rarg.lt.0) write(*,*) "Rarg.lt.0"
R=Sqrt(-((omega**2/beta1**2) + omega**2/c**2))
write(*,*) "R=", R

write(*,*) " beta1 = ", beta1," beta2 = ", beta2
Sarg= -((omega**2/beta1**2) + omega**2/c**2)
S=Sqrt(-((omega**2/beta1**2) + omega**2/c**2))
write(*,*) "S=", S
write(*,*) " alpha1=" ,alpha1," alpha2=" ,alpha2
r2arg= omega**2/alpha2**2 - omega**2/c**2
if(r2arg) 21,21,22
write(*,*) "r2arg .lt. 0; c must exceed alpha2"
goto 20
22 r2=Sqrt(omega**2/alpha2**2 - omega**2/c**2)
write(*,*) " r2=" ,r2
```

60
5371 Bytes  
anew202.for  
11:43  
12.11.97

\[ s2arg = \text{omega}^{**}2/\text{beta2}^{**}2 - \text{omega}^{**}2/\text{c}^{**}2 \]
\[ s2 = \text{sqr(omega}^{**}2/\text{beta2}^{**}2 - \text{omega}^{**}2/\text{c}^{**}2) \]

\[ c \]
wriite(",", " s2=", s2
\[ c \text{ now define the Love parameters} \]
wriite(",", " to here"

\[ X = c^{**}2*\text{mu2}/(\text{beta2}^{**}2*\text{mu1}) - 2*(-1 + \text{mu2}/\text{mu1}) \]
\[ Y = c^{**}2/\text{beta1}^{**}2 + 2*(-1 + \text{mu2}/\text{mu1}) \]
\[ Z = -(c^{**}2*\text{mu2}/(\text{beta2}^{**}2*\text{mu1}) - 2*(-1 + \text{mu2}/\text{mu1}) \]
\[ W = 2*(-1 + \text{mu2}/\text{mu1}) \]

\[ c \text{ now define the value of the determinant (= zero)} \]
\[ a = 2* \text{h}^{**}2 + 2*bb* \text{h} + cc = 0 \]
\[ aa = -4*k^{**}2 + 4*r2*s2 + 16*r2*s2*W/3 + 4*r2*s2*W^{**}2/3 + \]
\[ - 16*k^{**}2*X/3 - 4*k^{**}2*Y^{**}2/3 - 2*r2*s2*Y - 2*r2*s2*W*Y - \]
\[ - 4*k^{**}2*Z + 8*k^{**}2*X*Z/3 - k^{**}2*Z^{**}2 \]

\[ bb = \]
\[ - 2*r2*W - 2*s2*W/3 + 2*r2*X + 10*s2*X/3 + \]
\[ - 4*s2*W*X/3 - 2*s2*Y - s2*X*Y - 2*r2*Z - 2*s2*Z - r2*W*Z - \]
\[ - s2*W*Z \]

\[ cc = 4 - 4*r2*s2/\text{k}^{**}2 - \]
\[ - 4*r2*s2*W/2*k^{**}2 - r2*s2*W^{**}2/2/k^{**}2 - X^{**}2 \]

\[ c \]
wriite(",", " \text{X}^{**}Y + W^{**}Z = \text{X}^{**}Y + W^{**}Z \) a factor of <\text{bb}> \]
\[ c \]
wriite(",", " cc = "
\[ c \]
wriite("", " bb cc "
\[ c \]
hs = \text{-bb} + \text{sqr(bb}^{**}2 - 4*aa*cc)/(2*aa) \]
h1 = \text{-bb} - \text{sqr(bb}^{**}2 - 4*aa*cc)/(2*aa) \]
\[ c \]
wriite(",", " Thickness(1) Thickness(2) S* h = "
\[ c \]
goto 20
\[ c \]
end

5.4 Listing of EW202.FOR

The following is a listing of the program EW202.FOR. It leads to a table of reciprocal wavelength and the corresponding frequency, for a system consisting of a soft layer overlying a hard semi-infinite medium. The quantities \( r_1, r_2, s_1, s_2 \) are all real.

The reciprocal wavelength is calculated as \( \frac{\pi h}{\lambda} \). The output must be divided by \( \pi \) to obtain reciprocal wavelength, with the thickness \( h \) of the layer as unity.

The frequency is output as \( CB1/V \). That is \( \frac{c}{\lambda} \cdot \frac{\pi h}{\beta_1} \) or \( \frac{\pi h}{\beta_1} \) times the frequency \( \frac{c}{\lambda} \).

5.4.1 Program listing

\[
\begin{align*}
\text{C} & \quad 003G4 \\
\text{C} & \quad 001 \\
\text{C} & \quad 025 \text{ TTI BASEMENT EOT BUILDING WHC33F} \\
\end{align*}
\]
C THERE ARE SEVEN TEN COLUMN DATA FIELDS ON THE VELOCITY CARD.
C ANY FIELD MAY CONTAIN 999., WHICH CAUSES
C THE PROGRAM TO READ A FURTHER VELOCITY CARD.
C This is the real version of Ewing Jardetzky and Press equation (4-202).
C It is for a soft layer overlying a hard semi-infinite medium.
C See CTAC93 for output obtained from it.
C
C************************************************************************************
C************************************************************************************
C RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM. JANUARY 1968 WHC33F
C************************************************************************************
C************************************************************************************

PROGRAM WHC33F
DIMENSION CA(8)
C
THE INPUT VELOCITIES ARE CA(J): CB2 > C > CA1
101 FORMAT (4F10.0)
102 FORMAT(6X,'RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM.',
+5X'EWING EQU (4 - 202).',/25X'REAL VERSION. CB2 > C > CA1',
+1X'POISSON'S RATIO IN LAYER = ',
+F5.2,', IN HALF SPACE = ', F5.2,', MU2/MU1 = ', F7.2)
103 FORMAT(8F10.0)
104 FORMAT(1X 'VELOCITY WAVELENGTH WAVE NUMBER ',
+1X 'FREQUENCY COUNTER REG.'))
105 FORMAT(5X,G8.3,3X,G8.3,4X,G8.3,7X,G8.3,3X,13)
106 FORMAT(5X,G8.3,3X,G8.3,4X,G8.3,7X, G8.3, 3X, 13, 3X,
+1 'Y EQUALS ', F9.2)
107 FORMAT(1H1)
OPEN(1,FILE='EW202.DAT',STATUS='OLD',FORM='FORMATTED')
OPEN(3,FILE='EW202.FRN',FORM='FORMATTED')
201 READ(1, *,END=999) B1A1,A1A2,B2A2,XMU2M1
 IF((B1A1.EQ. 0.0) GO TO 999
 WRITE(*,102) SIGMA1,SIGMA2,XMU2M1
 WRITE(*,104)
 C V = LAMBDA / PI
 V=1.0
 write (*,*) ' enter V : '
 read (*,*) V
202 READ(1, *, ) (CA(J),J=1,8)
 C REF=10.V
 I=0
C************************************************************************************
C********************STATEMENT 203 IS RE-ENTRY POINT FOR NEW VALUE OF CB1************
C************************************************************************************
203 CONTINUE
 I=I+1
 IF((LGT.8) GO TO 201
KOUNT = 0
CA1=CA(I)
IF(CA1.EQ.0.) GO TO 201
IF(CA1.EQ.999.) GO TO 202

C***************************************************************
CB1=CA1/BI1A1
CA2=CB1*B1A1*A1A2
CB2=CB1*B1B2

C write(*,*), 'cb1 = ',cb1,' ca1 = ',ca1,' cb2 = ',cb2,' ca2 = ',ca2
C***************************************************************
R1K=SQR(R(-1.0+CA1xCA1)) !C > ALPHA1 AND C < BETA2
R2K=SQR(R(+1.-CA2xCA2))
S1K=SQR(R(-1.0+CBI1CB1))
S2K=SQR(R(+1.-CB2CB2))
W = 2.*(XMU2M1-1.)
