A Theoretical Model for Curve Squeal

D.J. Thompson and A.D. Monk-Steel

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by

D.J. Thompson and A.D. Monk-Steel

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Authorised for issue by
Professor M.J. Brennan
Group Chairman

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ABSTRACT

A railway wheel in a rigid bogie that is negotiating curved track is unable to align its rolling direction tangentially to the curve of the rail. In sharp curves, the forces at the wheel / rail interface that guide the bogie around the curve operate in a regime of high creepage. Due to the falling friction characteristic at large slip velocities this causes excitation of the wheels making them prone to curve squeal. There are two main types of curve squeal, characterised by the mechanisms of excitation: (i) stick/slip excitation due to creepage of the wheel perpendicular to the rolling direction (ii) squeal due to wheel flange contact with the rail. In the case of the former, the contact position on the wheel tread has been found to be important in determining the likelihood of squeal. In the latter case more complex equations coupling the wheel and rail apply. This paper presents theoretical models for the generation of curve squeal in each case and describes a theoretical investigation into the influence of contact parameters such as lateral contact position and contact angle on squeal.
1. INTRODUCTION

When a railway vehicle traverses a sharp curve, tonal noise of a high amplitude may be generated. This is known as 'curve squeal'. A consortium co-ordinated by ERRI and including DB, SBB, TNO and ISVR is currently conducting research into the phenomenon of curve squeal under contract to UIC. Within Phase 1 of this project, ISVR together with TNO are responsible for Work Package 4A, "Theoretical model for curve squeal". An interim report summarising progress made by ISVR was presented at the meeting on 25 October 2002. The current report presents the status as at February 2003, including the parts previously presented.

A theoretical model for curve squeal has previously been developed by TNO [1-4]. This model is based on excitation by unstable lateral creepage. It consists of two parts: a first part in the frequency domain can be used to determine instability and to predict which mode is most likely to be excited, and a second part in the time domain calculates the amplitude of the squeal noise. Within WP4A of Phase 1, TNO are due to implement these existing models within the TWINS program for rolling noise [5].

The main work to be carried out by ISVR under this contract is to extend the current theoretical models by including longitudinal and spin creepage into the model and to account for flange contact. It is not the intention at this stage to implement these models in software such as TWINS for third-party use. Rather, the development consists of

(i) theoretical development of the governing equations,
(ii) implementation in a preliminary program, not for general use. For this, the Matlab environment is used,
(iii) example calculations to indicate the behaviour of the model.

The model is limited to the question of stability at this stage. It does not include the time-domain calculation of squeal amplitudes.

2. BACKGROUND

Observations indicate that the highest squeal noise amplitude is usually generated by the leading inner wheel of a four-wheeled bogie or two-axle vehicle. This noise has been associated with stick-slip lateral motion at the contact between the wheel tread and the rail head [1, 6-10]. This is referred to here as squeal due to lateral creepage. The fundamental frequency of such squeal noise corresponds to a natural frequency of the wheel and is often in the range 200 to 2000 Hz.

Contact between the wheel flange and the rail, which occurs at the leading outer wheel (and possibly the trailing inner wheel) in sharp curves, has generally been found to reduce the likelihood of stick-slip squeal due to lateral creepage at this wheel. For example, Remington [6] concluded from laboratory experiments (due to Bleedorn and Johnstone [11]) that flange contact reduces the level of squeal noise.

However, it is thought that flange contact may generate a different form of squeal noise [7] that is referred to here as flange squeal. Compared with squeal due to lateral creepage, this generally has a considerably higher fundamental frequency, may have a lower level and is often more intermittent in nature. Nevertheless it can be a source of considerable annoyance. It is usually associated with flange contact, either with the outer running rail or with check
rails, in sharp curves or wing rails in points and crossings. Compared with stick-slip squeal due to lateral creepage, flange noise has received much less attention. The models in [1-4, 6-10] are all for noise generated at a simple wheel/rail contact with no flange contact.

This report describes a new model that has been developed. This allows for an arbitrary contact angle and includes lateral, longitudinal and spin creepages. It may therefore be considered to apply to flange squeal as well as squeal due to lateral creepage. It is the first such model to include excitation in directions other than lateral.

3. THEORETICAL DEVELOPMENT

3.1 Longitudinal creepage

Consider a wheel and a rail as shown in Figure 1 with the axis system x-y-z (x vertical, z along the track) and the axis system 1-2-3 local to the contact, with 3 normal to the contact, 2 transverse and 1 along the track\(^1\).

During rolling, the wheel does not move at exactly the velocity of the train. For example, if traction forces are applied, the wheel tends to rotate slightly faster than it would in an undriven state. A particle on the wheel surface therefore passes through the contact zone at a higher average speed than a corresponding particle on the rail. If \(v^W_1\) is the velocity of the wheel at the contact and \(v^R_1\) is that of the rail, the relative slip velocity is given by, \(v_1 = v^W_1 - v^R_1\). This relative slip velocity divided by the mean rolling velocity \(V\) is defined as the longitudinal creepage, \(\gamma_1 = v_1 / V\). According to the definitions in Figure 1, \(v_1\) and hence \(\gamma_1\) is negative when the wheel is under traction.

![Figure 1. Axis systems at the wheel/rail contact for a right-hand wheel.](image)

\(\text{1} \) It is usual in studies of creepage (e.g. [12]) to use \(x\) for the longitudinal direction, \(y\) for the lateral direction and \(z\) for the vertical direction. The notation used here is chosen for consistency with the TWINS model for rolling noise [5]. The 1-2-3 system is consistent with \((x,y,z)\) in [12].
This creepage gives rise to a friction force, $F_1$, known as the creep force, which acts on the wheel in the opposite direction to the slip velocity. An equal and opposite force $-F_1$ acts on the rail. The relationship between the creep force and the creepage has been investigated extensively by Kalker [12].

At low values of creepage, the creep force $F_1$ increases linearly in proportion to the creepage [12]:

$$F_1 = -G \, ab \, C_{11} \, \gamma$$

(1)

where $G$ is the shear modulus (the two materials are assumed here to be identical), $a$ and $b$ the contact patch semi-axis lengths, assuming the contact patch to be an ellipse ($a$ is in the direction of rolling). The creep coefficient $C_{11}$ depends on the Poisson's ratio of the wheel and rail material(s) and on the aspect ratio, $\nu = a / b$. It has been tabulated by Kalker [12]. Note that the normal load $N$ has an influence on the creep force through the product $ab$ which is proportional to $N^{\nu/3}$. The minus sign indicates that the creep force acts in the opposite direction to the creepage.

### 3.2 Lateral and spin creepage

As well as the relative velocity in the longitudinal direction, the wheel can have a lateral velocity relative to the rail, given by, $v_2 = v_{w2} - v_{r2}$. This gives rise to a lateral creepage, $\gamma_2 = v_2 / V$ and a corresponding lateral creep force.

When the contact patch is inclined to the horizontal, as shown in Figure 1, a difference exists in the effective rolling radius of the wheel within the contact patch. As a result, the part with the larger radius has to slip backwards and the part with the smaller radius has to slip forwards. The net result is 'spin' – a relative rotational velocity about the 3 axis perpendicular to the contact plane, $\Omega_z$. The corresponding spin creepage is defined as $\omega_3 = \Omega_z / V$ (radians/m).

For a contact angle $\phi$ between the plane of the contact and the horizontal, the spin creepage is given by

$$\omega_3 = \frac{\sin \phi}{r_0}$$

(2)

with $r_0$ the wheel radius. For the usual conicity of 1:40 or 1:20 the spin creepage is quite small. However, for flange contact the contact angle can be very large and then the spin creepage is significant. From equation (2) it can be seen that the maximum spin creepage is $1 / r_0$. This is equivalent to a spin velocity $\Omega_3 = V / r_0$, which is equal to the rotational speed of the wheelset. The spin velocity can be seen as the component of the wheel rotation vector in the direction normal to the contact patch.

