isvr Institute of Sound and Vibration Research

Computer Programs for the Discrete Wavenumber Finite/Boundary Element Model for Ground Vibration and Noise from Railway Tunnels

X. Sheng, C.J.C. Jones and D.J. Thompson

ISVR Technical Memorandum 915

July 2003

University of Southampton

SCIENTIFIC PUBLICATIONS BY THE ISVR

Technical Reports are published to promote timely dissemination of research results by ISVR personnel. This medium permits more detailed presentation than is usually acceptable for scientific journals. Responsibility for both the content and any opinions expressed rests entirely with the author(s).

Technical Memoranda are produced to enable the early or preliminary release of information by ISVR personnel where such release is deemed to the appropriate. Information contained in these memoranda may be incomplete, or form part of a continuing programme; this should be borne in mind when using or quoting from these documents.

Contract Reports are produced to record the results of scientific work carried out for sponsors, under contract. The ISVR treats these reports as confidential to sponsors and does not make them available for general circulation. Individual sponsors may, however, authorize subsequent release of the material.

UNIVERSITY OF SOUTHAMPTON

INSTITUTE OF SOUND AND VIBRATION RESEARCH

DYNAMICS GROUP

Computer Programs for the Discrete Wavenumber Finite/Boundary Element Model for Ground Vibration and Noise from Railway Tunnels

by

X. Sheng, C.J.C. Jones and D.J. Thompson

ISVR Technical Memorandum No: 915

July 2003

Authorised for issue by
Professor M.J. Brennan
Group Chairman

CONTENTS

1. INTRODUCTION

The problem of ground vibration and noise from surface and underground trains has received great attention in recent years. The sources are the moving axle loads of trains and the wheel/rail dynamical forces generated from the irregularities of the wheel-rail contact surface. The frequencies of vibration generated cover a range from about 4 Hz to 200 Hz. For surface trains, a semi-analytic model has been developed [1] which assumes a flat layered ground and includes the vehicle/track/ground dynamic interactions. For trains running in tunnels, however, numerical methods must be employed. Two numerical methods are used most commonly; one is the finite element method (FEM) [2] and the other is the boundary element method (BEM) [3]. A two-dimensional FE/BE model [4] and a three-dimensional FE/BE model [5] have been implemented for the study of ground vibration induced by given harmonic loads. However, the three-dimensional model is very complicated to use and, because of the extremely large computing resources and time that are required, cannot be run many times in order to carry out an investigative study for the whole frequency range of interest for structure-borne noise from tunnels. It is unrealistic currently to couple a vehicle model with the three-dimensional track/ground model for the prediction of vibration spectra produced by wheel/rail irregularities.

In many situations, the ground-tunnel structure can be assumed to be homogeneous in the tunnel direction. This also applies to track structures and many vibration mitigation devices, such as ditches and wave impedance blocks (WIBs) [6]. An approach, termed *the discrete wavenumber finite/boundary element method* (or *the two-and-half dimensional finite/boundary element method*) has been developed for the dynamics of such structures. For each discrete wavenumber in the tunnel direction, the finite cross-section of the built structure is modelled using the FEM and the wave propagation in the surrounding soil is modelled using the BEM. The global FE/BE equations are then coupled and solved, giving the component of the response at that wavenumber. The total response is then constructed from these components using an inverse Fourier transform scheme. The simplicity and efficiency of this approach comes from the fact that the discretization is only made over the cross-section of the ground-tunnel structure, and therefore the total number of degrees of freedom is greatly reduced compared to the corresponding three-dimensional model. To implement this approach, several FORTRAN programs (FE_2_5D.EXE, FE_BE_2_5D.EXE WHEEL_RAIL_FORCE.EXE and SPECTRUM.EXE) have been produced for predicting ground vibration from tunnels. The functions of the programs are described in Sections 6 to 9.

The theory of the discrete wavenumber finite/boundary element method is presented in reference [7]. This report presents a detailed description of the computer programs. The contents of the report are arranged as follows. Section 2 briefly outlines the discrete wavenumber FE/BE methods and shows how the vibration spectra are calculated from the power spectral density of the vertical rail irregularities. Section 3 defines the element types used in the programs. Section 4 explains the structure of the main input data file. Section 5 describes the models for vehicles and a track. Sections 6 to 9 summarise the outputs of the programs. And finally, Section 10 presents some example results.

2. OUTLINES OF THE DISCRETE WAVENUMBER FINITE AND BOUNDARY ELEMENT METHODS

2.1 THE FINITE ELEMENT EQUATION

Suppose an elastic body is infinitely long in the x-direction and its cross-sections normal to the x-axis are invariant with x (Figure 1). The x cross-section is discretized into a number of elements. The displacements of the nodes on the x cross-section are denoted by a $3n$ vector

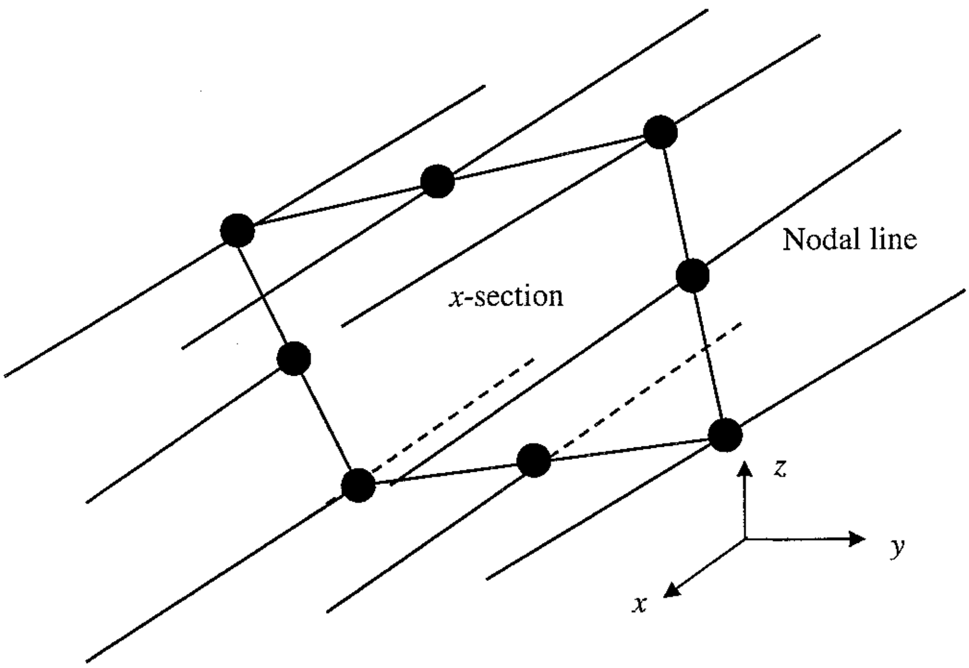


Figure 1. The elastic body and the coordinate system.