X = XMU2M1*CB1*CB1*B1B2+B1B2-W
Y = CB1*CB1+W
Z = X-CB1CB1

C***************************************************************
XKR1=1./R1K
XKS1=1./S1K
R2R1=R2K/R1K
S2S1=S2K/S1K

C***************************************************************START OF ITERATION LOOP***************************************************************

250 CONTINUE
R1H=(2./R)*R1K
KOUNT = KOUNT + 1
R2H=(2./R)*R2K
S1H=(2./R)*S1K
S2H=(2./R)*S2K

C***************************************************************
X11=2.-CB1*CB1)*(X*COS(R1H)+R2R1*Y*SIN(R1H))
1 +2.*S1K*(R2K*W*SIN(S1H)-XKS1*Z*COS(S1H))
X12=2.-CB1*CB1)*(S2K*W*COS(R1H)+XKR1*Z*SIN(R1H))
1 +2.*S1K*(X*SIN(S1H)-S2S1*Y*COS(S1H))

C***************************************************************
ETA1=2.-CB1*CB1)*(R2K*W*COS(S1H)+XKS1*Z*SIN(S1H))
1 +2.*R1K*(X*SIN(R1H)-R2R1*Y*COS(R1H))
ETA2=2.-CB1*CB1)*(X*COS(S1H)+S2S1*Y*SIN(S1H))
1 +2.*R1K*(S2K*W*SIN(R1H)-XKR1*Z*COS(R1H))

C***************************************************************
XITEM1=X11*ETA2
XITEM2=X12*ETA1

C***************************************************************DEL = XITEM1 - XITEM2
C write(*,*), 'del = ', DEL , ' V = ',V
IF (KOUNT - 1) 301, 301, 302
301 \texttt{XEVEN = V} \quad \texttt{V = \texttt{1.01} * V} \quad \texttt{!SET INCREMENT OF THE SEARCH} \\
\texttt{XODD = V} \\
\texttt{YEVEN = DEL} \\
3011 \texttt{GO TO 250} \\
C***************************************************************
302 \texttt{IF (KOUNT - 2*(KOUNT/2)) 305, 305, 304} \\
304 \texttt{YEVEN = DEL} \\
\texttt{XEVEN = V} \\
3041 \texttt{GO TO 306} \\
305 \texttt{YODD = DEL} \\
\texttt{XODD = V} \\
C***************************************************************
306 \texttt{IF(ABS(YODD-YEVEN),LE.,00000000001) GO TO 599} \\
\texttt{IF(ABS(XODD - XEVEN) .LE. 000000001) GO TO 600} \\
\texttt{V = XEVEN - YEVEN*(XODD - XEVEN)/(YODD- YEVEN)} \\
3061 \texttt{GO TO 250} \\
C
C******************************************************************************
C******************************************************************************
C******************************************************************************
599 \texttt{FREQ=CB1/V} \\
\texttt{VI = 1./V} \\
\texttt{WRITE(*,108) CA1, V,VI, FREQ, KOUNT,YODD} \\
\texttt{WRITE(3,106) CA1, V,VI, FREQ, KOUNT,YODD} \\
\texttt{GO TO 203} \\
600 \texttt{FREQ=CB1/V} \\
\texttt{VI=1./V} \\
\texttt{WRITE(*,105) CA1,V,VI,FREQ,KOUNT} \\
\texttt{WRITE(3,105) CA1,V,VI,FREQ,KOUNT} \\
\texttt{GO TO 203} \\
999 \texttt{WRITE(*,107)} \\
\texttt{STOP} \\
\texttt{END} \\
A sample data file follows.

\begin{verbatim}
0.4 0.4 100
1.1 1.5 1.7 1.9 2.1 2.3 999
2.6 3.1 3.3 3.5 3.53 3.57 999
3.6 3.63 3.66 3.69 3.72 3.74 3.77 0
\end{verbatim}

\section*{5.5 Listing of \texttt{REVW202.FOR}}

The following is a listing of the program \texttt{REVW202.FOR}. It attempts to reverse the output of the program \texttt{EW202.FOR}. It aims to yield the thickness of the surface layer for a medium consisting of a soft surface layer overlying a hard semi-infinite medium.
The series on which the program is based is divergent, so the results are accurate to an order of magnitude only.

5.5.1 Program listing

This program attempts to inverse Ewing Jardetzky and Press equation (202)
It does so by using the solution for the determinant of the system, expanded in terms of
the thickness h of the surface layer.
Expansion to the power of five leads to a polynomial which is reversible, using
REVERSE.NB However the resulting reversion is not convergent.
The first part of this program is the output of LEE2.NB, called LEE2.DAT
The second part of this program is the output of REVERSE.NB, called REVERSE.DAT

```fortran
REAL MU1,MU2,K1,K2,K
DIMENSION A(8)
   201 FORMAT(3G14.3)
   202 FORMAT(1X,1G14.3,1X,1G14.3)
CONTINUE
WRITE(*,*) 'Enter frequency, reciprocal wavelength.. '
READ(*,*) FREQ, XK
K = 6.28 * XK
W = 6.28 * FREQ
C = FREQ / XK

B1 = 0.1
B2 = 1.0
A1 = 2.5*B1
A2 = 2.5*B2
MU1 = 1.0
MU2 = 100.0

K = W / C
K1 = W / B1
K2 = W / B2

R1 = (-1/C**2 + 1/A1**2)**(1/2)
S1 = (-1/C**2 + 1/B1**2)**(1/2)
R2 = (-1/C**2 + 1/A2**2)**(1/2)
S2 = (-1/C**2 + 1/B2**2)**(1/2)

WW = 2 * (MU2/MU1 - 1)
XX = MU2/MU1 * K2**2/K**2 - WW
YY = K1**2/K**2 + WW
ZZ = MU2/MU1 * K2**2/K**2 - K1**2/K**2 - WW
Z0 = -(144*K**4*R2*S2*WW**2 + 144*K**2*K1**2*R2*S2*WW**2 -
      36*K1**4*R2*S2*WW**2 + 144*K**6*XX**2 - 144*K**4*K1**2*XX**2 +
       36*K1**4*XX**2 +

```

65
z = z0 / z1
a(0) = 0
a(1) = 1

a(2) = \((w^{*}(-A*k**4*r**2*(r**2 + s**2)\cdot s**2/ww**2)) + 60\)#

a(3) = \((-A*k**4*(s**2*2*(3*r + z**2) + r**2*(r + 2s + 3s)**2)) + \(A*k**4*(s**2*2*2*(3*r + z**2)) + A*k**4*(s**2*2*2*(3*r + z**2))\)#

a(4) = \((-A*k**4*(s**2*2*2*(3*r + z**2)) + A*k**4*(s**2*2*2*(3*r + z**2))\)#
\[ a(6) = (w**5*(4*r1**4*s1**4*(c2*s2*ww**2 - k**2*x*x)**2) - \]
\[ 2*(2*k**2 - k1**2)*r1**6*(-(c2*s2*ww*yy) + k**2*x*x*zz) - \]
\[ 2*(2*k**2 - k1**2)*s1**6*(-(c2*s2*ww*yy) + k**2*x*x*zz) - \]
\[ (-2*k**2*k1**2)*r1**2*s1**2*(-(c2*s2*yy*yy) + k**2*zz*zz))/\]
\[ (36*(4*k**4*(c2 + s2) + k1**4*(c2 + s2) - 100)\]
\[ 4*k**2*(-(c2*s1**2) + s1**2*s2 + k1**2*(c2 + s2)))*(x*x*yy-ww*zz)) \]

```c
write((*,201) (a(i),i=1,8)

bb1 = 1
bb2 = -(a(2)/a(1))
bb3 = (2*a(2)**2 - a(1)*a(3))/a(1)**2
bb4 = -((5*a(2)**3 - 5*a(1)*a(2)*a(3) + a(1)*a(4))/a(1)**3)
bb5 = (14*a(2)**4 - 21*a(1)*a(2)**2*a(3) + 3*a(1)**2*a(3)**2 + \]
\[ 6*a(1)**2*a(2)*a(4) - a(1)**3*a(5))/a(1)**4 \]
bb6 = (-42*a(2)**5 + 84*a(1)*a(2)**3*a(3) + \]
\[ -28*a(1)**2*a(2)**2*a(4) + 7*a(1)**2*a(2)*(-4*a(3)**2 + a(1)*a(5)) + \]
\[ a(1)**3*(7*a(3)*a(4) - a(1)*a(6)))/a(1)**5 \]
p = (p**2*a(2))/a(1) + \]
\[ (p**3*(2*a(2)**2 - a(1)*a(3)))/a(1)**2 - \]
\[ (p**4*(-5*a(2)**3 - 5*a(1)*a(2)*a(3) + a(1)**2*a(4)))/a(1)**3 + \]
\[ (p**5*(-14*a(2)**4 - 21*a(1)*a(2)**2*a(3) + 3*a(1)**2*a(3)**2 + \]
\[ 6*a(1)**2*a(2)*a(4) - a(1)**3*a(5)))/a(1)**4 + \]
\[ (p**6*(-42*a(2)**5 + 84*a(1)*a(2)**3*a(3) - 28*a(1)**2*a(2)**2*a(4) + \]
\[ 7*a(1)**2*a(2)*(-4*a(3)**2 + a(1)*a(5)) + \]
\[ a(1)**3*(7*a(3)*a(4) - a(1)*a(6)))/a(1)**5 \]
```

\[ p=z ! independent variable of the reversion \]
\[ h = p*bb1 + p**2*bb2 + p**3*bb3 + p**4*bb4 + p**5*bb5 + p**6*bb6 \]

```c
h = p*bb(1) + p**2*bb(2) + p**3*bb(3) + p**4*bb(4)
```

```c
write((*,202) z,h)
```

5.6 Listing of RAY.FOR

The following is a listing of the program RAY.FOR. It calculates the surface velocity as a function of the frequency for a multi-layered elastic system.