Both lateral and spin creepages generate a lateral force and a spin moment at the contact. At low values of creepage, linear theory gives these as
\[ F_2 = -G \, ab \, C_{22} \, \gamma_2 - G \, (ab)^{3/2} \, C_{23} \, \omega_3 \]  
(3)

\[ M_3 = G \, (ab)^{3/2} \, C_{23} \, \gamma_2 - G \, (ab)^2 \, C_{33} \, \omega_3 \]  
(4)

where \( C_{22}, \, C_{23} \) and \( C_{33} \) are creep coefficients, also tabulated by Kalker [12].

### 3.3 Saturated regime

Figure 2 shows a typical 'creep-curve' relating creep force to creepage. At low values of creepage the magnitude of the creep force increases linearly, as discussed above. At high values of creepage the force becomes 'saturated', with a maximum value of \( \mu_0 N \), where \( \mu_0 \) is the friction coefficient and \( N \) is the normal load. At still higher values of creepage the creep force reduces in amplitude, due to the dependence of the friction coefficient on sliding velocity ('dynamic' friction is different from 'static' friction). It is this falling amplitude at high creepage that is the main reason for the unstable dynamic behaviour leading to squeal noise.

![Figure 2. A typical creep force – creepage relationship.](image)

In the presence of only lateral creepage, it is convenient to write the quasi-static lateral force as

\[ F_2 = \mu_2 (\gamma_2) N \]  
(5)

where the 'rolling friction coefficient' \( \mu_2 \) represents the function of creepage \( \gamma_2 \) given in Figure 2 as opposed to the limiting value \( \mu_0 \). The sign convention of equation (5) is that the positive directions of \( F_2 \) and \( \gamma_2 \) coincide, while \( \mu_2 \) is negative for a positive value of \( \gamma_2 \). At low values of \( \gamma_2 \), \( \mu_2 \) is also dependent on \( N \) as \( F_2 \) is proportional to \( ab \) (see equation (3)) which is proportional to \( N^{2/3} \).

In [1,4] this function \( \mu_2 \) is given as a combination of two models. The first is the model of Vermeulen and Johnson [13] for unsaturated creepage,

\[ F_i = \begin{cases} -\mu_0 N \left( \Gamma_i - \frac{1}{3} \Gamma_i^2 + \frac{1}{5} \Gamma_i^3 \right) & \text{for } \Gamma_i < 3 \\ -\mu_0 N & \text{for } \Gamma_i > 3 \end{cases} \]  
(6)
where $F_l$ is the longitudinal or lateral creep force and $\Gamma_i$ is a normalised creep given by

$$\Gamma_i = \frac{GC_\mu ab}{\mu_0 N} \gamma_i \quad \text{for } i = 1, 2$$  \hspace{1cm} (7)

where $a$ and $b$ are the semi-axis lengths of the contact patch, $G$ is the shear modulus of the wheel and rail material (assumed identical) and $C_\mu$ is the creep coefficient given by Kalker [12]. This model, while not exact, gives a good approximation to the behaviour in the region where linear creep theory no longer applies.

In the TNO model [1,4], a model by Kraft (see [14]) is used to describe the dependence of the sliding friction coefficient, $\mu_0$ on the sliding velocity:

$$\mu_0(\gamma) = \mu_{stat} \left[ 1 - 0.5 e^{-0.133/\gamma V} - 0.5 e^{-0.9/\gamma V} \right]$$  \hspace{1cm} (8)

where $\mu_{stat}$ is the static coefficient of friction, $\gamma$ is either the lateral or longitudinal creepage (or a combination) and $V$ is the rolling velocity.

The combination of equations (6) and (8) gives the final expression for $\mu_2$. In practice, $\mu_2$ may also depend on the longitudinal and spin creepages, $\gamma_1$ and $\omega_2$, as these will affect the saturation of the lateral creep force.

3.4 Equations for linear excitation mechanism

From [1], for an unstable lateral force, the lateral contact force (tangential to the contact plane) can be written as $F_2 f_2(t)$ where $F_2$ is the quasi-static force and $f_2$ is the time-varying oscillatory force. This can be written as the product of the normal force $N + f_3(t)$, which again has a quasi-static part and a time-varying part, and the rolling friction coefficient $\mu_2(\gamma_2 + v_2(t)/V)$ with $v_2$ the time-varying relative velocity (due principally to wheel vibration). Thus

$$f_2(t) = \mu_2 \left( \gamma_2 + \frac{v_2(t)}{V} \right) [N + f_3(t)] - \mu_2(\gamma_2)N$$  \hspace{1cm} (9)

$$\approx \mu_2(\gamma_2) f_3(t) + \frac{N}{V} \frac{d\mu_2}{d\gamma_2} v_2(t)$$

where this has been linearised by assuming that time-varying quantities are small and by ignoring terms of second order in small quantities. This can be extended, for example by applying a similar equation to the time-varying longitudinal force $f_1(t)$ in terms of the corresponding rolling friction coefficient $\mu_1$:

$$f_1(t) = \mu_1 \left( \gamma_1 + \frac{v_1(t)}{V} \right) [N + f_3(t)] - \mu_1(\gamma_1)N$$  \hspace{1cm} (10)

$$\approx \mu_1(\gamma_1) f_3(t) + \frac{N}{V} \frac{d\mu_1}{d\gamma_1} v_1(t)$$
For a more general situation, where lateral, longitudinal and spin creepages are present simultaneously, both rolling friction coefficients \( \mu_1 \) and \( \mu_2 \) are non-linear functions of all three creepages, \( \gamma_1 \), \( \gamma_2 \) and \( \omega_3 \). A third rolling friction coefficient, \( \mu_3 \) can be introduced to describe the spin moment \( M_3 = \mu_3 N \). This moment may also be split into a steady component \( M_3 \) and a time-varying component \( m_3 \). \( \mu_3 \) also depends on all three creepages, \( \gamma_1 \), \( \gamma_2 \) and \( \omega_3 \).

These friction coefficients depend on the normal contact force \( N \), although this dependence is ignored in [1,4]. This can be seen from the Vermeulen-Johnson relations, equation (6), for the situations where the normalised creepage \( \Gamma < 3 \). \( N \) is explicitly included in the definition of \( \Gamma \) and is also present implicitly through the dependence of \( a \) and \( b \) on \( N \).

Thus, the fuller version of equations (9, 10) is

\[
\begin{align*}
\{ f_1 \} &= \mu_1 \left( \frac{\gamma_1 + v_1}{V}, \frac{\gamma_2 + v_2}{V}, \frac{\omega_3 + \Omega_3}{V}, N + f_3 \right) \\
\{ f_2 \} &= \mu_2 \left( \frac{\gamma_1 + v_1}{V}, \frac{\gamma_2 + v_2}{V}, \frac{\omega_3 + \Omega_3}{V}, N + f_3 \right) \\
m_3 &= \mu_3 \left( \frac{\gamma_1 + v_1}{V}, \frac{\gamma_2 + v_2}{V}, \frac{\omega_3 + \Omega_3}{V}, N + f_3 \right)
\end{align*}
\]

As above, this can be linearised by assuming that time-varying quantities are small and ignoring terms of second order in small quantities:

\[
\begin{align*}
\{ f_1 \} &= \mu_1(\gamma_1, \gamma_2, \omega_3, N) + \frac{N}{V} \left[ \frac{\partial \mu_1}{\partial \gamma_1} v_1 + \frac{\partial \mu_1}{\partial \gamma_2} v_2 + \frac{\partial \mu_1}{\partial \omega_3} \Omega_3 \right] \\
\{ f_2 \} &= \mu_2(\gamma_1, \gamma_2, \omega_3, N) + \frac{N}{V} \left[ \frac{\partial \mu_2}{\partial \gamma_1} v_1 + \frac{\partial \mu_2}{\partial \gamma_2} v_2 + \frac{\partial \mu_2}{\partial \omega_3} \Omega_3 \right] \\
m_3 &= \mu_3(\gamma_1, \gamma_2, \omega_3, N) + \frac{N}{V} \left[ \frac{\partial \mu_3}{\partial \gamma_1} v_1 + \frac{\partial \mu_3}{\partial \gamma_2} v_2 + \frac{\partial \mu_3}{\partial \omega_3} \Omega_3 \right]
\end{align*}
\]