$$\{q(x,t)\} = (u_1, \upsilon_1, w_1, u_2, \upsilon_2, w_2, \cdots, u_n, \upsilon_n, w_n)^T \tag{1}$$

where, n denotes the number of the nodes. The corresponding nodal force vector is denoted by $\{F(x,t)\}$, the units of which are N/m. The finite element equation is given by

$$[M]\{\ddot{q}(x,t)\} + [K]_0\{q(x,t)\} + [K]_1 \frac{\partial}{\partial x}\{q(x,t)\} - [K]_2 \frac{\partial^2}{\partial x^2}\{q(x,t)\} = \{F(x,t)\} \tag{2}$$

where, $[M]$, $[K]_0$ and $[K]_2$ are $3n \times 3n$ symmetric matrices, and $[M]$, $[K]_2$ are positive definite and $[K]_0$ is non-negative. $[K]_1$ is an anti-symmetric matrix. All these matrices are independent of x.

Now, it is assumed that the nodal forces all are harmonic with radian frequency Ω and their action points move along the nodal lines in the positive x-direction at speed c. The Fourier transform of any quantity $f(x,t)$ of a node is denoted by $\bar{f}(\beta,t)$, where β is the wavenumber (in rad/m) in the x-direction, and

$$\bar{f}(\beta,t)=\int_{-\infty}^{\infty}f(x)e^{-i\beta x}dx\,,\quad f(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(\beta,t)e^{i\beta x}d\beta \tag{3}$$

It is shown in reference [1] that

$$\bar{f}(\beta,t)=\tilde{f}(\beta)e^{i(\Omega-\beta c)t} \tag{4}$$

Thus equation (2) becomes

$$-\omega^2[M]\{\tilde{q}(\beta)\}+\left([K]_0+i\beta[K]_1+\beta^2[K]_2\right)\{\tilde{q}(\beta)\}=\{\tilde{F}(\beta)\} \tag{5}$$

where, the equivalent radian frequency ω is given by

$$\omega=\Omega-\beta c \tag{6}$$

From equation (5) the Fourier transformed displacement vector $\{\tilde{q}(\beta)\}$ (with the factor $e^{i(\Omega-\beta c)t}$ dropped) can be worked out and then the actual displacements may be obtained using a Fourier transform algorithm.

2.2 THE BOUNDARY ELEMENT EQUATION

For the elastic body shown in Figure 1, two elasto-dynamic states (displacement and stress) are chosen. A reciprocal relationship exists between these two states. The first state is chosen to be the displacement and stress of the body due to harmonic loads of frequency Ω, with their action points moving in the positive x-direction at speed c. The displacement and stress in the second state are chosen to be those of a whole-space (the elastic body is a part of the whole-space) due to a unit harmonic load of frequency Ω. This unit load moves at speed *c in the negative x-direction* along a straight line which passes through (y_0, z_0) and is parallel to the x-axis. Thus the body force for the second state is given by

$$\rho b_k^*(x,y,z,t)=\delta(x+ct)\delta(y-y_0,z-z_0)\delta_{kl}e^{i\Omega t} \tag{7}$$

where, $k,l=1,2,3$ correspond to x, y and z, δ_{kl} is the Kronecker's delta and ρ is density of the elastic body. Fourier transforming equation (7) with respect to x yields

$$\rho\bar{b}_k^*=\delta(y-y_0,z-z_0)\delta_{kl}e^{i(\Omega+\beta c)t}$$

from which, according to equation (4), it follows that

$$\rho \tilde{b}_k^* = \delta(y - y_0, z - z_0)\delta_{kl} \tag{8}$$

Then the reciprocal relation yields

$$\begin{aligned} &\delta_{kl}\tilde{u}_k(\beta, y_0, z_0) \\ &= \int_\Gamma [\tilde{p}_k(\beta, y, z)\tilde{u}_{kl}^*(-\beta, y, z; y_0, z_0) - \tilde{p}_{kl}^*(-\beta, y, z; y_0, z_0)\tilde{u}_k(\beta, y, z)]d\Gamma \\ &+ \int_A \rho\tilde{b}_k(\beta, y, z)\tilde{u}_{kl}^*(-\beta, y, z; y_0, z_0)dA \end{aligned} \tag{9}$$

where, A denotes the cross-section of the elastic body and Γ denotes the boundary of A in its own plane. Equation (9) is an integral equation which expresses the (transformed) displacements at point (y_0, z_0) in terms of the displacements and tractions on the boundary, and the externally applied body forces. In equation (9), $\tilde{u}_{kl}^*(-\beta, y, z; y_0, z_0)$ and $\tilde{p}_{kl}^*(-\beta, y, z; y_0, z_0)$ are used to denote the Fourier transformed moving Green's functions (displacement and traction in the second state) due to the moving load defined in equation (7). The first subscript indicates the response direction while the second denotes the source direction. The summation convention over the repeated index applies. Equation (9) may be expressed in matrix form

$$\begin{aligned} &\{\tilde{u}(\beta, y_0, z_0)\} \\ &= \int_\Gamma \left([\tilde{U}^*(-\beta, y, z; y_0, z_0)]^T \{\tilde{p}(\beta, y, z)\} - [\tilde{P}^*(-\beta, y, z; y_0, z_0)]^T \{\tilde{u}(\beta, y, z)\}\right)d\Gamma \\ &+ \int_A [\tilde{U}^*(-\beta, y, z; y_0, z_0)]^T \{\rho\tilde{b}(\beta, y, z)\}dA \end{aligned} \tag{10}$$

where, $\{\tilde{u}\} = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)^T$ is the displacement vector at $(y_0, z_0) \in A$, $\{\tilde{p}\} = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)^T$ is the surface traction vector at $(y, z) \in \Gamma$,

$$[\tilde{U}^*] = \begin{bmatrix} \tilde{u}_{11}^* & \tilde{u}_{12}^* & \tilde{u}_{13}^* \\ \tilde{u}_{21}^* & \tilde{u}_{22}^* & \tilde{u}_{23}^* \\ \tilde{u}_{31}^* & \tilde{u}_{32}^* & \tilde{u}_{33}^* \end{bmatrix} \tag{11}$$

is the displacement Green's function matrix, and

$$[\tilde{P}^*] = \begin{bmatrix} \tilde{p}_{11}^* & \tilde{p}_{12}^* & \tilde{p}_{13}^* \\ \tilde{p}_{21}^* & \tilde{p}_{22}^* & \tilde{p}_{23}^* \\ \tilde{p}_{31}^* & \tilde{p}_{32}^* & \tilde{p}_{33}^* \end{bmatrix} \tag{12}$$

is the traction Green's function matrix.