5.6.1 Program listing

```
C ** N-LAYERED RAYLEIGH WAVE PROGRAM. MOD LEVEL 3. MAR 1970
C THIS PROGRAM COMPUTES THE VALUES OF THE SURFACE PHASE VELOCITY OF
C WAVES OF THE RAYLEIGH TYPE AT SELECTED WAVELENGTHS. ALL UNITS OF
C LENGTH ARE DIMENSIONLESS, AND ARE MULTIPLES OF THE THICKNESS OF THE
C FIRST(TOP) SURFACE LAYER. TEST DATA FOLLOWS:
C
C comparison with single-layer el=1,e2=100,nu1=nu2=0.4
C 002 1.0 .40 100.0 4
C 2.0 .2 2.0
C 1.0
C 1.4 0.2 0.1 0.01 0.19 0.26
C
C PROGRAM RAYL.
C
C X00G4 *10 025 COGILL TTI TEXAS TRANSPORTATION INSTI
C X00G4 *30 025 COGILL TTI TEXAS TRANSPORTATION INSTI
REAL*8 FREQ(100), PVEL(100), PWMTGH(100), PXN(100)
REAL*8 EM(6), V(6), XMU(6), BETA(6), ALPHA(6), RHO(6), HH(6),
1 FREINC
REAL*8 SQRT, XMIDA(6)
COMPLEX*16 XK, A(6), R(6), S(6), T(6), E(6, 4, 4),
1 PM(6, 4, 4), CINV(6, 4, 4), XI(4, 4), DELXJ(2, 2),
2 T6, T7, G(4, 4)
COMPLEX*16 CB1RES, CINRES, WLNTHM, DELEX
COMPLEX*16 CB1, CB1TRL, CINCR, WLNTH, CV1, CV2, CMPLX, XN,
1 DELTA1, DELTA2, DELTAN, DELTAP, FREQ, CVP, CVN
C COMMON ALPHA, BETA, XK, XMU, XMIDA, PM, T1, T7
COMMON CB1, DELTA1, DELTA2, CV1, CV2, CB1TRL, CINCR, DELTAN, DELTAP, 30
1 CVN, CVP, WLNTH, WSWTCH, NTRL, ICYCLE, ISWTC, T6, K, M, EM, V,
2 XN, XI, FREQ, FREINC, SLOPE, CINV, RHO
C CHARACTER*4 TITL(18)
C CHARACTER*4 ASTER
101 FORMAT(13, 3X, 12G8.0)
102 FORMAT(6X, 12F6.0)
103 FORMAT(10F6.0)
104 FORMAT(8F6.0)
105 FORMAT(18A4, F8.0)
C CALL ERSET (20B8.0,-1,1,1,207)
   open (1, file='ray.dat', status='old', form='formatted')
   open (3, file='ray.pxn', status='unknown', form='formatted')
   rewind 1
   rewind 3
1001 READ(1,105, end = 999) (TITLH(l), l=1,18), WSWTCH
C
C **WSWTCH** IS AN OUTPUT SWITCH WHICH CONTROLS THE
C PRINTING OF THE CONVERGENCE PARAMETER 'VALUE'.

```
C IF WSWITCH = 0 (OR LEFT BLANK) 'VALUE' IS NOT PRINTED
C WSWITCH = 1.0 'VALUE' IS PRINTED WITH THE FINAL OUTPUT
C WSWITCH = 2.0 'VALUE' AND ITS CORRESPONDING TRIAL VELOCITY
C 'CB1' ARE PRINTED AFTER EACH CYCLE OF CALCULATION.
C

NPAGE = 1
ASTER = '****'
WRITE (*,204) (ASTER,I = 1,5), (TITLE(I),I=1,18), (ASTER,I=1,5).
2 NPAGE
WRITE (3,204) (ASTER,I = 1,5), (TITLE(I),I=1,18), (ASTER,I=1,5).
2 NPAGE
READ(1,101) NS,(EM(I),V(I), I=1,6)
IF(NS.EQ.0) GO TO 9992
READ(1,102) (RHO(I), I = 1,8)
C201 FORMAT('TRIAL VELOCITY OUTSIDE ALLOWABLE RANGE')
C202 FORMAT (IH1)
204 FORMAT(1H1//1H0,5A4,1X18A4,1X5A4,1X7H PAGE,(I3))
205 FORMAT(1H0,4X,'THE PROBLEM PARAMETERS ARE'//
  2 (1H,3X,'LAYER',I3,') HAS MODULUS ',F10.0,
  3 ' POISSON'S RATIO ',F5.3,' DENSITY ',F5.1,
  4 ' AND THICKNESS ',F6.3,' UNITS')
206 FORMAT(1H,3X,'LAYER',I3,' HAS MODULUS ',F10.0,
  2 ' POISSON'S RATIO ',F5.3,' DENSITY ',F5.1,
  3 ' AND IS SEMI-INFINITE. '/)
207 FORMAT(2X,'WAVELENGTH VELOCITY ',11X,'FREQUENCY',9X,
  2 'WAVENUMBER COUNTER REG.'//)
209 FORMAT(1X,F8.4,8X,F8.4,4XF8.4,8X, 2F8.4,3X4,E18.4)
C NLINE = 17 + NS
IF(NS=6) 10,10,1
1 READ(1,102) (EM(I),V(I),I=7,NS)
C READ(1,102) (RHO(I), I = 7,NS)
10 N=NS-1
DO 1011 J = 1,6
1011 HH(I) = 0.0
READ(1,103) (HH(I),I=1,N)
WRITE(*,205) (I,EM(I),V(I),RHO(I),HH(I), I = 1,N)
WRITE(*,206) NS,EM(NS),V(NS),RHO(NS)
WRITE(*,207)
WRITE(3,205) (I,EM(I),V(I),RHO(I),HH(I), I = 1,N)
WRITE(3,206) NS,EM(NS),V(NS),RHO(NS)
WRITE(3,207)
DO 120 I =-1,100
PFREQ(I) =0.0
PVEL(I) =0.0
PWNGTH(I) =0.0
PXN(I) =0.0
120 CONTINUE
C
C THE FIVE CARDS STARTING WITH STATEMENT 126 ARE INSERTED
C IN ORDER TO PERMIT THE OPERATION OF THE FREE PLATE OPTION.