This terms relating to \( f_3 \) can be brought together to give,

\[
\begin{align*}
\{ f_1 \} &= \mu_1 + \frac{N}{V} \frac{\partial \mu_1}{\partial f_3} f_3 \\
\{ f_2 \} &= \mu_2 + \frac{N}{V} \frac{\partial \mu_2}{\partial f_3} f_3 \\
m_3 &= \mu_3 + \frac{N}{V} \frac{\partial \mu_3}{\partial f_3} f_3
\end{align*}
\]

3.5 Inclusion of wheel and track dynamics

The above expressions relate the time-varying components of the forces to the time-varying relative velocities. In addition, the forces acting on the wheel, the track and the contact spring induce vibration. Assuming harmonic motion, the vibration velocity amplitudes can be derived according to their mobilities:
\[
\begin{align*}
\begin{bmatrix} v_1^w \\ v_2^w \\ \Omega_3^w \end{bmatrix} &= \begin{bmatrix} Y_{11}^w & Y_{12}^w & Y_{16}^w \\ Y_{21}^w & Y_{22}^w & Y_{26}^w \\ Y_{61}^w & Y_{62}^w & Y_{66}^w \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} + \begin{bmatrix} Y_{13}^w \\ Y_{23}^w \\ Y_{63}^w \end{bmatrix} f_3 \\
(14) \\
\begin{bmatrix} v_1^r \\ v_2^r \\ \Omega_3^r \end{bmatrix} &= - \begin{bmatrix} Y_{11}^r & Y_{12}^r & Y_{16}^r \\ Y_{21}^r & Y_{22}^r & Y_{26}^r \\ Y_{61}^r & Y_{62}^r & Y_{66}^r \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} - \begin{bmatrix} Y_{13}^r \\ Y_{23}^r \\ Y_{63}^r \end{bmatrix} f_3 \\
(15) \\
\begin{bmatrix} v_1^c \\ v_2^c \\ \Omega_3^c \end{bmatrix} &= \begin{bmatrix} Y_{11}^c & Y_{12}^c & Y_{16}^c \\ Y_{21}^c & Y_{22}^c & Y_{26}^c \\ Y_{61}^c & Y_{62}^c & Y_{66}^c \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} + \begin{bmatrix} Y_{13}^c \\ Y_{23}^c \\ Y_{63}^c \end{bmatrix} f_3 \\
(16)
\end{align*}
\]

where \( v^w \) and \( v^r \) are the wheel and rail velocities which are positive for vibration in the positive coordinate directions and \( v^c \) is the contact deflection velocity which is positive for a compression of the contact spring. \( Y^w \) are the wheel mobilities, \( Y^r \) are the rail mobilities and \( Y^c \) are the contact spring mobilities. Indices 1-3 indicate translations and 4-6 indicate rotations (see Figure 1). The minus signs in equation (15) come from the fact that the forces acting on the rail are \(-F_h\). It should be noted that, although contact velocities and mobilities have been introduced in equation (16), these relate to the contact spring but omit the creep force ‘damper’ (see [15]). The velocities at the contact must obey the relations (modified from Sections 3.1 and 3.2 to include the local deformation of the contact spring)

\[
\begin{align*}
v_1 &= v_1^w - v_1^r + v_1^c \\
v_2 &= v_2^w - v_2^r + v_2^c \\
\Omega_3 &= \Omega_3^w - \Omega_3^r + \Omega_3^c \\
(17)
\end{align*}
\]

where \( v_i \) is positive for a positive creepage (i.e. the wheel moves in the positive coordinate direction relative to the rail). This can be used to write

\[
\begin{align*}
\begin{bmatrix} v_1 \\ v_2 \\ \Omega_3 \end{bmatrix} &= \begin{bmatrix} Y_{11} & Y_{12} & Y_{16} \\ Y_{21} & Y_{22} & Y_{26} \\ Y_{61} & Y_{62} & Y_{66} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} + \begin{bmatrix} Y_{13} \\ Y_{23} \\ Y_{63} \end{bmatrix} f_3 \\
(18)
\end{align*}
\]

where \( Y_{ij} = Y_{ij}^w + Y_{ij}^r + Y_{ij}^c \) is the sum of wheel, rail and contact spring mobilities. In addition, there is no relative motion (creepage) in the normal direction, so

\[
0 = v_3 = v_3^w - v_3^r + v_3^c \\
(19)
\]

In the same way as in equation (18), the displacement \( v_3 \) can be related to the forces acting:

\[
0 = v_3 = \begin{bmatrix} Y_{31} & Y_{32} & Y_{36} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} + Y_{33} f_3 \\
(20)
\]
which can be rearranged to give \( f_3 \) in terms of \( f_1, f_2 \) and \( m_3 \),

\[
    f_3 = - \frac{1}{Y_{33}} \begin{bmatrix} Y_{31} & Y_{32} & Y_{36} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix}
\]

(21)

This allows \( f_3 \) to be eliminated from equation (18)

\[
    \begin{bmatrix} v_1 \\ v_2 \\ \Omega_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{13}Y_{31} & Y_{16} \\ Y_{21} & Y_{23}Y_{31} & Y_{26} \\ Y_{61} & Y_{63}Y_{31} & Y_{66} \end{bmatrix}^{-1} \begin{bmatrix} Y_{12} & Y_{13}Y_{32} & Y_{16} \\ Y_{22} & Y_{23}Y_{32} & Y_{26} \\ Y_{62} & Y_{63}Y_{32} & Y_{66} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} = [G] \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix}
\]

(22)

Similarly \( f_3 \) can be eliminated from equation (13)

\[
    \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} = \frac{N}{V} \begin{bmatrix} \frac{\partial \mu_1}{\partial \gamma_1} & \frac{\partial \mu_1}{\partial \omega_1} & \frac{\partial \mu_1}{\partial \omega_3} \\ \frac{\partial \mu_2}{\partial \gamma_1} & \frac{\partial \mu_2}{\partial \omega_1} & \frac{\partial \mu_2}{\partial \omega_3} \\ \frac{\partial \mu_3}{\partial \gamma_1} & \frac{\partial \mu_3}{\partial \omega_1} & \frac{\partial \mu_3}{\partial \omega_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \Omega_3 \end{bmatrix} + \begin{bmatrix} \mu_1 + \frac{N}{V} \frac{\partial \mu_1}{\partial \gamma_3} \\ \mu_2 + \frac{N}{V} \frac{\partial \mu_2}{\partial \gamma_3} \\ \mu_3 + \frac{N}{V} \frac{\partial \mu_3}{\partial \gamma_3} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix} = [H_1] \begin{bmatrix} v_1 \\ \Omega_3 \end{bmatrix} + [H_3] \begin{bmatrix} f_1 \\ f_2 \\ m_3 \end{bmatrix}
\]

(23)

### 3.6 Interpretation in terms of lateral creepage model

The above derivations lead to matrix relations. For a case where only lateral creepage is important in producing squeal noise, the terms involving \( v_1, \Omega_3, f_1 \) and \( m_3 \) can be eliminated. This leads to the following reduced equations:

\[
    v_2 = Y_{22} - Y_{23}Y_{32} \begin{bmatrix} f_2 \end{bmatrix} = Gf_2
\]

(24)

and

\[
    f_2 = \frac{N}{V} \frac{\partial \mu_2}{\partial \gamma_2} v_2 - \left( \mu_2 + \frac{N}{V} \frac{\partial \mu_2}{\partial \gamma_3} \right) Y_{32} f_2 = H_1 v_2 + H_2 f_2
\]

(25)

Comparing these equations with the model given in [1], equation (24) is identical to the model given in Section 3 of [1] apart from some simplifications there allowing the rail or contact mobilities to be neglected in some cases which are not assumed here. The factor \( G \) contains information relating to the wheel and rail dynamics and the contact stiffness.