Following the general development of the BEM, the boundary integral equation is established by taking point (y_0, z_0) onto the boundary Γ in equation (10). A discretization is made for equation (10) with N nodes on the boundary, giving the boundary element equation as follows

$$[H]\{\tilde{\mathbf{u}}(\beta)\} = [G]\{\tilde{\mathbf{p}}(\beta)\} + \{\tilde{\mathbf{B}}(\beta)\} \tag{13}$$

where, if the displacement and traction vectors of node i are denoted by $\{\tilde{u}(\beta)\}^i$ and $\{\tilde{p}(\beta)\}^i$, then

$$\{\tilde{\mathbf{u}}(\beta)\} = \begin{Bmatrix} \{\tilde{u}(\beta)\}^1 \\ \vdots \\ \{\tilde{u}(\beta)\}^N \end{Bmatrix}, \ \{\tilde{\mathbf{p}}(\beta)\} = \begin{Bmatrix} \{\tilde{p}(\beta)\}^1 \\ \vdots \\ \{\tilde{p}(\beta)\}^N \end{Bmatrix} \tag{14}$$

$[H]$ and $[G]$ are $3N \times 3N$ matrices depending on β, and

$$\{\tilde{B}(\beta)\}^i = \int_A [\tilde{U}^*(-\beta, y, z; y_0, z_0)]^T \{\rho\tilde{b}(\beta, y, z)\} dA, \text{ with } (y_0, z_0) = \text{node } i \tag{15}$$

constructs the vector $\{\tilde{\mathbf{B}}(\beta)\}$.

The finite and boundary element equations (5) and (13) and a track equation may be coupled to produce a global equation for the whole system. Solution of this global equation gives the Fourier transformed displacements generated by the harmonic loads at the given frequency and moving uniformly along the track.

2.3 VIBRATION POWER SPECTRA

To produce results which are comparable with measured data, prediction of vibration spectra is required. As derived in reference [1], the vertical displacement power spectrum of the track/tunnel/ground structure produced by trains running uniformly at speed c along a track is given by

$$\begin{aligned} P_w(x, y, z, f) = \frac{1}{2\pi} \sum_{k=1}^{\infty} [| S_w^0(x, y, z, f; \Omega_k) |^2 + | S_w^0(x, y, z, f; -\Omega_k) |^2] P_z(\beta_k) \Delta\beta \\ + | S_w^0(x, y, z, f; 0) |^2 \end{aligned} \tag{16}$$

where, $P_w(x, y, z, f)$, in m²/(Hz)², denotes the vertical displacement power spectrum of point (x, y, z) at frequency f; $S_w^0(x, y, z, f; \Omega_k)$, in units of 1/Hz, denotes the vertical displacement spectrum of (x, y, z) at frequency f *due to a unit vertical rail irregularity of wavenumber* β_k, *i.e.*,

$$z(x) = e^{i\beta_k x} \tag{17}$$

At this wavenumber, the radian frequency of the generated wheel/rail forces are given by $\Omega_k = \beta_k c$; $S_w^0(x, y, z, f; -\Omega_k)$ accounts for wavenumber $-\beta_k$ and for train speed $c \neq 0$, is not equal to $S_w^0(x, y, z, f; \Omega_k)$; $P_z(\beta_k)$, in m²/(cycle/m), denotes the power spectral density of the rail irregularities at wavenumber β_k; $\Delta\beta$ is the wavenumber spacing, in rad/m; and finally,

$S_w^0(x,y,z,f;0)$ denotes the vertical displacement spectrum of (x, y, z) at frequency f due to the moving axle loads (zero frequency) of the train. When divided by a chosen period of time, which normally is the time needed for a train to pass a fixed point, equation (16) gives the power spectral density of the track/tunnel/ground structure. Equation (16) may be written as an integral,

$$P_w(x,y,z,f) = \frac{1}{2\pi}\int_0^\infty [|S_w^0(x,y,z,f;\Omega)|^2 + |S_w^0(x,y,z,f;-\Omega)|^2] P_z(\beta)d\beta + |S_w^0(x,y,z,f;0)|^2 \tag{18}$$

where, $\Omega = \beta c$ is the excitation frequency corresponding to wavenumber β.

For a unit vertical rail irregularity of wavenumber β, the complex amplitude of the lth wheel/rail force is denoted by $\tilde{P}_l(\Omega)$. Except for $\Omega = 0$ which corresponds to the axle loads, $\tilde{P}_l(\Omega)$ must be determined by considering the interactions between the vehicle and the track/tunnel/ground structure. At time $t = 0$, this wheel/rail force is assumed to be located at $x = a_l$, where, $l = 1,2,\cdots,M$, M being the number of the wheel/rail forces. The quantity

$$S_P(f;\Omega) = \sum_{l=1}^{M} \tilde{P}_l(\Omega)e^{-ia_l(\Omega-2\pi f)/c} = \sum_{l=1}^{M} \tilde{P}_l(\Omega)e^{-ia_l(\beta-2\pi f/c)} \tag{19}$$

is termed *the load spectrum* at frequency f in reference [1] since it reflects the harmonic components of the excitation produced by the passage of the axles of a train. It can be shown that a shift of the train position does not change of the magnitude of the load spectrum. If the Fourier transformed vertical displacement of the track/tunnel/ground structure due to *a single stationary unit vertical harmonic load of frequency f* on the rails is denoted by $\tilde{w}^0(\beta,y,z)$, then

$$|S_w^0(x,y,z,f;\Omega)| = \frac{1}{c}|S_P(f;\Omega)||\tilde{w}^0(\beta^*,y,z)| \tag{20}$$

where,

$$\beta^* = (\Omega - 2\pi f)/c = \beta - 2\pi f/c \tag{21}$$

which is the wavenumber difference between the wavenumber of the irregularity and $2\pi f/c$. Equations (18) and (20) show that $P_w(x,y,z,f)$ and $S_w^0(x,y,z,f;\Omega)$ are independent of x as well as the position of the train, though x has been included in these notations.

For ground vibration from trains running in tunnels, the important frequencies range from about 4 to 200 Hz. For conventional train speeds between 10 and 70 m/s (36 and 250 km/h), this frequency range corresponds to rail irregularities of wavelengths from 0.05 to 17.5 m (or wavenumbers from 0.36 to 126 rad/m). The evaluation of the power spectrum using formula (16) or

(18) requires the dynamic wheel/rail interaction forces to be calculated first by coupling a vehicle model and the track/tunnel/ground model. Equation (16) also shows that calculations must be performed not only for β_k, but also for $-\beta_k$; however, the wheel/rail forces for $-\beta_k$ are the conjugates of those for β_k. In order to get an accurate and smooth power spectrum, the wavenumber spacing $\Delta\beta$ should be small and the frequencies at which the spectrum is evaluated must be as many as possible. The whole procedure may take too much time to produce results. Therefore some simplifications must be adopted to make the length of the computational time reasonable.