C    ISWITCH = 1 FOR SEMI-INFINITE SYSTEM
C    ISWITCH = 2 FOR COMPOUND PLATE
C
126   IF(EM(NS)) 127,127,128
127    NS = NS - 1
       ISWITCH = 2
       GO TO 1111
128    ISWITCH = 1
1111   DO 11 I=1,NS
       XLMDA(I)=V(I)*EM(I)/((1.4+V(I))*(1.-2.*V(I)))
       XMU(I)=EM(I)/(2.*(1.+V(I)))
       CONTINUE
C
C       WE SHALL HOLD THE WAVELENGTH CONSTANT DURING EACH ITERATION, AND
C       INTERPOLATE TO FIND THE PHASE VELOCITY 'CB1'.
C
DO 111 I=1,NS
       BETA(I)=SQRT(XMU(I)/XMU(1))**SQRT(RHO(1)/RHO(I))
111   ALPHAI=SQRT((XLMDA(I)+2.*XMU(I))/XLMDA(1)+
       12.*XMU(1)))**SQRT((XLMDA(1)+2.*XMU(1))/XMU(1)**SQRT(RHO(1)/RHO(I))
C
C       THIS (DO 111) ROUTINE DETERMINES BETA(I), ALPHAI AS MULTIPLES
C
 OF BETA(1)
       DELTAP = CMPLX(0.0,0.0)
       DELTAN = CMPLX(0.0,0.0)
       CVNP = CMPLX(0.0,0.0)
       CVN  = CMPLX(0.0,0.0)
C/212 READ(1,104) WLNTH,FREQ,FREINC
C       CB1TRL = WLNTH * FREQ1
C       CINCR = WLNTH * FREINC
C
READ (1,104) WLNTH,CB1TRL,CINCR,WLFAC,DELFAC
C       'WLNTH' IS THE MAXIMUM WAVELENGTH TO BE CALCULATED
C       'WLFAC' IS THE RATIO OF THE MINIMUM TO THE MAXIMUM WAVELENGTH.
C       'DELFAC' IS THE WAVENUMBER INTERVAL AT WHICH POINTS ARE TO BE
C       CALCULATED.  THE QUOTIENT 1/DELFAC GIVES THE NUMBER OF
C       POINTS TO BE CALCULATED, IN EACH UNIT OF WAVENUMBER.  FOR EXAMPLE,
C       IF THE RANGE FROM THE MINIMUM TO THE MAXIMUM WAVE NUMBER IS 6.0,
C       THE PROGRAM WILL RETURN 6.0/DELFAC POINTS ON THE FREQUENCY-DISPERSION
C       CURVE.
C
       IPILOT = 0
       CB1RES = CB1TRL
       CINRES = CINCR
       WLNTHM = WLFAC * WLNTH
       DELXN = DELFAC
       GO TO 1213
1212   WLNTH = WLNTH/(1 + DELXN*WLNTH)
       IF(REAL(WLNTH),LE, REAL(WLNTHM)) GO TO 9991
       CB1TRL = CB1RES
       CINCR = CINRES
1213 CONTINUE  
   IF(REAL(WLNGTH) .EQ.0.0 .AND. AIMAG(WLNGTH) .EQ. 0.0) GO TO 1001  
   CALL TRAVEL(N,NS,HH,A,R,S,T,E,G,DELXI,ITRACK)  
C900  CB1 = 0.5*(CV1+CV2)  
   XN = 1./WLNGTH  
   FREQ = CB1 * XN  
C    THE FOLLOWING 'IF' HAS BEEN DE-ACTIVATED TO SHORT CIRCUIT A  
C    LOOP THROUGH THE SPIRAL SEARCH, WHICH CAN OCCUR IF THE  
C    IMAGINARY PART OF 'CB1' IS ZERO.  WHC 71.024  
C    IF(ABS(AIMAG(CB1)).LT. 0.0001 .AND. NTRLS .LT. 10) GO TO 1213  
   I PLOT = IPLOT+ 1  
   PFREQ(IPLOT) = FREQ  
   PVEL(IPLOT) = CB1  
   PWNGTH(IPLOT) = WLNGTH  
   PXN(IPLOT) = XN  
C    OUTPUT FORMAT BASED ON WHC33B  
   IF (WSWITCH .EQ. 1.0) GO TO 901  
   GO TO 902  
901  DIFF=AMN1(CABS(DELTA1),CABS(DELTA2),CABS(DELTA3),CABS(DELTA4))  
   WRITE (*,209) REAL(WLNGTH),CB1,REAL(FREQ),XN,NTRLS,D1FF  
   WRITE (3,209) REAL(WLNGTH),CB1,REAL(FREQ),XN,NTRLS,D1FF  
   GO TO 1212  
902  WRITE (3,209) REAL(WLNGTH),CB1,REAL(FREQ),XN,NTRLS  
   WRITE (*,209) REAL(WLNGTH),CB1,REAL(FREQ),XN,NTRLS  
   GO TO 1212  
C999  WRITE (3,201)  
9991  CONTINUE  
9992  CONTINUE  
C9991  CALL FPLOT (PVEL, PWNGTH, PFREQ, PXN, TITLE, IPLOT)  
C9992  CALL CLOSE  
GO TO 1001  
999  stop  
END  
SUBROUTINE TRAVEL(N,NS,HH,A,R,S,T,E,G,DELXI,ITRACK)  
REAL*8 EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),HH(6),  
   FREINC  
REAL*8 ABS,XMLDA(6)  
COMPLEX*16  
   XK,A(6),R(6),S(6),T(6),E(6,4,4),  
1 PM(6,4,4),CINV(6,4,4),XI(4,4),VALUE,DELXI(2,2),  
2 T6,T7,G(4,4)  
COMPLEX*16 CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CMPLX,XN,  
1 DELTA1,DELTA2,DELTA3,DELTA4,FREQ,CVP,CVN  
COMPLEX*16 CELTA,Celta1,CElTa2  
COMPLEX*16 CB1REC  
COMPLEX*16 CDSSQR,CDCOS,CDSSQ,CDSIN,CHR,CSH,SRH,SSH  
COMMON ALPHA,BETA,XK,XMU,XMLDA,PM,T1,T7  
COMMON CB1,DELTA1,DELTA2,DELTA3,DELTA4,FREQ,CVP,CVN  
1 CVN,CVP,WLNGTH,WSWITCH,NTRLS,ICYS,ICYSW,T6,K,M,EM,V,  
2 XN,XJ,FREQ,FREINC,SLOPE,CINV,RHO  
203 FORMAT( ' CB1 = ', 2E14.4)
208 FORMAT(20X,'DELTA = ',2D14.4,',' CBIREC = ',2E14.4)
210 FORMAT(' RESIGNED AFTER ',J4, ' TRIALS')
211 FORMAT(' RESIGNED NCIRC = ',J4, ' ATTEMPTS')
NINTP = 0
NTRLS=0
ITRACK = 1
NREV = 5
C
C     IF THE PROGRAM OSCILLATES, MAKE THE PREVIOUS CARD 'NREV = 7'.
C     IF 'NREV = 1', THE PROGRAM IS SENSITIVE AND OSCILLATES READILY.
C
121 CONTINUE 210
IF(MOD(ITRACK,4).EQ. 1) ICYCLE = 1
IF(MOD(ITRACK,4).EQ. 2) ICYCLE = 2
IF(MOD(ITRACK,4).EQ. 3) ICYCLE = 3
IF(MOD(ITRACK,4).EQ. 0) ICYCLE = 4
C IF(REAL(CB1).EQ. 0.0) GO TO 1201
GO TO (1203,1204,1203,1204),ICYCLE
1203 CBI = CMPLX(REAL(CB1TRL)+REAL(CINCR),AIMAG(CB1TRL))
GO TO 1205
1204 CBI = CMPLX(REAL(CB1TRL),AIMAG(CB1TRL)+AIMAG(CINCR))
1205 CONTINUE 220
C GO TO 122
C     STATEMENT 1201 IS A BACKTRACKER IN CASE EITHER A(I), R(I) OR
C     S(I) IS ZERO, IF R(I) OR S(I) IS ZERO, A ZERO DIVIDE OCCURS IN
C     SUBROUTINE 'GMATRIX'. IF A(I) IS ZERO, THE RESULT IS NOT OF
C     INTEREST FOR THE PRESENT.