The factors \( H_1 \) and \( H_2 \) in equation (25) are due to the creepage. \( H_1 \) is the usual feedback term between lateral velocity and lateral force, also used by Heckl [10], which depends on the slope of Figure 2. \( H_2 \) contains a term that is given in [1] as a relation between vertical and
lateral forces ($-Y_{32}/Y_{33}$) and multiplied by $\mu_2$, and another term that is new here. However, this latter term is only relevant in the unsaturated regime.

Equations (24) and (25) form a feedback loop with two branches. This has an open loop transfer function given by $H_1 G + H_2$. As is conventional in control theory, the stability of this loop can be tested by plotting the open loop transfer function on the Nyquist plane (i.e. plotting the imaginary part against the real part). If this encircles the point +1, the system is unstable and the frequency at which it crosses the real axis with the largest positive value is the frequency at which it will tend to become unstable.

### 3.7 Stability of full model

Equations (22) and (23) also form a feedback loop, as shown in Figure 3, but this differs from the model of [1] in having a vector/matrix form. Therefore, unlike in Section 3.6, there is no single parameter that can be used to test the feedback gain. Instead, the open loop gain is formed by a $3 \times 3$ matrix, $[T] = [H_1] [G] + [H_2]$.

![Figure 3. Feedback loop based on equations (22) and (23).](image)

The criterion for stability of such a multiple input, multiple output system is that the determinant of the matrix $[I - T]$ should not enclose the point 0 on the Nyquist plane. If it does enclose this point, the loop of the locus that crosses the real axis furthest from the origin on the negative axis corresponds to the frequency with the greatest instability.

This criterion can be further refined by finding the eigenvalues of the matrix $[T]$ and plotting these on the Nyquist plane. The eigenvalues that encircle the point +1 are unstable. However, this requires a procedure to track each eigenvalue as a function of frequency, i.e. to identify eigenvalues at one frequency with the correct eigenvalues at the previous frequency.

### 3.8 Discussion

It may be noted that the terms in $[G]$ are all of the form $Y_y - Y_{32}Y_{33}$. At a wheel resonance, the wheel mobility will usually be the largest contributor to $Y_y$. Moreover, the wheel mobility will be dominated by a single mode and can be approximated by
\[ Y^W_{ij} = \frac{\psi_k(x_i)\psi_k(x_j)}{2m_k\zeta_k\omega_k} \]  

where \( \psi_k \) is the mode shape of the \( k^{th} \) mode for direction \( i \) or \( j \), \( m_k \) is the modal mass, \( \zeta_k \) is the modal damping ratio, \( \omega \) is the excitation frequency which is equal to the resonance frequency \( \omega_k \). It can then be seen that the terms of the form \( Y^W_{ij} - Y^W_{ik}Y^W_{jk}Y^W_{33} \) are equal to zero. This is only the case for the terms of [G], however, if \( Y^W_{33} \) is large compared with \( Y^R_{33} \) and \( Y^C_{33} \). Consequently, only the modes in which the component of the mode shape normal to the surface is small and that tangential to the surface is large are excited in squeal.

4. WHEEL, TRACK AND CONTACT MOBILITIES

4.1 Introduction

In order to implement the above model, including longitudinal and spin creep, the matrices [G], [H1] and [H2] must be determined. The matrix [G] is considered in this section, [H1] and [H2] are considered in the next section. [G] consists of terms that are the sum of wheel, rail and contact mobilities. Within TWINS, the separate mobilities are calculated for the wheel and rail and transferred to the module lynx for coupling together with the contact mobilities. In principle, all six degrees of freedom (36 point mobilities) are available and this matrix can be obtained for the wheel, rail and contact. In practice there are limitations to the data available, and mostly TWINS is used only with vertical and lateral degrees of freedom. In the present implementation, a Matlab program has been written to carry out the calculations.

4.2 Wheel mobilities

The wheel mobility is calculated using the same method as in TWINS [5]. This reads a modal parameter file (TWINS format) that contains natural frequencies, mode shapes and damping values for each mode of vibration. This file is created from the results of a finite element calculation. The mobility matrix at the contact point is calculated from the data in the modal parameter file by a modal superposition [15]. The contact point may be offset laterally and/or vertically on the wheel. In principle, it is also possible to include wheel rotation, which leads to splitting of the resonance frequencies into pairs.

The wheel modal parameters are usually only available for the \( x \), \( y \) and \( \theta \) directions (see Figure 1). This allows only \( Y^W_{22}, Y^W_{23}, Y^W_{33}, Y^W_{24}, Y^W_{34} \) and \( Y^W_{44} \) to be determined of those contained in [G]. To extend this, new finite element analyses of wheels are required. In the present project a new model of the standard UIC 920 mm freight wheel has been produced. The opportunity has also been taken to extend the frequency range covered by this modal parameter file up to 13 kHz in order to include the whole of the 10 kHz one-third octave band.

Figure 4 shows the FE mesh used. The model is axisymmetric and has a plane of symmetry at the centre of the axle. Both symmetry and anti-symmetry conditions at the centre of the axle are considered. Figure 5 compares the lateral mobility from the new model with that from the previous modal parameter file. The new model includes some additional peaks at low frequencies due to modes of the axle, but otherwise the wheel modes can be seen to agree quite well. The new model also includes modes up to higher frequencies than the previous model (for which no modes were calculated above 5 kHz). Figure 6 shows the modes shapes of
some typical wheel modes. From this it can be seen that the second mode for each number of nodal diameters is predominantly radial and the third is predominantly axial with 1 nodal circle. At \( n = 6 \) the distinction is less clear. The natural frequencies of all modes below 8 kHz are listed in Table 1.

Figure 4. Finite element mesh of UIC 920 mm freight wheel used to calculate wheel mobilities. Symmetry or anti-symmetry boundary conditions are applied at the centre of the axle.

![Finite element mesh of UIC 920 mm freight wheel](image)

Figure 5. Lateral wheel mobility of UIC 920 mm freight wheel. —— new model including axle, —— previous model of wheel only.

![Lateral wheel mobility](image)
Figure 6. Examples of modeshapes of UIC 920 mm freight wheel.
Table 1. Natural frequencies of all modes of UIC 920 m freight wheel calculated using FEM. 
*n is the number of nodal diameters, R indicates predominantly radial modes, C 
predominantly circumferential modes, (n,m) indicates predominantly axial modes 
with m nodal circles, S indicates symmetric axial modes and A anti-symmetric axial 
modes.*

<table>
<thead>
<tr>
<th>n</th>
<th>Symmetric axle modes</th>
<th>Anti-symmetric axle modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>90.354 (S)</td>
<td>3201.8 (S)</td>
</tr>
<tr>
<td></td>
<td>840.27 (S)</td>
<td>4063.6 (S)</td>
</tr>
<tr>
<td></td>
<td>1437.5 (S)</td>
<td>5057.2 (S)</td>
</tr>
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<td>1839.8 (S)</td>
<td>6245.7 (S)</td>
</tr>
<tr>
<td></td>
<td>2255.8 (S)</td>
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<td></td>
<td>2607.0 (S)</td>
<td>7513.6 (S)</td>
</tr>
<tr>
<td></td>
<td>153.94 (A)</td>
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<td></td>
<td>553.34 (A)</td>
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<td>4610.2 (A)</td>
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<td>1561.7 (A)</td>
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<tr>
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</table>

<table>
<thead>
<tr>
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<th>(n,3)</th>
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<td>1594.2 (A)</td>
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<td>4017.1 (S)</td>
<td>4073.3 (S)</td>
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<td>5416.6 (A)</td>
<td>6134.0 (S)</td>
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<td>4450.3</td>
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<td>7944.5</td>
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<td>10</td>
<td>7454.2</td>
<td></td>
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</table>

The wheel mobilities are calculated initially in the (x,y,z) coordinate system (see Figure 1). 
They should first be converted to a local 1-2-3 coordinate system by reordering the matrix 
elements. In addition, the longitudinal axis (along the track) is orientated in the opposite 
direction. Therefore, the corresponding velocities and forces in the 1 direction (and in the 4 
direction) should be multiplied by −1. Mobilities involving one such component, e.g. *Y*12, 
should be multiplied by −1, while mobilities involving two such components, e.g. *Y*11 or *Y*14, 
should be multiplied by (−1)² = +1.