First, the dynamic wheel/rail forces and therefore the load spectrum defined in equation (19) are calculated by considering only the unsprung masses of the vehicles and the track structure. For passenger vehicles, the unsprung masses become decoupled from the rest of the carriage at a frequency of only about 2 Hz. For freight vehicle, this would be about 4 or 5 Hz. However the unspung masses are coupled through the rails. The track rests (via ballast or bearings) on a rigid foundation. This simplification is justified by the fact that for underground trains, the track is supported by a concrete tunnel lining which is much stiffer than the ballast or the bearings. According to the circumstances, a small finite element model including the track and part of the surrounding medium may be required for the calculation of the wheel/rail forces. Program WHEEL_RAIL_FORCE.EXE is produced for the calculations of the wheel/rail forces (see Section 8).

Second, the Fourier transformed displacement $\tilde{w}^0(\beta, y, z)$ (which is due to a single stationary unit harmonic load exerted on the rails) is calculated only at certain load frequencies (*e.g.* the one-third octave band centre frequencies which are termed *sample frequencies*). These frequencies are specified in the **frequency* block in the input data file (see Section 4). The Fourier transformed displacement at other frequencies is evaluated from those at the sample frequencies using interpolation. Since for conventional ground conditions, peaks of $\tilde{w}^0(\beta, y, z)$ occur at low frequencies, therefore the frequency spacing should be narrower for low frequency bands than for high frequency bands. For the frequency range of 4 to 200 Hz, the frequency limits and the centre frequency of each band are listed in Table 1.

TABLE 1. UPPER LIMIT, LOWER LIMIT AND CENTRAL FREQUENCIES OF ONE-THIRD OCTAVE BANDS

Central frequency (Hz)	Band width (Hz)	Central frequency (Hz)	Band width (Hz)	Central frequency (Hz)	Band width (Hz)
4	3.6 – 4.5	16	14.3 – 17.8	63	56.2 – 70.8
5	4.5 – 5.6	20	17.8 – 22.4	80	70.8 – 89.1
6.3	5.6 – 7.1	25	22.4 – 28.2	100	89.1 – 112
8	7.1 – 8.9	31.5	28.2 – 35.5	125	112 – 141
10	8.9 – 11.1	40	35.5 – 44.7	160	141 – 178
12.5	11.1 – 14.3	50	44.7 – 56.2	200	178 – 224

For each sample frequency $\tilde{w}^0(\beta, y, z)$ in equation (20) is calculated by the program SPECTRUM.EXE. This takes most of the computational time. The following facts can be used to simplify the calculation of $\tilde{w}^0(\beta, y, z)$.

(1) As pointed above, $\tilde{w}^0(\beta, y, z)$ is due to a single stationary unit vertical harmonic load of frequency f (at which the displacement power spectrum is evaluated), therefore $\tilde{w}^0(\beta, y, z) = \tilde{w}^0(-\beta, y, z)$. (For the longitudinal displacement $\tilde{u}^0(\beta, y, z)$ and the lateral displacement $\tilde{\upsilon}^0(\beta, y, z)$, $\tilde{u}^0(\beta, y, z) = -\tilde{u}^0(-\beta, y, z)$, $\tilde{\upsilon}^0(\beta, y, z) = \tilde{\upsilon}^0(-\beta, y, z)$).

(2) Since damping exists in the ground, $|\tilde{w}^0(\beta, y, z)|$ is a regular function of β and has significant values only for $|\beta| \le \beta_0$. For wavenumber greater than β_0, $|\tilde{w}^0(\beta, y, z)|$ decreases quickly with β. However, $|\tilde{w}^0(\beta, y, z)|$ has several peaks within this range, making it not a simple function. For the ground surface, β_0 is believed to be much less than the Rayleigh wavenumber of the soil, due partly to the presence of the built tunnel which, as a whole, vibrates like a beam, and partly to the fact that the excitation is not on, but below, the ground surface. At 200 Hz and for most types of soil, the Rayleigh wavenumber is less than 2π rad/m (corresponding to a Rayleigh wave speed of 200 m/s). Figures showing $|\tilde{w}^0(\beta, y, z)|$ are presented in Section 10.

(3) With the aforementioned range of wavelengths (0.05 to 17.5 m), the power spectral density of the vertical rail irregularities, $P_z(\beta)$, can be approximated by [8]

$$P_z(\beta) = \frac{A}{(\beta^2+B^2)(\beta^2+C^2)} \tag{22}$$

where, A, B and C are positive constants. On the other hand, for a rail irregularity of unit amplitude and for frequency less than the unsprung-mass-on-track resonance frequency (beyond which, the wheel/rail forces decrease with frequency, see Section 10), the magnitudes of the wheel/rail forces increase with β at a rate of about power 2. Thus the product of the squared load spectrum and the power spectral density of the rail irregularities can be expressed as

$$|S_P(f;\Omega)|^2 \; P_z(\beta) = \frac{D\beta^4}{(\beta^2+B^2)(\beta^2+C^2)} \tag{23}$$

(D is a new constant) which approaches a constant as $\beta \to \infty$. Due to the property of $|\tilde{w}^0(\beta, y, z)|$ identified in (2), the integral over the semi-infinite interval in equation (18) converges and may be approximated by an integral over a finite interval 0 to β_{max}. In other words

$$P_w(x,y,z,f) =$$

$$\frac{1}{2\pi c^2}\int_0^{\beta_{max}}[|S_P(f;\Omega)|^2|\tilde{w}^0(\beta-\tfrac{2\pi f}{c},y,z)|^2+|S_P(f;-\Omega)|^2|\tilde{w}^0(-\beta-\tfrac{2\pi f}{c},y,z)|^2]P_z(\beta)d\beta \quad (24)$$