C     THE SEARCH FOR A VELOCITY BACKTRACKS TO THE MOST RECENT 'CBI'
C     YIELDING NON-ZERO A(I), R(I), S(I). IT RE-COMMENCES FORWARD
C     TRACKING USING AN INTERVAL ONE TENTH OF THE PREVIOUS ONE.
1201 CBI = CB1TRL + 0.1 * CINCR
C CINCR = 0.1 * CINCR 230
122 CONTINUE
WLNTH = CMPLX(REAL(WLNTH), REAL(WLNTH)*AIMAG(CB1)/REAL(CB1))
XK = 6.2831852/WLNTH
IF(NTRLS .GT.240) GO TO 8901
IF(WSWTCH .EQ. 0.0) GO TO 123
WRITE(3,203) CB1
WRITE(3*,203) CB1
C IF('CB1.GT.1.0,0.0 .OR.CB1.LT.BETA(NS),0.0) GO TO 999
123 CONTINUE
C DO 129 I = 1, NS
A(I) = 0.0
R(I) = 0.0
S(I) = 0.0
T(I) = 0.0
129 CONTINUE
131 CONTINUE
DO 130 I = 1,NS

72
A(I)=XK*XR*(1.-CB1*CB1/((BETA(I)/BETA(1))**2)))
R(I)=CDSQRT( XK*XR*(1.-CB1*CB1/((ALPHA(I)/BETA(1))**2)))
S(I)=CDSQRT( XK*XR*(1.-CB1*CB1/((BETA(I)/BETA(1))**2)))
T(I)=XMU(I)/XMU(1)
C THE FOLLOWING 'IF' HAS BEEN DE-ACTIVATED TO OBViate ERROR MD-3
C IF(A(I),EQ,CMLX(0,0,0,0),OR,R(I),EQ,CMLX(0,0,0,0),OR.
C I S(I),EQ,CMLX(0,0,0,0)) GO TO 1201
130 CONTINUE
C CALL GMATRX (A,R,S,T,G,N)
C
C 'C' 'S.
C
C CALL EMATRX (HH,A,R,S,T,E,N)
C CALL PROMAT(N,E,G,DEI)
C
C WE NOW HAVE THROWER'S 'I' MATRIX COMPLETE, FOR THE PARTICULAR
C WAVELENGTH 'W1NGHT' AND THE TRIAL VELOCITY 'CB1'.
C
C CALL CHECK(DEI,VALUE)
138 CONTINUE
IF(WSWITCH.EQ.2.0) GO TO 1371
GO TO 139
1371 WRITE (3,208) VALUE,ICYCLE,CB1REC
WRITE (*,208) VALUE,ICYCLE,CB1REC
139 IF(NTRL,NE.0) GO TO 140
DELTA1=VALUE
CV1 = CB1
GO TO 160
140 DELTA2=VALUE
CV2 = CB1
DIFVEL = CABS(CV2-CV1)
SUMVEL = CABS(CV1+CV2)
GO TO (1412,1412,1411,1411),ICYCLE
1411 PHI = PHI + PINCR
C
C ENTER SPIRAL SEARCH FOR IMPROVED ROOT
C
C EJECT IF CB1 IS PURELY REAL
204 FORMAT( RAD = ,E14.4)
IF(ABS(AIMAG(CB1)).LT.0.001) GO TO 9009
IF(NCIRC) 1416,1415,1416
1415 RAD = AIMAG(CB1REC)
1416 CONTINUE
CELTA = VALUE
IF(NCIRC,EQ.0) GO TO 1420
CELTA1 = CELTA2
1420 CELTA2 = CELTA
IF(NCIRC .EQ. 0) GO TO 1429
C SIGN CHANGE TEST FOLLOWS
IF(SIGN(1.,REAL(CELT1))-SIGN(1.,REAL(CELT2))) 1421, 1429, 1421
1421 IF(SIGN(1.,AIMAG(CELT1)) - SIGN(1.,AIMAG(CELT2))) 1422, 1429, 1422
1422 PHI = PHI - 1.5*PINC
CINC = CMPLX(XKONST*RAD*COS(PHI1),XKONST*RAD*SIN(PHI1))
PHI = 2 * PI
C
'CV1', 'CV2' ARE NEEDED AT STATEMENT 900, IN THE MAIN PROGRAM.
CV1 = CB1REC
CB1REC = CB1REC + 0.3 * CINC
CV2 = CB1REC
PINC = -PINC
1429 NCIRC = NCIRC + 1
IF(NCIRC .GT. 40) GO TO 9008
IF(PHI = 1.9 * PI) 1417, 1413, 1413
1417 IF(PHI+1.9*PI) 1413, 1413, 1414
1413 XKONST = 0.7 * XKONST
IF(XKONST*RAD .LT. 0.002) GO TO 9009
PHI = 0.0
1414 CINC = CMPLX(XKONST*RAD*COS(PHI),XKONST*RAD*SIN(PHI))
CB1 = CB1REC + CINC
GO TO 122
1412 CONTINUE
IF(DIFVEL .LT.0.0005 * SUMVEL) GO TO 900
IF(NINTP .NE. 0) GO TO 300
GO TO (1401,1402,1402,1401), ICYCLE
1401 IF(SIGN(1.,REAL(DELTA2)) -SIGN(1.,REAL(DELTA1))) 300, 150, 300
1402 IF(SIGN(1.,AIMAG(DELTA2)) -SIGN(1.,AIMAG(DELTA1))) 300, 150, 300
C1403 CONTINUE
150 GO TO (1501,1502,1502,1501), ICYCLE
1501 IF(ABS(REAL(DELTA1)) -ABS(REAL(DELTA2))) 151, 152, 1512
1502 IF(ABS(AIMAG(DELTA1)) -ABS(AIMAG(DELTA2))) 151, 152, 1512
C1503 CONTINUE
C
ROUTINE TO OVERCOME SIGNIFICANCE ERROR
151 NREV = NREV - 1
IF(NREV) 1511, 152, 152
1511 CINC = -2.*CINC
152 DELTA1 = DELTA2
CV1 = CV2
160 NTRLS = NTRLS + 1
CB1TRL = CB1
C
ITRACK = ITRACK + 1
GO TO 121
C
C INTERPOLATE ROUTINE, RETAINING THE MOST RECENT DELTAS
C OF OPPOSITE SIGN
C
C PREPARE FOR INTERPOLATION ROUTINE. THE PROGRAM PERFORMS THE
C FOLLOWING OPERATIONS IN A CYCLIC FORM.
C(1)TRACK 'CB1' ALONG ITS REAL*8 AXIS, SEEK ZERO ON REAL*8 AXIS OF 'DELTA'.
C(2)TRACK 'CB1' ALONG ITS IMAG AXIS, SEEK ZERO ON IMAG AXIS OF 'DELTA'.

74
C(3)TRACK 'CBI' ALONG ITS REAL*8 AXIS, SEEK ZERO ON IMAG AXIS OF 'DELTA'.
C(4)TRACK 'CBI' ALONG ITS IMAG AXIS, SEEK ZERO ON REAL*8 AXIS OF 'DELTA'.
C
C
C FIRST CONNECT THE REAL*8 AND IMAGINARY PARTS OF THE INTERPOLATED
C 'DELTA1'. THE SEQUENCE IS CONTROLLED BY 'ICYCLE', AND IS SET
C BY THE 'MOD' SWITCH FOLLOWING STATEMENT 121.
C
300 CONTINUE
   GO TO (3001,3002,3002,3001),ICYCLE
3001 IF(REAL(Delta1)) 301,302,302
3002 IF(AIMAG(Delta1)) 301,302,302
3003 CONTINUE
301 CONTINUE
   GO TO (3011,3012,3012,3011),ICYCLE
3011 DELTAN = CMPLX(REAL(Delta1),AIMAG(Delta1))
   GO TO 3013
3012 DELTAN = CMPLX(REAL(Delta1),AIMAG(Delta1))
3013 CONTINUE
C NOW CONNECT THE PARTS OF 'CVI' AND 'CVN'.