Figure 1 shows the wheel as a right-hand wheel. To convert the mobilities to the correct axis 
system for a left-hand wheel, the components of velocity or force in the axis directions 2, 4 
and 6 should next be multiplied by −1.

Having obtained a 6×6 matrix of wheel mobilities, these should then be translated to the 
corresponding contact position and rotated by the contact angle *ϕ*, so that they are in the local 
coordinate system of the contact plane (see Figure 1). These features are available within 
TWINS, but in the present case they have been implemented in the Matlab software, as 
follows. Consider the nominal contact point O (as used in the FE calculation) and the actual 
contact point P, shown in Figure 7.
Figure 7. Effect of vertical and lateral offsets of the contact zone on the deflections of the wheel for a right-hand wheel.

If \( v^0 \) are the velocities at the nominal contact position O in a coordinate system with 1 along the track, 2 lateral and 3 vertical, and the actual contact position P is offset by a lateral distance \( y \) and a vertical distance \( z \), the velocities at the position P are

\[
\begin{align*}
    v_1^p &= v_1^0 - yv_y^0 + zv_z^0 \\
    v_2^p &= v_2^0 - zv_z^0 \\
    v_3^p &= v_3^0 + yv_y^0
\end{align*}
\]  

(27)

where \( y \) is defined as positive for a displacement outwards on the wheel and \( z \) is positive for a displacement downwards.

If \( Y_{ij}^0 \) are the mobilities at the nominal contact position in the coordinate system 1-2-3 and the actual contact position is offset as above, the mobilities at this position are

\[
\begin{align*}
    Y_{11}^p &= Y_{11}^0 - 2yY_{16}^0 + y^2Y_{66}^0 + 2zY_{15}^0 + z^2Y_{55}^0 - 2yzY_{56}^0 \\
    Y_{ij}^p &= Y_{ij}^0 - yY_{j6}^0 + zY_{5j}^0 \quad \text{for } j \neq 1 \\
    Y_{j1}^p &= Y_{j1}^0 - yY_{j6}^0 + zY_{5j}^0 \quad \text{for } j \neq 1 \\
    Y_{22}^p &= Y_{22}^0 - 2zY_{44}^0 + z^2Y_{44}^0 \\
    Y_{2j}^p &= Y_{2j}^0 - zY_{4j}^0 \quad \text{for } j \neq 2 \\
    Y_{j2}^p &= Y_{j2}^0 - zY_{j4}^0 \quad \text{for } j \neq 2 \\
    Y_{33}^p &= Y_{33}^0 + 2yY_{54}^0 + y^2Y_{44}^0 \\
    Y_{3j}^p &= Y_{3j}^0 + yY_{4j}^0 \quad \text{for } j \neq 3 \\
    Y_{j3}^p &= Y_{j3}^0 + yY_{j4}^0 \quad \text{for } j \neq 3
\end{align*}
\]  

(28)
To rotate the velocities through a contact angle $\phi$ (positive for a clockwise rotation, the positive direction of coordinate 4), the following transformation is applied

\[
\begin{align*}
\nu_2 &= \nu_2^p \cos \phi + \nu_3^p \sin \phi \\
\nu_3 &= \nu_3^p \cos \phi - \nu_2^p \sin \phi \\
\nu_5 &= \nu_5^p \cos \phi + \nu_6^p \sin \phi \\
\nu_6 &= \nu_6^p \cos \phi - \nu_5^p \sin \phi
\end{align*}
\]

(29)

from which the rotated mobilities are

\[
\begin{align*}
Y_{22} &= Y_{22}^p \cos^2 \phi + 2Y_{23}^p \cos \phi \sin \phi + Y_{33}^p \sin^2 \phi \\
Y_{2j} &= Y_{2j}^p \cos \phi + Y_{j3}^p \sin \phi & \text{for } j \neq 2 \\
Y_{j2} &= Y_{j2}^p \cos \phi + Y_{j3}^p \sin \phi & \text{for } j \neq 2 \\
Y_{33} &= Y_{33}^p \cos^2 \phi - 2Y_{23}^p \cos \phi \sin \phi + Y_{55}^p \sin^2 \phi \\
Y_{3j} &= Y_{3j}^p \cos \phi - Y_{2j}^p \sin \phi & \text{for } j \neq 3 \\
Y_{j3} &= Y_{j3}^p \cos \phi - Y_{j2}^p \sin \phi & \text{for } j \neq 3 \\
Y_{55} &= Y_{55}^p \cos^2 \phi + 2Y_{56}^p \cos \phi \sin \phi + Y_{66}^p \sin^2 \phi \\
Y_{5j} &= Y_{5j}^p \cos \phi + Y_{6j}^p \sin \phi & \text{for } j \neq 5 \\
Y_{j5} &= Y_{j5}^p \cos \phi + Y_{j6}^p \sin \phi & \text{for } j \neq 5 \\
Y_{66} &= Y_{66}^p \cos^2 \phi - 2Y_{56}^p \cos \phi \sin \phi + Y_{66}^p \sin^2 \phi \\
Y_{6j} &= Y_{6j}^p \cos \phi - Y_{5j}^p \sin \phi & \text{for } j \neq 6 \\
Y_{j6} &= Y_{j6}^p \cos \phi - Y_{j5}^p \sin \phi & \text{for } j \neq 6
\end{align*}
\]

(30)

In the absence of wheel rotation, the coordinate directions 2, 3, and 4 are not coupled to the directions 1, 5 and 6, i.e. the mobilities $Y_{12}^W$, etc. are zero. If wheel rotation is included, these cross-terms are skew-symmetric, i.e. $Y_{12}^W = -Y_{21}^W$, etc [15].

4.3 Track mobilities

The track mobility is calculated using two models. For the vertical direction a model is used that is identical to the rodel model in TWINS [5]. This represents the rail as a Timoshenko beam on a continuous support of springs, masses and springs (a discretely supported track could also be used). For the lateral and torsional directions a model is used that is based on two beams (the head and the foot) connected by an array of beams representing the rail web [16]. This is again on a two-layer continuous resilient support, although it would also be possible to include a discretely supported track model [17]. Coupling between lateral and vertical directions is introduced by a lateral offset of the contact position from the centreline of the rail, according to equation (28).

Example results are shown in Figures 8 and 9. The lateral mobility is considerably greater in magnitude than the result obtained from a simple beam model due to the introduction of torsional flexibility and web bending. When the rail is excited at its centreline the cross
mobility is zero, but for a non-zero offset a cross mobility is obtained. Note that, for a non-zero offset, the vertical mobility also contains peaks at lateral track resonances.

Figure 8. Track mobility for a contact position at the rail centreline.

Figure 9. Track mobility for a contact position 20 mm from the centreline.
Within TWINS [5], the full 6x6 matrix of rail mobilities can only be obtained if the *perm* module is used [5]. The above beam models normally allow only $Y_{22}^e$, $Y_{23}^e$ and $Y_{33}^e$ to be obtained, although the rotation about the rail axis (direction 4) can also be determined from the model of [16].

For the longitudinal (1) direction, the rail can be represented as a rod in extensional vibration, mounted on an elastic layer [18]. The mobility $Y_{11}^e$ can be calculated from this model, although this has not yet been implemented. It can be assumed that the components $Y_{ij}^e = Y_{ji}^e$ = 0 for $j \neq 1$ (although strictly the longitudinal vibration at the railhead couples with the rotation 5 due to the rotation of the rail cross-section during vertical bending). The two remaining directions (5 and 6) represent the gradients of the vertical and lateral bending respectively and could be obtained by a modification of the above beam models. These have also not been implemented at this stage.

For the rail, as for the wheel, it should be noted that these mobilities should be rotated by the contact angle $\phi$ according to equation (30).