$$+\frac{1}{c^2}|S_P(f;0)|^2|\tilde{w}^0(\tfrac{2\pi f}{c},y,z)|^2$$

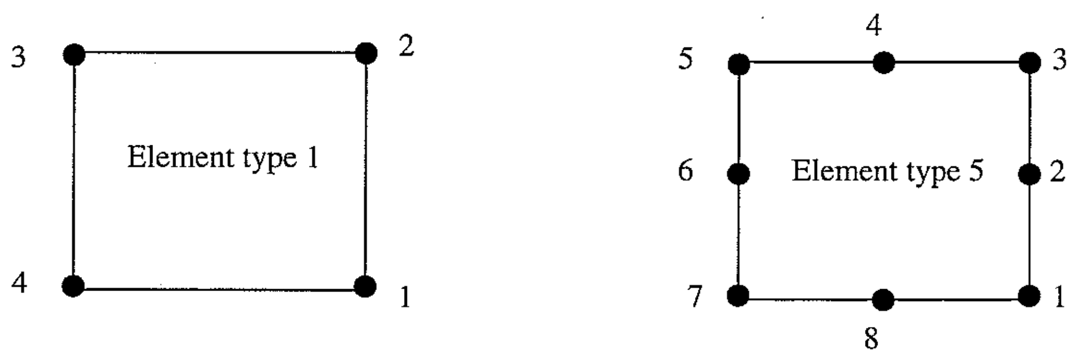
As has been identified, β_{max} is determined by the property of $|\tilde{w}^0(\beta,y,z)|$. Suppose for $|\beta|>\beta_0$ the value of $|\tilde{w}^0(\beta,y,z)|$ is negligible, then equation (24) indicates that $\beta_{max}=\beta_0+\frac{2\pi f}{c}$. The maximum β_{max} occurs at the highest frequency (200 Hz) and the slowest train speed (10 m/s) and $\beta_{max}=125$ rad/m (β_0 is in general much smaller than this value and is neglected). Since the excitation frequency for the wheel/rail forces is given by $\Omega=\beta c$, the maximum frequency below which the wheel/rail forces are calculated is given by $\Omega_{max}=(\beta_0+\frac{2\pi f}{c})c=\beta_0 c+2\pi f$, which is less than 300 Hz for the highest train speed of 70 m/s and for the highest frequency of 200 Hz.

(4) In actual evaluation of the integral in equation (24), a numerical method must be employed. To do so, the 'step' of β must be chosen properly. From equation (19) it can be seen that $|S_P(f;\Omega)|^2$ is a highly oscillatory function of Ω or β. The step of β should be less than 2π divided by the distance from the first axle to the last.

3. FINITE AND BOUNDARY ELEMENTS USED IN THE PROGRAMS

3.1 FINITE ELEMENTS

There are four types of finite element being included (Table 2 and Figure 2) in the FE program FE_2_5D.EXE. These are the four-noded quadrilateral element, the three-noded triangle element, the eight-noded quadrilateral element and the six-noded triangle element. Two numbering systems are applied to the nodes of a FE element, *i.e.*, the global numbering system and the local numbering system. The local numbering must be anticlockwise, as indicated in Figure 2 (coordinate system is shown in Figure 1).



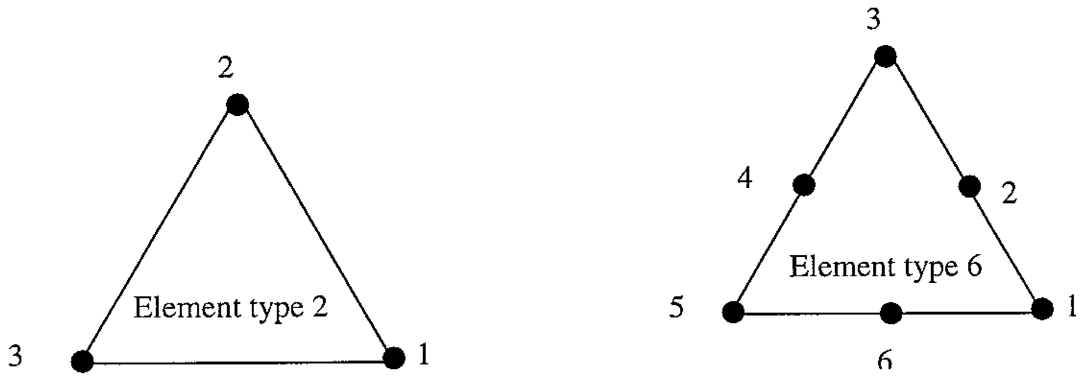


Figure 2. Finite element type numbers and node numbering.

TABLE 2. FINITE ELEMENT TYPES

Element description	Type Number	Order of approximation used
Four-noded quadrilateral	1	Linear shape functions
Three-noded triangle	2	Linear shape functions
Eight-noded quadrilateral	5	Quadratic shape functions
Six-noded triangle	6	Quadratic shape functions

3.2 BOUNDARY ELEMENTS

Only one type of boundary element has been implemented in the coupled FE/BE programs FE_BE_2_5D.EXE and SPECTRUM.EXE. This is the three-noded, quadratic shape function boundary element, shown in Figure 3. For the three nodes of a boundary element, there are two numbering systems, *i.e.*, the global numbering system and the local numbering system. The local numbering must be such that when moving along the boundary element from the locally first node to the locally third node, the material is on the left-hand side.

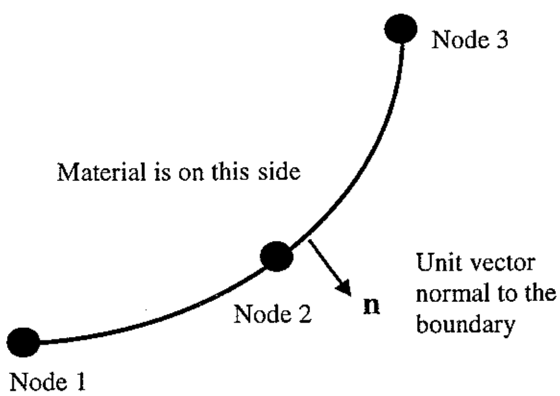


Figure 3. Three-noded, quadratic shape function boundary element.

3.3 ELEMENTS ON THE INTERFACE OF TWO SUB-DOMAINS

If a structure is divided into several sub-domains, then any of the interfaces of the sub-domains must be discretized into boundary elements in such a way that each of the elements must totally belong to the interface, and each node on the elements must be common to the two neighbouring sub-domains. This is illustrated in Figure 4, where, the nodes on the boundary elements (Γ_1 and Γ_2) form part of the nodes of the two eight-noded quadrilateral finite elements. This requirement is imposed by the program rather than by the FE/BE theory. The program allows, at most, four BE sub-domains, one FE sub-domain and one track model. The FE or BE equations of the sub-domains, and that of the track, are coupled to form the global equation for the whole system.

3.4 USE OF THE SYMMETRY OF A STRUCTURE

If a structure is symmetric about the *xz* plane (see Figure 1), then only half the structure is required to be discretized (for a track structure, see Section 5). Note that each element must totally belong to the chosen half-structure. A mirror discretization is formed by the program for the other half-structure. The mirror element of the *j*th element in the chosen half-structure is numbered as $NE + j$, where *NE* is the number of the elements in the chosen half. Similarly, the mirror node of the *j*th node in the chosen half-structure is numbered as $N + j$, where *N* is the number of the nodes in the chosen half-structure. If the loads applied are symmetric or anti-symmetric about the *xz* plane, then the response of the structure is also symmetric or anti-symmetric. Use of the symmetry property of the structure, loads and response yields a global equation which governs the nodal displacements of the chosen half-structure. Further, if the loads are stationary and symmetric about the *yz* plane, then the calculation is only required to be carried out for $\beta \leq 0$ (or $\beta \geq 0$). This is because $\tilde{u}_1(-\beta) = -\tilde{u}_1(\beta)$, $\tilde{u}_2(-\beta) = \tilde{u}_2(\beta)$, $\tilde{u}_3(-\beta) = \tilde{u}_3(\beta)$.