   GO TO (3014,3015,3014,3015),ICYCLE
3014 CVN = CMPLX(REAL(CVI),AIMAG(CVN))
   GO TO 3016
3015 CVN = CMPLX(REAL(CVI),AIMAG(CVI))
3016 CONTINUE
   GO TO 303
302 CONTINUE
   GO TO (3021,3022,3022,3021),ICYCLE
3021 DELTAP = CMPLX(REAL(Delta1),AIMAG(DELTAP))
   GO TO 3023
3022 DELTAP = CMPLX(REAL(DELTAP),AIMAG(Delta1))
3023 CONTINUE
C NOW CONNECT THE PARTS OF 'CVI' AND 'CVP'.
   GO TO (3024,3025,3024,3025),ICYCLE
3024 CVP = CMPLX(REAL(CVI),AIMAG(CVP))
   GO TO 3026
3025 CVP = CMPLX(REAL(CVP),AIMAG(CVI))
3026 CONTINUE
C
C NOW CONNECT THE REAL*8 AND IMAGINARY PARTS OF THE INTERPOLATED
C 'DELTA2'
C
303 CONTINUE
   GO TO (3031,3032,3032,3031),ICYCLE
3031 IF(REAL(Delta2)) 305,306,306
3032 IF(AIMAG(Delta2)) 305,306,306
3033 CONTINUE
305 CONTINUE
   GO TO (3051,3052,3052,3051),ICYCLE
3051 DELTAN = CMPLX(REAL(Delta2),AIMAG(Delta1))
GO TO 3053
3052 DELTAN = CMPLX(REAL(DELTAN),AIMAG(DELTAN))
3053 CONTINUE
C    NOW CONNECT THE PARTS OF 'CV2' AND 'CVN'.
    GO TO (3054,3055,3054,3055),ICYCLE
3054 CVN = CMPLX(REAL(CV2),AIMAG(CV2))
    GO TO 3056
3055 CVN = CMPLX(REAL(CV2),AIMAG(CV2))
3056 CONTINUE
    GO TO 307
306 CONTINUE
    GO TO (3061,3062,3062,3061),ICYCLE
3061 DELTAP = CMPLX(REAL(DELTAP),AIMAG(DELTAP))
    GO TO 3063
3062 DELTAP = CMPLX(REAL(DELTAP),AIMAG(DELTAP))
3063 CONTINUE
C    NOW CONNECT THE PARTS OF 'CV2' AND 'CVP'.
    GO TO (3064,3065,3064,3065),ICYCLE
3064 CVP = CMPLX(REAL(CV2),AIMAG(CV2))
    GO TO 3066
3065 CVP = CMPLX(REAL(CV2),AIMAG(CV2))
3066 CONTINUE
307 NINTP = 1
    GO TO (3071,3072,3072,3071),ICYCLE
3071 DP = REAL(DELTAP)
    DN = REAL(DELTAN)
    GO TO 3073
3072 DP = AIMAG(DELTAP)
    DN = AIMAG(DELTAN)
3073 CONTINUE
    GO TO (3074,3075,3074,3075),ICYCLE
3074 CP = REAL(CVP)
    CN = REAL(CVN)
    GO TO 3076
3075 CP = AIMAG(CVP)
    CN = AIMAG(CVN)
3076 CONTINUE
    IF(CP.EQ.0.0.AND.CN.EQ.0.0.OR.DP.EQ.0.0.AND.DN.EQ.0.0)GO TO 900
    FN = XTERPL(DP,DN,CP,CN)
    GO TO (3077,3078,3077,3078),ICYCLE
3077 CBI = CMPLX(FN,AIMAG(CB1))
    GO TO 3079
3078 CBI = CMPLX(REAL(CB1),FN)
3079 CONTINUE
    NREV = 3
    NTRLS = NTRLS + 1
    DELTA1 = DELTA2
    CV1 = CV2
    GO TO 122
C
900 CONTINUE
   NINTP = 0
   GO TO (3082,3081,3082,3082), ICYCLE
3081 CBIREC = CB1
   CBITRL = CB1
   NCIRC = 0
   XKONST = 1.0
   PHI = 0
   PI = 3.14159265
   PINCR = PI/4
3082 CONTINUE
   GO TO (9081,9084,9081,9082), ICYCLE
9081 CINCR = CMPLX(-.5*REAL(CINCR),AIMAG(CINCR))
   GO TO 9083
9082 CINCR = CMPLX(REAL(CINCR),-.5*AIMAG(CINCR))
   GO TO 9083
9084 CINCR = CMPLX(AIMAG(CBIREC),AIMAG(CINCR))
9083 CONTINUE
   GO TO (9091,9092,9091,9092), ICYCLE
9091 CBITRL = CMPLX(REAL(CB1),AIMAG(CBITRL))
   GO TO 9093
9092 CBITRL = CMPLX(REAL(CBITRL),AIMAG(CB1))
9093 CONTINUE
   GO TO (9001,9001,9001,9009), ICYCLE
9001 ITRACK = ITRACK + 1
   GO TO 121
9008 WRITE(3,211) NCIRC
   WRITE(*,211) NCIRC
   RETURN
9009 CONTINUE
   RETURN
8901 WRITE(3,210) NTRLS
   WRITE(*,210) NTRLS
   RETURN
C       DEBUG TRACE, SUBCHK, SUBTRACE, INIT(CV1,CV2,CBIREC, CBITRL, VALUE, NCIRC
C       *, CELTA, CELTA1, CELTA2, PHI, CINCR, PHI2, DIFVEL, SUMVEL)
C       AT 121
C       TRACE ON
C       AT 123
C       TRACE OFF
C       AT 140
C       TRACE ON
C       AT 1412
C       TRACE OFF
C       AT 900
C       TRACE ON
C       AT 9009
C       TRACE OFF
END
SUBROUTINE GMATRX (A,R,S,T,G,NS)
GMATRX
REAL*8 EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),
1 FREINC
500
COMPLEX*16 XK,A(6),R(6),S(6),T(6),
1 PM(6,4,4),CINV(6,4,4),XI(4,4),
2 T6,T7,G(4,4)
REAL*8 XLMDA(6)
COMPLEX*16 CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,XN,
1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CYN
COMMON ALPHA,BETA,XK,XMU,XLMDA,PM,T1,T7
COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
1 CYN,CVP,WLNGTH,WSWTCH,NTRL,S,ICYCLE,ISWTCH,T6,K,M,EM,V,
2 XN,XI,FREQ,FREINC,SLOPE,CINV,RHO

C
C DEFINITION OF MATRIX ‘G’, AS GIVEN IN THROWER’S PAPER

C
DO 1 J = 1,4
DO 1 I = 1,4
1 G(I,J) = 0.0
11 DO 10 I = NS,NS
I=NS
C
C
G(1,1) = 1/(2.*T(I)*(A(I)-XK**2))
G(1,2) = XK/(2.*T(I)*R(I)*(A(I)-XK**2))
G(1,3) = A(I)/(R(I)*(A(I)-XK**2))
G(1,4) = XK/(A(I)-XK**2)
G(2,1) = XK/(2.*T(I)*S(I)*(A(I)-XK**2))
G(2,2) = G(1,1)
G(2,3) = G(1,4)
G(2,4) = A(I)/(S(I)*(A(I)-XK**2))
10 CONTINUE
RETURN
END

SUBROUTINE EMATRIX (HH,A,R,S,T,E,N) EMATRIX
REAL*8 EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(5),HH(6),
1 FREINC
REAL*8 XLMDA(6)
COMPLEX*16 XK,A(6),R(6),S(6),T(6),E(6,4,4),
1 PM(6,4,4),CINV(6,4,4),XI(4,4),
2 T6,T7
COMPLEX*16 CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,XN,
1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CYN
COMPLEX*16 CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
1 CYN,CVP,WLNGTH,WSWTCH,NTRL,S,ICYCLE,ISWTCH,T6,K,M,EM,V,
2 XN,XI,FREQ,FREINC,SLOPE,CINV,RHO

C SET UP THE ELEMENTS OF THROWER’S “E” MATRIX