### 4.4 Contact mobilities

The contact mobilities are calculated using Hertz theory as in TWINS [5]. For the present application the terms containing the creep force dampers need to be removed, leaving only the contact stiffness terms:

$$
Y_{11}^c = \frac{i\omega \kappa_1}{k_H}, \quad Y_{22}^c = \frac{i\omega \kappa_2}{k_H}, \quad Y_{33}^c = \frac{i\omega}{k_H}
$$

(31)

where $k_H$ is the Hertzian contact stiffness, and $\kappa_i$ is a factor between about 1 and 1.2, depending on the aspect ratio of the contact patch $a/b$ [15].

The TWINS module *lynx* allows all 36 components of contact mobility $Y_{ij}^c$ to be determined, although for the present application the terms containing the creep force dampers need to be removed, leaving only the contact stiffness terms, as in equation (31). The combination of a creep force damper and a contact stiffness corresponds to a frequency-dependent creep term for lateral and longitudinal creepage [9, 15]. However, for $C_{23}$ and $C_{33}$ the frequency-dependent creep coefficients are more complicated and may require a different solution. For the present implementation the corresponding terms $Y_{26}^c$, $Y_{62}^c$ and $Y_{66}^c$ are taken as zero.

### 5. ROLLING FRICTION COEFFICIENTS

#### 5.1 Introduction

The matrices $[H_1]$ and $[H_2]$ are rather more difficult to obtain than $[G]$. For these, the three components of rolling friction coefficient, $\mu_1$, $\mu_2$ and $\mu_3$ are required. TWINS contains formulae which implement linear creep theory, equations (1,3,4), including tables of creep coefficients. However, this will have to be extended to saturated creep, for example using the Vermeulen and Johnson formula from equation (6), together with the velocity-dependent friction coefficient from equation (8). Unfortunately, in the presence of large spin creepage together with the translation components, there is no simple formula available.
The derivatives of $\mu$ with respect to $\gamma_1$, $\gamma_2$, $\omega_3$ and $f_3$ are also difficult to implement. The creep forces depend on the various components of creepage in a complex way. The options for implementing this into a calculation model for curve squeal are:

(a) Generate a look-up table of many values of normalised creep force against normalised creepages. The amount of work required to produce this would be rather large and some automation of the Contact software [19] would be advisable.

(b) Use Contact [19] directly to calculate the components of creep force for the given values of creepage and for incremental changes to obtain the various derivatives (some derivatives, or sensitivities, are output directly). Only a small number of runs of the program would be required for a given situation. This is the most suitable method for an initial implementation of the model, but would be impractical for a final model as it would require a licence for Contact for each installation of the software.

(c) Derive a curve-fit to calculated values and use this to obtain the derivatives. This is unlikely to be feasible.

Although the Contact software can be made to produce a falling creep curve for saturated creepage, as in Figure 2, this requires two different values of friction coefficient (static and dynamic), applied to the 'stick' and 'slip' zones of the contact patch. Once the whole contact patch is slipping the creep forces reach a constant value. The method adopted here is therefore to use Contact with a single friction coefficient and to derive the effect of the velocity-dependence of the friction coefficient separately, as described below.

5.2 Use of Contact to obtain rolling friction coefficients

According to approach (b), the Contact software has been used to derive the rolling friction coefficients and their derivatives for two situations. These represent the leading inner and outer wheels of a bogie traversing a curve of radius 300 m at a speed 12.25 m/s. These two situations were calculated initially by AEA Technology bv using the Simpack vehicle dynamics software [20]. The creepages, contact patch dimensions, normal forces, contact angles and contact positions are listed in Table 2. These values were input to the Contact software which was used to calculate the longitudinal and transverse forces $F_1$ and $F_2$ and the spin moment, $M_3$. (Although the creep forces are output from Simpack, this uses a simpler method of calculating them than Contact). The results are listed in Table 3. These are expressed here in the form

$$\beta_1 = \frac{F_1}{\mu_{\text{stat}} N}$$

$$\beta_2 = \frac{F_2}{\mu_{\text{stat}} N}$$

$$\beta_3 = \frac{M_3}{\mu_{\text{stat}} N}$$

where $\mu_{\text{stat}}$ is the static friction coefficient input to the Contact program and $N$ is the normal force.
Table 2. Results from vehicle dynamics simulations [20]

<table>
<thead>
<tr>
<th></th>
<th>Leading inner wheel (right-hand)</th>
<th>Leading outer wheel (left-hand)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal force, $N$ (kN)</td>
<td>42.0</td>
<td>90.5</td>
</tr>
<tr>
<td>Contact patch semi-length, $a$ (mm)</td>
<td>5.46</td>
<td>12.1</td>
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<tr>
<td>Contact patch semi-width, $b$ (mm)</td>
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<td>1.37</td>
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<tr>
<td>Longitudinal creepage, $\gamma_1$ (°)</td>
<td>$5.91 \times 10^{-3}$</td>
<td>$-3.96 \times 10^{-3}$</td>
</tr>
<tr>
<td>Lateral creepage, $\gamma_2$ (°)</td>
<td>$9.94 \times 10^{-3}$</td>
<td>$13.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>Spin creepage, $\omega_3$ (radians/m)</td>
<td>$-18.1 \times 10^{-3}$</td>
<td>1.73</td>
</tr>
<tr>
<td>Contact angle (radians)</td>
<td>0.012</td>
<td>0.932</td>
</tr>
<tr>
<td>Lateral offset of contact from nominal position, $y$ (mm)</td>
<td>13.4</td>
<td>34.8</td>
</tr>
</tbody>
</table>

Table 3. Rolling friction coefficients obtained from Contact.

<table>
<thead>
<tr>
<th></th>
<th>leading inner wheel</th>
<th>leading outer wheel</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>-0.5168</td>
<td>-0.8560</td>
</tr>
<tr>
<td>$\frac{\partial \beta_i}{\partial \gamma_1}$</td>
<td>-22.1</td>
<td>13.7</td>
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<td>$\frac{\partial \beta_i}{\partial \gamma_2}$</td>
<td>13.7</td>
<td>-8.05</td>
</tr>
<tr>
<td>$\frac{\partial \beta_i}{\partial \omega_3}$</td>
<td>-7.0 x 10^{-4}</td>
<td>-5.60 x 10^{-4}</td>
</tr>
<tr>
<td>$\frac{\partial \beta_i}{\partial N}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The derivatives with respect to the creepages were obtained either directly or by carrying out a second run with one creepage changed slightly. The derivatives with respect to the normal load were obtained by varying this load, but these were found to be zero since the contact in both cases is fully saturated (there is no stick zone, but instead all parts of the contact are in slip).

It will be observed that $\beta_2$ is negative for both wheels, indicating that the lateral creep force on the wheel acts towards the outside of the curve in each case. The creepage is positive so that the wheel is sliding inwards relative to the rail. However, due to the large contact angle on the outer wheel, the net lateral force on this wheel is actually inwards, since the normal force has a large component in the lateral direction, as shown in Figures 10 and 11. The longitudinal creepage is positive for the inner wheel and negative for the outer wheel; the corresponding longitudinal creep forces act backwards on the inner wheel and forwards on the outer wheel (tending to rotate the wheelset to orientate it radially in the curve), see Figure 10.
5.3 Static friction coefficients

The static friction coefficients are calculated using the same model as de Beer [4], taken from Fingberg [14]:

\[
\mu_{stat} = \frac{\tau_r \tau_w}{\tau_r + \tau_w} \frac{\tau_{ab}}{N} 
\]

(33)

where \(\tau_r\) and \(\tau_w\) are the shear strength of the rail and wheel material, taken to be \(4 \times 10^8\) and \(6 \times 10^8\) N/m\(^2\). This leads to the value 0.398 for the leading inner wheel and 0.138 for the outer wheel.