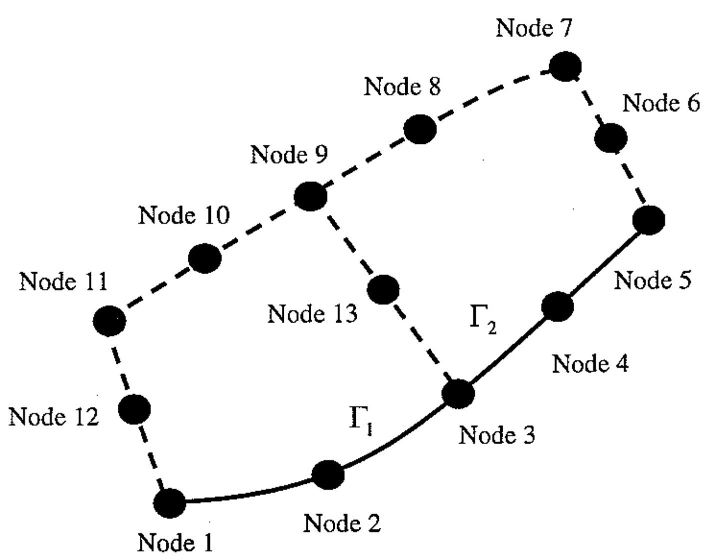


Figure 4. The coupling of finite elements and boundary elements.

4. THE MAIN INPUT DATA FILE

4.1 THE STRUCTURE OF THE MAIN INPUT DATA FILE

The finite element program FE_2_5D.EXE and the coupled FE/BE programs FE_BE_2_5D.EXE and SPECTRUM.EXE use the same input data file (the main input data file) which provides all the data needed except for those of the vehicles and the track. The vehicle/track data file is explained in Section 5. The main input data file contains several data blocks. The heading of each block begins with the symbol *. The order of the blocks, except for the **end* block which must appear last in the file, is arbitrary. The data records within each data block consist of several numbers separated by commas and are explained in Section 4.2. The data blocks can contain blank records, and/or comment lines which must begin with **. The headings of the data blocks are as follows.

**title*

**material*

**global node coordinates*

**boundary element topology (domain 1)*

**boundary element topology (domain 2)*

......

**edge element*

**track*

**finite element topology*

**frequency*

**load speed*

**structure symmetry*

**load symmetry*

**load position and amplitude on BE*

**load position and amplitude on FE*

**end*

It should be noted that the program FE_2_5D.EXE only reads the data in the **title*, **material*, **global node coordinates*, **finite element topology* and **structure symmetry* blocks.

4.2 DATA RECORDS IN EACH DATA BLOCK

4.2.1 *THE *title BLOCK*

This is an optional block. If it is present, a title describing the job may be given in this block, and when the program is started, the program will display on the screen the title in a line just below *Found title*. If it is not present, then the program will display on the screen '***Warning – no title*'.

4.2.2 *THE *material BLOCK*

This block defines the material property. Each data record corresponds to one set of material parameters and contains five numbers. The first is an integer, specifying the material number. The other four are real numbers of double-precision, specifying the Young's modulus, Possion ratio, density and loss factor of the material. The order of the records can be arbitrary. If this block is present and contains no error, then the screen will display

Read material properties for each domain

Number of sets of material property read:

to show how many sets of material have been read. If this block is not present, or if it contains errors, then an error message appears and the program stops.

4.2.3 *THE *global node coordinates BLOCK*

This block contains the global coordinates of the nodes. The data records must follow the natural order of the node numbers. For example, the 20^{th} record defines the coordinates of the 20^{th} node. Each record contains two double-precision numbers, *i.e.* the y and z-coordinates of the corresponding node. If this block is present and contains no error, then the screen reads

Read global node coordinates

Number of sets of coordinates read:

If this block is not present, or if it contains errors, then an error message appears on the screen and the program stops.

4.2.4 *THE *boundary element domain topology (domain 1) BLOCK*

Each data record in this block contains three integers. They are, respectively, the global node numbers of the locally first, second and third nodes on a boundary element in the first boundary element domain. The program sequentially numbers the elements as it reads the records, defining the element numbers local to the domain. If this block is present and contains no error, then the screen reads

Read boundary element topology for domain 1

If this block contains errors, then an error message will be displayed and the program stops. *The material number of a boundary element domain must be defined to be the same as the domain number.*

4.2.5 *THE *edge element BLOCK*

In the boundary element model, the ground surface and interfaces of a layered ground must be truncated. In the programs FE_BE_2_5D.EXE and SPECTRUM.EXE, a facility is devised to minimise the effects of the truncation. This data block provides information for the facility. It is assumed that at the edge element, its neighbouring element and beyond the edge element, the boundary is flat. Each data record in this block contains seven integers. The first one indicates the domain number. The second is either +1 or –1, indicating the integral direction along the edge element; if the integral direction is in the positive y-direction, then +1 is set; if the integral direction is in the negative y-direction, then –1 is set. The other five integers are the global numbers of the nodes on the edge element and its neighbouring element. They must be in such an order that begins with the number of the edge node and ends with the number of the node farthest from the edge node.

If no truncation is made for the BE model, then this block must not be present.

4.2.6 *THE *track BLOCK*

This must be an empty block. It is to inform the programs FE_BE_2_5D.EXE and SPECTRUM.EXE that a track is coupled to the tunnel. If no track is present, this block must not be present.

4.2.7 *THE *finite element domain topology BLOCK*

This block must not be present if no FE domain is under analysis. Each data record in this block contains, at most, fourteen integers. The first two integers are, respectively, the finite element type number (see Section 3) and the material number. The other integers are the global node numbers of the locally first, second ... nodes of the FE element. The program FE_2_5D.EXE sequentially numbers the elements as it reads the records, defining the element numbers local to the FE domain. If this block is present, the programs FE_BE_2_5D.EXE and SPECTRUM.EXE display

A finite element domain is present

Read information of the FE domain

Instead of reading the records in this block, these two programs read information from files *FE_topology.out*, *m.out*, *k0.out*, *k1.out*, *k2.out*, *c0.out*, *c1.out*, and *c2.out*, which must be produced in advance by running the finite element program FE_2_5D.EXE. These files must be stored in the user's output directory. If the reading is successful, then a message appears summarising the information for the finite element domain (see Section 6). It should be noted that *the finite element domain must be defined as the last domain having the maximum domain number*.