C
DO 401 I = 1,6

78
DO 401 J = 1, 4
DO 401 K = 1, 4
401   E(I,J,K) = 0.0
402   DO 499 I = 1, N
   AIXK = A(I) - XK*XK
   R1 = CEXP(R(I)*HH(I))
   R2 = CEXP(-R(I)*HH(I))
   CRH = 0.5*(R1 + R2)
   SRH = 0.5*(R1 - R2)
   S1 = CEXP(S(I)*HH(I))
   S2 = CEXP(-S(I)*HH(I))
   CSH = 0.5*(S1+S2)
   SSH = 0.5*(S1-S2)
   E(I,1,1) = (A(I)*CRH - XK*XK*CSH/AIXK
   E(I,1,2) = XK*A(I)*SRH/R(I) - S(I)*SSH/AIXK
   E(I,1,3) = 2.*T(I)*(A(I)*SRH/R(I) - XK*XK*S(I)*SSH/AIXK
   E(I,1,4) = 2.*T(I)*XK*A(I)*(CRH - CSH/AIXK
   E(I,2,1) = -XK*(R(I)*SRH - A(I)*SSH/S(I))/AIXK
   E(I,2,2) = (A(I)*CSH - XK*XK*CRH)/AIXK
   E(I,2,3) = -2.*T(I)*XK*A(I)*(CRH - CSH /AIXK
   E(I,2,4) = -2.*T(I)*XK*XK*SRH - A(I)*A(I)*SSH/S(I))/AIXK
   E(I,3,1) = (R(I)*SRH - XK*XK*SSH/S(I))/(2.*T(I)*AIXK
   E(I,3,2) = XK*(CRH - CSH)/(2.*T(I)*AIXK)
   E(I,3,3) = (A(I)*CRH - XK*XK*CSH)/AIXK
   E(I,3,4) = XK*(R(I)*SRH - A(I)*SSH/S(I))/AIXK
   E(I,4,1) = -XK*(CRH - CSH)/(2.*T(I)*AIXK
   E(I,4,2) = -(XK*XK*SRH/R(I) - S(I)*SSH)/(2.*T(I)*AIXK)
   E(I,4,3) = -XK*(A(I)*SRH/R(I) - S(I)*SSH)/AIXK
   E(I,4,4) = -(XK*XK*CRH-A(I)*CSH)/AIXK
   CONTINUE
   RETURN
END
SUBROUTINE PROMAT(N,E,G,DELXI)
REAL*8 EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),
1    FREINC
REAL*8 XLMDA(6)
COMPLEX*16    XK,E(6,4,4),
1    PM(6,4,4),CVN(6,4,4),XJ(4,4),DELXJ(2,2),
2    T5,T7,G(4,4)
COMPLEX*16    CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,XN,
1    DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CVN
COMMON ALPHA,BETA,XK,XMU,XLMDA,PM,T1,T7
COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
1    CVN,CVP,WLNGTH,WSWTCH,NTRL5,ICYCLE,ISWTCH,T6,K,M,EM,V,
2    XN,XJ,FREQ,FREINC,SLOPE,CINV,RHO
C201 FORMAT('XH12.'*,12.) = 'D14.8'
C
C    CONTINUED PRODUCT OF THE 'E' MATRICES.
START SETTING UP THE PRODUCT MATRICES "PM". THEY ARE OBTAINED USING THE CONTINUED PRODUCT OF THE 'E' MATRICES.
IS PRE-MULTIPLIED BY THE 'G' MATRIX FOR THE SEMI-INFINITE MEDIUM.

\[ \begin{align*}
\text{NS} &= \text{NO. OF MEDIA} \\
N &= \text{NO. OF LAYERS} \\
\text{K} &= \text{NS - 1} \\
\end{align*} \]

DO 26 K = 1,N 
DO 20 J = 1,4 
DO 20 M = 1,4 
PM(K,M,J) = 0.0 
IF(K-1) 21,21,23 
PM(I,J,J) = E(I,1,J) IS THE 'E' MATRIX FOR THE TOP LAYER 
DO 22 J = 1,2 
DO 22 I = 1,4 
PM(K,I,J) = E(K,I,J+2) 
GO TO 26 
CONTINUE 

PRE-MULTIPLY THE PRODUCT MATRIX THUS FAR ESTABLISHED BY THE 'E' MATRICES FOR THE SUCCESSIVELY LOWER LAYERS 

FOR AN EXPLANATION OF THE STRANGE APPEARANCE OF 'PM' AT THIS STAGE SEE THROWER'S EQUATION (19). AS THE INITIAL 'PM' MATRIX (THE CONTRIBUTION FROM THE TOP LAYER) IS ONLY A 4(ROW) X 2(COLUMN) MATRIX, SUBSEQUENT 'PM' MATRICES ARE ONLY 4(ROW) X 2(COLUMN) MATRICES. THIS CONSTITUTES THE RATHER ASTOUNDING ECONOMY OF THE METHOD.

DO 25 J = 1,2 
DO 25 I = 1,4 
T6 = E(K,I,1)*PM(K-1,1,J) + E(K,I,2)*PM(K-1,2,J) 
+ E(K,I,3)*PM(K-1,3,J) + E(K,I,4)*PM(K-1,4,J) 
PM(K,I,J) = T6 
CONTINUE 

NOW PRE-MULTIPLY THE PRODUCT MATRIX 'PM' BY THE 'G' MATRIX, A 2 X 4 RECTANGULAR MATRIX DEFINED IN THROWER'S PAPER. THIS YIELDS THE 'TEST' DETERMINANT DENOTED BY 'DELXJ'.

CONTINUE 
GO TO (27,36), ISWITCH 
DO 33 I = 1,2 
DO 33 J = 1,2 
DELXJ(I,J) = 0.0 
DO 35 J = 1,2 
DO 35 I = 1,2 
T7 = G(I,1)*PM(N,1,J) + G(I,2)*PM(N,2,J) 
+ G(I,3)*PM(N,3,J) + G(I,4)*PM(N,4,J) 
DELXJ(I,J) = T7
5.7 Listing of PREP.FOR

The following is a listing of the program PREP.FOR. It prepares the datafile RAY.DAT as input to the program RAY.FOR.

5.7.1 Program listing

```
PROGRAM PREP
C    THIS PROGRAM PREPARES DATA FOR RAY. THE INPUT IS IN FILE PREP.DAT
C    THE DATA LINES INPREP.DAT ARE TITLE,
C    NS,(EM(I),V(I), I=1,NS),SWITCH
C    (RHO(I),I=1,NS)
C    (HH(I),I=1,NS)
```
C WLENGTH,CDITRA,CINCR ALLCOMPLEX; WLFAC,DELFAC BOTH REAL
C
DIMENSION EM(5),V(5),RHO(5),HH(5),TITLE(20)
COMPLEX WLENGTH,CB1TRL,CINCR

OPEN(1,FILE='PREP.DAT',STATUS='OLD')

OPEN(3,FILE='RAY.DAT',STATUS='UNKNOWN')
REWIND 3

101 FORMAT(20A4)
300 WRITE(*,*) 'ENTER TITLE'
READ(1,101,END=999) (TITLE(I), I = 1,19)
WRITE(*,*) 'NS, (EM(I),V(I),I=1,NS), SWITCH'
READ(1,*) NS,(EM(I),V(I),I=1,NS),SWITCH
WRITE(*,*) (REO(I), I = 1, NS)
READ(1,*) (RHO(I), I = 1, NS)
WRITE(*,*) 'HH(I),I=1,N'
N = NS - 1
READ(1,*) (HH(I), I = 1, N)

WRITE(*,*) 'WLENGTH,CB1TRL,CINCR,COMPLEX,WLFAC,DELFAC,REAL'
WRITE(*,*) 'ENTER WLENGTH,CB1TRL,CINCR ALL COMPLEX'

C 'WLFAC,DELFAC BOTH REAL'
C 'WLENGTH IS THE MAXIMUM WAVELENGTH TO BE CALCULATED'
C 'WLFAC IS THE RATIO OF THE MINIMUM TO THE MAXIMUM WAVELENGTH.
C 'DELFAC IS THE WAVENUMBER INTERVAL AT WHICH POINTS ARE TO BE CALCULATED. THE QUOTIENT 1/DELFAC GIVES THE NUMBER OF
C POINTS TO BE CALCULATED, IN EACH UNIT OF WAVENUMBER. FOR EXAMPLE,
C IF THE RANGE FROM THE MINIMUM TO THE MAXIMUM WAVELENGTH IS 60,
C THE PROGRAM WILL RETURN 60/DELFAC POINTS ON THE FREQUENCY-DISPERSION
C CURVE.'