5.4 Inclusion of Kraft model for falling friction coefficients

From equation (8), the sliding friction coefficient depends on the sliding velocity according to

\[
\mu_s(y) = \mu_{stat} \left\{ -0.5e^{-0.138/\nu|v|} - 0.5e^{-6.9/\nu|v|} \right\} = \mu_{stat} \alpha(y) 
\]

(34)
where \( \gamma > 0 \) represents the combined longitudinal and lateral creepage

\[
\gamma = \left( \gamma_1^2 + \gamma_2^2 \right)^{\frac{1}{2}}
\]  
(35)

It is assumed here that the spin creepage has a negligible effect on the overall creepage, \( \gamma \). This is reasonable provided that the spin pole is outside the contact zone. For a combination of translation and spin creepage, the spin pole can be defined as

\[
x_1 = \frac{\gamma_2}{\omega_3}, \quad x_2 = -\frac{\gamma_1}{\omega_3}
\]  
(36)

where \( x_1 \) and \( x_2 \) are the distances in the 1 and 2 directions from the centre of the contact zone. This is the point around which the spin rotates. For the present cases, the leading inner wheel has a spin pole of \((-0.55 \text{ m}, 0.33 \text{ m})\) which is well outside the contact area. The leading outer wheel has its spin pole at \((-0.0076 \text{ m}, -0.0023 \text{ m})\) which is much closer to the contact area but is still outside.

Equation (34) is combined with the above results from Contact to give the rolling friction coefficients

\[
\mu_i(\gamma_1, \gamma_2, \omega_3) = \mu_{stat} \alpha(\gamma) \beta_i(\gamma_1, \gamma_2, \omega_3)
\]  
(37)

From this the various derivatives can be obtained from

\[
\frac{\partial \mu_i}{\partial \gamma_j} = \mu_{stat} \left( \alpha \frac{\partial \beta_i}{\partial \gamma_j} + \frac{\partial \alpha}{\partial \gamma_j} \frac{\partial \gamma_i}{\partial \gamma_j} \beta_i \right)
\]  
(38)

where \( \frac{\partial \beta_i}{\partial \gamma_j} \) is given in Table 3,

\[
\frac{\partial \alpha}{\partial \gamma} = -0.5 \left\{ \frac{0.138}{\gamma^2} e^{-0.138 \delta \gamma} + \frac{6.9}{\gamma^2} e^{-6.9 \delta \gamma} \right\}
\]  
(39)

\[
\frac{\partial \beta_i}{\partial \gamma_i} = 2\gamma_i \times \frac{1}{2} \left\{ \frac{1}{\gamma_1^2 + \gamma_2^2} \right\}^{1/2} = \frac{\gamma_i}{\gamma} \quad \text{(for } i=1, 2 \text{ and } \gamma_i > 0) \]  
(40a)

\[
\frac{\partial \beta_i}{\partial \gamma_i} = -2\gamma_i \times \frac{1}{2} \left\{ \frac{1}{\gamma_1^2 + \gamma_2^2} \right\}^{1/2} = -\frac{\gamma_i}{\gamma} \quad \text{(for } i=1, 2 \text{ and } \gamma_i < 0) \]  
(40b)

and

\[
\frac{\partial \gamma}{\partial \omega_3} = 0
\]  
(41)

The resulting derivatives are listed in Table 4. This also indicates the contribution of the two terms in equation (38).
Table 4. Rolling friction coefficients obtained from Contact combined with model of Kraft.

For the derivatives, the first row is $\mu_{\text{stat}} \frac{\partial \gamma_1}{\partial \gamma_1}$, the second is $\mu_{\text{stat}} \frac{\partial \alpha}{\partial \gamma} \frac{\partial \gamma}{\partial \gamma_1}$, and the third is the sum, that is $\partial \mu_j / \partial \gamma_j$.

<table>
<thead>
<tr>
<th></th>
<th>Leading inner wheel, $\mu_{\text{stat}} = 0.398$</th>
<th>Leading outer wheel, $\mu_{\text{stat}} = 0.138$</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_i )</td>
<td>( \mu_1 )</td>
<td>( \mu_2 )</td>
</tr>
<tr>
<td>( \frac{\partial \mu_i}{\partial \gamma_1} )</td>
<td>-17.93</td>
<td>11.11</td>
</tr>
<tr>
<td></td>
<td>4.20</td>
<td>7.06</td>
</tr>
<tr>
<td></td>
<td>-13.73</td>
<td>18.18</td>
</tr>
<tr>
<td>( \frac{\partial \mu_i}{\partial \gamma_2} )</td>
<td>11.11</td>
<td>-6.53</td>
</tr>
<tr>
<td></td>
<td>6.96</td>
<td>11.70</td>
</tr>
<tr>
<td></td>
<td>18.07</td>
<td>5.17</td>
</tr>
<tr>
<td>( \frac{\partial \mu_i}{\partial \omega_3} )</td>
<td>-5.68x10^4</td>
<td>-4.54x10^4</td>
</tr>
<tr>
<td></td>
<td>-2.41x10^4</td>
<td>-4.06x10^4</td>
</tr>
<tr>
<td></td>
<td>-8.09x10^4</td>
<td>-8.60x10^4</td>
</tr>
<tr>
<td>( \frac{\partial \mu_i}{\partial N} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that the sign of $\frac{\partial \mu_j}{\partial \gamma_2}$ is positive in each case. Since $\mu_2$ itself is negative, this indicates that an increase in creepage leads to a reduction in the magnitude of the lateral force. Such a situation can be expected to lead to instability. However, in both cases it is only the second term (the falling value of sliding friction coefficient according to Kraft) that leads to the instability; the first term from Contact has the opposite sign and would be stable.

The longitudinal creep has a gradient, $\frac{\partial \mu_j}{\partial \gamma_1}$ that is negative in each case. Since $\mu_1$ itself is also negative for the inner wheel, this will not lead to instability. For the outer wheel, however, $\mu_1$ is positive and $\frac{\partial \mu_j}{\partial \gamma_1}$ may also lead to instability.

In practice the various directions will interact with one another in a way that this simple analysis cannot show.

6. RESULTS

6.1 Results for leading inner wheel - contact on top of rail head

For the leading inner wheel the contact is on the top of the railhead. According to [20] it is located 13.4 mm towards the outer edge of the wheel (away from the flange), with a contact angle that is small (less than 1 degree).

The solution of the open loop transfer function (equations (22) and (23)) leads to three eigenvalues. Figure 12 shows the Nyquist plot (imaginary part plotted against real part) of the eigenvalue with the largest real part at each frequency. Calculations have been performed in the range 50 to 10,000 Hz. This figure corresponds to a contact at the nominal position on the wheel (70 mm from the flangeback). The contact position on the rail is fixed as +10 mm from
its centreline in this and subsequent results. Several large loops are seen that encircle the point +1, although one is considerably larger than the others.

![Nyquist plot](image)

**Figure 12.** Nyquist plot of the eigenvalue with the largest real part for the inner wheel with a lateral contact position at 0 mm on the wheel.

To determine the unstable frequency corresponding to each loop, the real part of the largest eigenvalue of the open loop transfer function (the 'loop gain') is plotted against frequency in Figure 13. The peak with the highest amplitude is at about 350 Hz, which corresponds to the wheel mode with 2 nodal diameters and 0 nodal circles, i.e. mode (2,0). This is therefore the mode that is most likely to squeal in this case. Other peaks occur at other 0-nodal-circle modes.

The loop gain at each crossing of the real axis in Figure 12 has been determined for this and a series of other lateral contact positions on the wheel. These are plotted in Figure 14. This shows that the (2,0) mode has the largest loop gain for many lateral contact positions, although the (4,0) mode has a larger loop gain for contact positions more than +15 mm (relative to the nominal position) and less than −13 mm. The maximum loop gain occurs at a contact position of +5 mm, so that this design of wheel can be expected to be particularly prone to squeal for contact positions between 0 and +10 mm, i.e. between 70 and 80 mm from the flangeback.

The Nyquist plots of Figure 12 cross the real axis at frequencies that are close to wheel resonance frequencies. Figure 15 plots the difference between the frequency of the crossing point (the frequency at which the system is unstable) and the corresponding resonance frequency. This shows that the system is unstable at frequencies that are slightly higher than the corresponding wheel natural frequency, but the difference is less than 5 Hz in each case.
Figure 13. Spectrum of real part of the loop gain (eigenvalue with the largest real part) for the inner wheel with a contact position at 0 mm.