4.2.8 *THE *frequency BLOCK*

This block contains records of a single number of double-precision. Each record specifies a load frequency in Hz. When the data in this block are read successfully, the programs will display how many frequencies are specified.

4.2.9 *THE *load speed BLOCK*

This block contains only one record of a single number of double-precision specifying the load speed in the x-direction. If this block is not present, then an error message will be displayed and the program stops.

4.2.10 *THE *structure symmetry BLOCK*

This block contains only one record of a single integer specifying the symmetry property of the structure. If the structure is not symmetric about the xz plane, this integer is set to be 0. On the other hand, if the structure is symmetric about the xz plane, then this integer is set to be 1. Only half the structure ($y \geq 0$) needs to be discretized if it is symmetric about the xz plane. The use of the symmetry of a track is further explained in Section 5.

4.2.11 *THE *load symmetry BLOCK*

This block contains only one record of a single integer specifying the symmetry property of the loads. This block should be present only when the structure is symmetric about the xz plane. If the loads are symmetric about the xz plane, then this integer is set to be 1. If the loads are anti-symmetric about the xz plane, then this integer is set to be 0. In these two cases, only the loads applied where $y \geq 0$ need to be specified in the load-blocks (see below). If the loads are not symmetric about the xz plane, then this integer is set to be 2. In this case, all the loads are assumed to be applied where $y \geq 0$ and the program will decompose the loads into symmetric and anti-symmetric parts.

4.2.12 *THE *load position and amplitude on BE BLOCK*

This block defines the point loads applied on the boundary element domains. Each record contains nine numbers and corresponds to one of the loads. The first is an integer, specifying the boundary element domain number. The second and third are double-precision numbers, specifying the y and z coordinates of the loading point of the load. Note that the program allows the loading point to be coincided with a node. In this case, results at nodes near this node are not reliable. The other six numbers, being of double-precision, are the real and imaginary parts of the *x*-, *y*- and *z*-components of the amplitude of the load. If this block is not present, then the program gives ***Warning no loads on BE domain.*

4.2.13 *THE *load position and amplitude on FE BLOCK*

This block defines the nodal forces applied on the finite element domain. Each record contains eight numbers and corresponds to one of the nodal forces. The first is an integer, specifying the finite element domain number. The second is also an integer, specifying the global number of the loaded node. The other six numbers, being of double-precision, are the real and imaginary parts of the *x*-, *y*- and *z*-components of the amplitude of the nodal force. If this block is not present, then the program gives ***Warning no loads on FE domain or no FE domain.*

If loads are applied via a track, then both the **load position and amplitude on BE* and **load position and amplitude on FE* blocks must not be present. Instead, load specification is given in the vehicle/track data file.

4.2.14 *THE *end BLOCK*

This is an empty block, indicating the end of the input data file.

5. TRACK EQUATION AND VEHICLE/TRACK DATA FILE

5.1 TRACK EQUATION

A track is modelled as a system consisting of multiple Euler beams. A distributed spring is inserted between each pair of adjacent beams, and between the lowest beam and the track support. Only the vertical displacements of the track are considered. The track equation in the wavenumber-frequency domain is given by

$$\left([K]_0 + \beta^4[K]_2 - \omega^2[M]\right)\{\tilde{w}(\beta)\} = \{\tilde{F}(\beta)\} \qquad (25)$$

where, $[M]$, $[K]_0$ and $[K]_2$ are symmetric matrices and ω is defined in equation (6). The degrees of freedom of the track are numbered after the degrees of freedom of the tunnel/ground system. In other words, the *i*th local degree of freedom of the track corresponds to the $(i+3N)$ th global degree of freedom, where *N* denotes the number of the nodes of the tunnel/ground system.

5.2 VEHICLE/TRACK DATA FILE

If a track is present, the track data file which must be in the user's input directory is required by the programs FE_BE_2_5D.EXE and SPECTRUM.EXE. This file contains the following data blocks. If the whole structure is symmetric about the *xz*-plane, then the parameters given in these blocks are for half the track (one rail, half s sleeper, etc.).

5.2.1 *THE *vehicle parameter BLOCK*

Each record in this block contains three numbers of double-precision and corresponds to one axle. The first number is the axle load. The second number is the unprung mass of the axle. The last number is the relative *x*-coordinate of the axle. The program FE_BE_2_5D.EXE does not access this block.

5.2.2 *THE *element in matrix [m] BLOCK*

This block contains the elements of matrix $[M]$ in equation (25) in the same order as in this matrix.

5.2.3 *THE *element in matrix [k]0 BLOCK*

This block contains the real parts of the elements in matrix $[K]_0$ in equation (25) in the same order as in this matrix.

5.2.4 *THE *element in matrix [c]0 BLOCK*

This block contains the imaginary parts of the elements in matrix $[K]_0$.

5.2.5 *THE *element in matrix [k]2 BLOCK*

This block contains the real parts of the elements in matrix $[K]_2$.

5.2.6 *THE *elements in matrix [c]2 BLOCK*

This block contains the imaginary parts of the elements in matrix $[K]_2$.

5.2.7 *THE *track/tunnel position in connection, stiffness and damping BLOCK*

This block consists of records of four numbers. Each record corresponds to a connection between a local degree of freedom of the track and a node on the tunnel. The number of the records indicates the number of the connections between the track and the tunnel/ground system. In each record (connection), the first number, an integer, specifies the *local* degree of freedom of the track, and the second number, also an integer, indicates the *global* number of the node on the tunnel. The last two are real numbers of double precision, giving the real part and the imaginary part of the vertical stiffness between the track and its support at this connection. Note that this stiffness is for a unit length of track, therefore having units of N/m^2.

5.2.7 *THE *load position and amplitude on track BLOCK*

Each record in this block contains three numbers. The first is an integer indicating the local degree of freedom of the track. The last two are real numbers of double precision, giving the real and imaginary parts of the load of that degree of freedom. If no load is applied on the track, this block must not be present.

When run the program SPECTRUM.EXE to calculate $\tilde{w}^0(\beta, y, z)$, the load on the rails must be unit in magnitude and single. If half the track is considered, the magnitude of the single load is 0.5. See the example presented in Section 10.2.

5.2.8 *THE *end BLOCK*

This is an empty block, indicating the end of the track data file.

6. OUTPUTS FROM PROGRAM FE_2_5D.EXE

This program is to calculate and save the FE matrices defined in equation (2).

6.1 OUTPUTS ON THE SCREEN

The outputs on the screen from the program FE_2_5D.EXE run for the example presented later in Section 10 are listed in Table 3.