C
2001 FORMAT(19A4,F4.0)
201 FORMAT(13,3X,6(F6.0,F6.2))
202 FORMAT(6X,12F6.0)
203 FORMAT(10F6.2)
204 FORMAT(8F6.4)
WRITE (3,2001) (TITLE(I),I=1,19),SWITCH
WRITE (3,201) NS,(EM(I),V(I),I=1,NS)
WRITE (3,202) (RHO(I),I=1,NS)
N = NS - 1
WRITE (3,203) (HH(I),I=1,N)
WRITE (3,204) WLENGTH,CB1TRL,CINCR,WLFAC,DELFAC

C GO TO 300

999 CONTINUE
CLOSE(UNIT=3)
STOP 'END OF FILE'
END
5.7.1.1 Program listing

Sample data follows for program PREP.FOR.

MARTINZEC55
E1=11100, E2=8400, E3=280; H1=0.13, H2=0.25, NU1=NU2=NU3=0.25 5 11100.
.25 84000. .25 20000. .25 40000. .25 280. .25 0.0
2. 2. 2. 2. 2. 2. 2.
0.05 0.08 0.25 0.10 (5, 0.0) (0.4, 0.00) (0.05, 0.01) 0.0001 0.1

5.8 Listing of HARDK.MWS

The following is a listing of the Maple worksheet HARDK.MWS. It can be used to
plot the response of a system consisting of a layer of material having a high stiffness
overlying a semi-infinite medium containing a material having a lower stiffness.

5.8.1 Listing of worksheet

This file uses implicitplot to plot the relationship between <k> and <w> given that
xi1*eta2=xi28eta1=0.
> eq1:=[a1 = 25, a2 = 2.5, b1 = 10.0, b2 = 1.0, mu1 = 100.0, mu2 = 1.0]:
> eq2:=[c=w/k, kb1 = w / b1, kb2 = w / b2]:
> Use <k> as a dependent variable.

> eq3:=[r1 = w*(1/c**2 - 1/a1**2)**(1/2), s1 = w*(1/c**2 - 1/b1**2)**(1/2),
> r2 = w*(-1/c**2 + 1/a2**2)**(1/2), s2 = w*(-1/c**2 + 1/b2**2)**(1/2),
> xx = mu2/mu1 * kb2**2/k**2 - 2 * (mu2/mu1 - 1),
> yy = kb1**2/k**2 + 2 * (mu2/mu1 - 1), zz = mu2/mu1 *
> kb2**2/k**2 - kb1**2 /k**2 - 2 * (mu2/mu1 - 1)]:
> eq3:=subs(eq1, eq2, eq3):
> eq4 := subs(eq2, eq1, eq4):
> Substitute the independent variables in equation (4)
> h := 0.1:
> Set the thickness of the layer equal to unity
> x1 := ((2-kb1^2/k^2) * (xx* cosh( r1*h) + r2/r1* yy* sinh( r1*h) )
>   -2* s1/k* (r2/k* ww* sinh( s1*h) + k/s1* zz* cosh( s1*h) )
> )
> x2 := ((2-kb1^2/k^2) * (s2/k* ww* cosh( r1*h) + k/r1* zz* sinh( r1*h) )
>   -2* s1/k* (xx* sinh( s1*h) + s2/s1* yy* cosh( s1*h) )
> )
> e1 := ((2-kb1^2/k^2) * (r2/k* ww* cosh( s1*h) + k/s1* zz* sinh( s1*h) )
>   -2* r1/k* ( xx* sinh(w* r1*h) + r2/r1* yy* cosh( r1*h) )
> )
> e2 := ((2-kb1^2/k^2) * (xx* cosh( s1*h) + s2/s1* yy* sinh( s1*h) )
>   -2* r1/k* (s2/k* ww* sinh( r1*h) + k/r1* zz* cosh( r1*h) )
> )
> x1 := subs(eq4,eq3,eq2,eq1,x1):
> Substitute the independent variables in xi and the eta
> x2 := subs(eq4,eq3,eq2,eq1,x2):
> e1 := subs(eq4,eq3,eq2,eq1,e1):
> e2 := subs(eq4,eq3,eq2,eq1,e2):
> simplify(x1*e2-x2*e1):
> Use this statement as a debugger
> with(plots):

Warning, the name changecoords has been redefined
> implicitplot(x1*e2=x2*e1,w=0.1..2.0,k=0.002..0.4,font=[TIMES,BOLD,1
> ,title="Plot of <k> vs <w> for hard over
> soft",labelfont=[HORIZONTAL,VERTICAL],labels=["Frequency
> omega","Reciprocal wavelength k"]);
> Plot of x1*e2-x2*e1=0, showing <k> and <w>
5.9 Listing of SOFTK.MWS

The following is a listing of the Maple worksheet SOFTK.MWS. It can be used to plot the response of a system consisting of a layer of material having a low stiffness overlying a semi-infinite medium containing a material having a higher stiffness.

5.9.1 Listing of worksheet

```maple
> eq1:=\{a1 = 2.5, a2 = 25, b1 = 1.0, b2 = 10.0, mu1 = 1.0, mu2 = 100.0\};
> eq2:=\{c=k/w, kbl = w / b1, kb2 = w / b2\};
> eq3:=\{r1 = w \cdot (-1/c**2 + 1/a1**2)**(1/2), s1 = w*\n        1/c**2 + 1/b1**2**2)**(1/2),
     r2 = w*(1/c**2 - 1/a2**2)**(1/2), s2 = w*(1/c**2
     - 1/b2**2**2)**(1/2)\};
> eq4:=\{ww = 2 * (mu2/mu1 - 1), xx = mu2/mu1 * kb2**2/k**2 - 2*
        (mu2/mu1 - 1),
     yy = kb1**2/k**2 + 2 * (mu2/mu1 - 1), zz = mu2/mu1 *
     kb2**2/k**2 - kb1**2/k**2 - 2 * (mu2/mu1 - 1)\};
> eq4:=subs(eq2,eq1,eq4): # This validates eq4
> eq3:=subs(eq2,eq1,eq3);

> x1 := ((2-kbl**2/k**2)**(1/2) * (xx* cos(r1* h) + r2/r1* yy* sin(r1* h))
    +2* s1/k* (r2/k* ww* sin(s1* h) - k/s1* zz* cos(s1* h)))
> x2 := ((2-kbl**2/k**2)**(1/2) * (s2/k* ww* cos(r1* h) + k/r1* zz* sin(r1* h))
    +2* s1/k* (xx* sin(s1* h) - s2/s1* yy* cos(s1* h))):
```
> e1 := ((2-kb1^2/k^2) * (r2/k* \( w \)\( w \) cos( s1* h) + k/s1* zz* sin( s1* h))
> )
> ) +2* r1/k* ( xx* \( \sin(\( w \) \( r1 \)* h) - r2/r1* yy* cos( r1* h))
> ));
> e2 := ((2-kb1^2/k^2) * (xx* \( \cos(\( s1 \)* h) +s2/s1* yy* sin( s1* h))
> ) +2* r1/k* (s2/k* \( w \)* \( \sin(\( r1 \)* h) - k/r1* zz* cos( r1* h)
> )
> );
> x1 := subs(eq4,eq3,eq2,eq1,x1):
> Update the \( x1 \) and the eta
> x2 := subs(eq4,eq3,eq2,eq1,x2):
> e1 := subs(eq4,eq3,eq2,eq1,e1):
> e2 := subs(eq4,eq3,eq2,eq1,e2):
> h:=1.0:
> eq=x1*e2-x2*e1:
> #solve(eq,c);
> with(plots):
> implicitplot(x1*e2=x2*e1,\( w \)=0.001..0.03,\( k \)=0.005..0.1,font=[TIMES,BOLD
> 14],title="Plot of \( k \) vs \( w \) for soft over
> hard",labeldirections={HORIZONTAL,VERTICAL},labels="Frequency
> omega","Reciprocal wavelength \( k \)"));
> Plot of x1*e2-x2*e1=0, showing \( k \) and \( w \)