Figure 14. Real part of the loop gain (eigenvalue with the largest real part) for the inner wheel at various contact positions on the wheel.
Figure 15. Difference between frequency of unstable response and natural frequency of wheel for the inner wheel.

For the case of the leading inner wheel considered in this section, the predominant excitation is in the lateral direction. In Figure 16 the result corresponding to Figure 14 is shown where only the coupling in the lateral direction is included, i.e., equations (22) and (23) are replaced by equations (24) and (25). This can be seen to give very similar results to those in Figure 14, indicating that the addition of other direction has little effect for this case. This is also shown by the comparison of the maximum loop gain, given in Figure 17.

Nevertheless, the inclusion of all three coupling degrees of freedom in the present model has had an effect on the results through the derivatives of the rolling friction coefficients (Table 4). If longitudinal and spin creepage were ignored, the value of $\partial \mu / \partial \gamma_z$ would differ.

The position of contact on the head of the rail is also allowed to vary about the middle of the crown of the UIC 60 rail section. Contact on the wheel is taken to be at the nominal contact position. Figure 18 shows the effect on maximum loop gain and this is totally dominated by the 2 nodal diameter mode. In the case under investigation, the effect of torsional flexibility and bending of the rail web was found not to change the results significantly if contact shifts laterally across the head of the rail.
Figure 16. Real part of the loop gain (eigenvalue with the largest real part) for the inner wheel at various contact positions on the wheel - model based only on coupling in the lateral direction.

Figure 17. Real part of the maximum loop gain for the inner wheel at various contact positions on the wheel.
Figure 18. Real part of the maximum loop gain for the inner wheel at various contact positions on the head of the rail.

6.2 Results for leading outer wheel - flange contact

For the leading outer wheel, which is in flange contact, the Nyquist plot of the open loop transfer function (the eigenvalue with the largest real part) is given in Figure 19. Calculations have again been performed in the range 50 to 10,000 Hz. This figure corresponds to a lateral contact position of +40 mm, which in this case is 30 mm from the flangeback (as it is a left-hand wheel), and a vertical offset of 5 mm. Compared with the case of contact on top of the rail head (Figure 12), there are many more loops of similar magnitude, although the magnitude is smaller in the present case.

In order to determine the squeal frequencies associated with these loops, the spectrum of the real part of the ‘loop gain’ is shown in Figure 20. From this it can be seen that the peaks with the highest loop gain occur at about 1700, 2400, 3100 and 4000 Hz. These correspond to the predominantly radial modes with 2, 3, 4 and 5 nodal diameters (see Table 1). These modes are excited as their modeshapes tend to have a large component in the tangential direction at the wheel flange (for this contact angle) and a small normal component. Figure 21 shows the direction of the modeshape vector at the contact position on the flange. This shows that the 0-nodal-circle axial modes vibrate mainly in the axial direction and the radial modes vibrate mainly in the radial direction. The frequencies associated with the instabilities are all within less than 1 Hz of the corresponding resonance frequencies.
Figure 19. Nyquist plot of the eigenvalue with the largest real part for the outer wheel with a lateral contact position at +40 mm on the wheel.

Figure 20. Spectrum of real part of the loop gain (eigenvalue with the largest real part) for the outer wheel with a contact position at +40 mm.
Figure 21. Normalised modeshape vectors of the most important wheel modes associated with flange squeal at the flange contact position indicated.

Figure 22 shows the loop gain associated with each 1-nodal-circle and radial mode for various lateral contact positions. For simplicity here, the contact angle is kept constant at 53.4° (0.932 radians) as in Table 2 and the vertical position at +5 mm. The relative instability of the various modes does not change greatly with changing lateral position.

Varying the contact angle would also require varying the spin creepage and the associated derivatives, which would mean rerunning the Contact program. For simplicity here, however, the contact angle has been varied only in the formulation for the mobilities. The contact position is also kept constant at +40 mm laterally and a vertical offset of 5 mm. This leads to the results given in Figure 23. The results in Figure 21 show that the radial modes with between 2 and 5 nodal diameters appear to vibrate mainly in the vertical direction at this position. These radial modes have an increasing loop gain as the contact angle increases. The mode with 6 nodal diameters has a maximum loop gain at a contact angle of about 0.8 radians, close to its direction to the horizontal. The mode (6,1) is also excited more with increasing contact angle as its modeshape is close to the vertical direction at this contact point.
Figure 22. Real part of the loop gain (eigenvalue with the largest real part) for the outer wheel at various lateral contact positions on the wheel (contact angle kept fixed).

Figure 23. Real part of the loop gain (eigenvalue with the largest real part) for the outer wheel at various contact angles (lateral contact position on the wheel flange kept fixed).
7. CONCLUSIONS

A model for the excitation of curve squeal has been developed that includes lateral, longitudinal and spin creepage terms. This is the first such model to allow for contact conditions other than a horizontal contact plane on the top of the railhead. The new model has been implemented as preliminary software in the Matlab environment.

A new finite element model of the standard UIC 920 mm diameter freight wheel has been produced including the axle. The results have been written to a modal parameters file including all six degrees of freedom at the contact and all modes up to 13 kHz (i.e. covering the 10 kHz one-third octave band).

A combination of two models for the track has been used to obtain the track mobilities in the vertical, lateral and torsional directions. A simple model of the longitudinal mobility has been identified that could also be introduced, but this has not yet been implemented.

The Contact program has been used to derive the rolling friction coefficients and their derivatives with respect to the creepages for example cases corresponding to the leading inner and leading outer wheels from earlier vehicle dynamics results of the attitude and creepages of a bogie.

Simulations of squeal have been carried out for the contact conditions of these two cases. The inner wheel is found to squeal in its O-nodal-circle axial modes, as found in earlier squeal models that account only for lateral creepage. The mode with the highest loop gain varies as the contact position moves laterally across the wheel but is mostly that with 2 nodal diameters occurring at 353 Hz. The largest squeal is expected for a contact position on the wheel between 70 and 80 mm from the flangeback.

The outer wheel, which is in flange contact, is found to squeal in its predominantly radial modes. This explains the fact that flange squeal generally occurs at higher frequencies than squeal due only to lateral creepage. The loop gains are smaller than for the leading inner wheel, although several modes are found to have similarly large loop gains. These results help to explain why flange squeal appears more intermittent and multi-tonal. It is expected, moreover, that the squeal frequency may depend strongly on the contact angle during flange contact, which will tend to vary considerably during curving.
8. FURTHER WORK

Further work is required to improve the model in the following areas:

- inclusion of additional coordinate directions in the rail mobility matrix (a model for longitudinal vibration and the rotations corresponding to vertical and lateral bending),
- inclusion of additional terms in the contact mobility matrix (to allow for ‘contact stiffness’ effects in the spin mobility and spin-lateral cross mobility),
- automation of the derivation of rolling friction coefficients and their derivatives from the Contact program,
- derivation of the effect of the falling friction coefficient with increasing rolling velocity on situations involving large spin creepage,
- calculation of squeal amplitudes (time-domain model) and radiated noise,
- validation experiments,
- implementation in the TWINS program.

In addition the model can be used to study the effect of changing wheel, track and contact parameters on the squeal noise generation. Effects that should be considered include:

- varying the creepages and derivatives to correspond to contact positions,
- including wheel rotation,
- varying the slope of the sliding friction coefficient,
- varying the train speed and curve radius,
- considering the relative effects of \([H_1][G]\) and \([H_2]\) on the instability.

It would be possible at a future stage to develop a module that will determine the lateral contact positions on the wheel and the rail and the corresponding radii of curvature of the surface starting from wheel and rail transverse profiles. Moreover, it would be desirable to develop a (steady-state) curving model to allow the effects of vehicle dynamics parameters on the squeal noise to be calculated within the same program. However, these have not been included in the present phase of the project.
REFERENCES