TABLE 3. OUTPUTS ON THE SCREEN FROM THE PROGRAM FE_2_5D.EXE

(Sans Serif front indicates user input)

On the screen	Explanation
Input path and name of the main input data file d:\working\lined_tun.dat *Input path and directory for the outputs* d:\working *Read input data* == *THIS PROGRAM IS DEVELOPED BY X SHENG, ISVR 2003* == *Found title* *Calculate the response of a half-space including a lined tunnel* *Read material properties* *Sets of material property read:* *2* *Read global node coordinates* *How many nodes are specified?* *254* *Read finite element topology* *Data read successfully* ******************************** *Structure is symmetric* *Number of on-z-nodes* *6* *Number of finite elements* *60*	See Section 7

Number of nodes of the finite elements 300 ********************************** *Finite element matrices are in calculation, wait...*	

6.2 OUTPUT DATA FILES

The program FE_2_5D.EXE produces eight data files named *m.out*, *k0.out*, *k1.out*, *k2.out*, *c0.out*, *c1.out*, *c2.out*, and *FE_topology.out*. The files *m.out*, *k0.out*, *k1.out*, *k2.out* contain the elements in matrices $[M]$, $[K]_0$, $[K]_1$ and $[K]_2$ (see Section 2.1). When damping is considered, the elements in matrices $[K]_0$ and $[K]_1$ and $[K]_2$ are complex. Thus *k0.out*, *k1.out*, *k2.out* contain the real parts while *c0.out*, *c1.out*, *c2.out* contain the imaginary parts. Due to the limited band width and the symmetry or anti-symmetry of these matrices, only elements which are on or below the diagonal and within the band are written into the files. To illustrate the structure of the stored matrices, the following Matlab program can be used to reconstruct the matrices from the files (Note: *totdof* represents the total number of degree of freedom of the finite element model and *hband* represents the half-band of the matrices. These two parameters are found in the file *FE_topology.out*):

```
%Open files
fid1 = fopen('m.out');
fid2 = fopen('k0.out');
fid3 = fopen('k1.out');
fid4 = fopen('k2.out');
fid5 = fopen('c0.out');
fid6 = fopen('c1.out');
fid7 = fopen('c2.out');

%Read in the half-band data
temp1 = fscanf(fid1,'%f',[totdof, hband]);
temp2 = fscanf(fid2,'%f',[totdof, hband]);
temp3 = fscanf(fid3,'%f',[totdof, hband]);
temp4 = fscanf(fid4,'%f',[totdof, hband]);
temp5 = fscanf(fid5,'%f',[totdof, hband]);
temp6 = fscanf(fid6,'%f',[totdof, hband]);
temp7 = fscanf(fid7,'%f',[totdof, hband]);

%Form the matrices [M], [K]0, [K]1, [K]2 and [C]0, [C]1 and [C]2.
%Note that [M], [K]0 and [K]2 are symmetric while [K]1 and [C]1 are anti-symmetric.
for i = 1: totdof
        for j = 1: i
                m0(i, j) = 0;
                k0(i, j) = 0; k1(i, j) = 0; k2(i, j) = 0;
                c0(i, j) = 0; c1(i, j) = 0; c2(i, j) = 0;
                ij = hband + j – i;
                if ij > 0
                        m0(i, j) = temp1(i, ij);
```

```
                k0(i, j) = temp2(i, ij);
                k1(i, j) = temp3(i, ij);
                k2(i, j) = temp4(i, ij);
                c0(i, j) = temp5(i, ij);
                c1(i, j) = temp6(i, ij);
                c2(i, j) = temp7(i, ij);
            end;
            m0(j, i) = m0(i, j);
            k0(j, i) = k0(i, j);
            k1(j, i) = – k1(i, j);
            k2(j, i) = k2(i, j);
            c0(j, i) = c0(i, j);
            c1(j, i) = – c1(i, j);
            c2(j, i) = c2(i, j);
        end;
    end;
```

With these matrices, solution can be performed for the finite element equation (5) for each wavenumber β. The sign of the imaginary parts of the elements in matrices $[K]_0$ and $[K]_1$ and $[K]_2$ must be changed when ω, given by equation (6), is negative.

The file *FE_topology.out* contains information of the finite element model which will be read by the program FE_BE_2_5D.EXE or SPECTRUM.EXE if the finite element domain is coupled with boundary element domains. This file also provides an opportunity to identify if there are errors in the main input data file. The file contains four data blocks.

The first is the **Totdof, hband, nztl and Totels* block, in which there is only one record having four integers separated by commas, *e.g.*, 900, 474, 6, 60. The first integer indicates the total number of degrees of freedom of the finite element model, the second indicates the half-band width, the third indicates the number of nodes on the z-axis (if the structure is not symmetric, then $nztl = 0$ and it will not be used by the programs), and the last indicates the number of the finite elements of the whole FE domain (rather than a half of it).

The second block is **Local/global/mirror node number; coordinates*, in which each record corresponds to a node of the FE domain and has five numbers separated by commas, for instance, 1, 102, 1, 0.000, –12.900. The first three numbers are integers, representing the local, global numbers of the node and the global number of the mirror image (node) of that node. Only for a symmetric structure are the mirror node numbers meaningful, otherwise they are all zero. The last two numbers in a record represent the y and z coordinates of the corresponding node.

The third block is the **element and its local node numbers* block, describing the topology of the finite element model. Each record in this block corresponds to a finite element and contains a number of integers with the first representing the element number and the remainder the local nodal numbers on this element following the order defined in Section 3.1.

The last block is the **end* block, indicating the end of the file.

7. OUTPUTS FROM PROGRAM FE_BE_2_5D.EXE

This program calculates the displacements of a structure due to moving or stationary harmonic loads.

7.1 OUTPUTS ON THE SCREEN

The outputs on the screen from the program FE_BE_2_5D.EXE run for the example in Section 10 are listed in Table 4.

TABLE 4. OUTPUTS ON THE SCREEN FROM THE PROGRAM FE_BE_2_5D.EXE
(Sans Serif front indicates user input)

On the screen	Explanation
Input number of wavenumbers & dbeta/2pi(cycle/m) 512, 0.005 *Input path and name of the main input data file* d:\working\lined_tun.dat *Input path and directory for the outputs* d:\working	Input the number of discrete wavenumbers and the wavenumber spacing in cycle/m. The number of discrete wavenumbers must be a power of 2.
====================================== *THIS PROGRAM IS DEVELOPED BY X SHENG, ISVR 2003* ======================================	
Found title *Calculate the response of a half-space including a lined tunnel* *Read material properties for each domain*	
Number of sets of material property read: 2	Two material sets are specified.
Read global node coordinates	
Number of sets of coordinates read: 254	254 nodes are specified.
Read boundary element topology for domain 1 *Read edge BE information* *Read frequency* *Read load speed* *Read position and amplitudes of node loads on FE* *Warning No load on the BE domain* *Data read successfully* *****Information for boundary element domains*****	
Number of on-z-nodes of domain 1 : 3	On the BE domain 1 there 3 nodes on the *z*-axis.
Number of elements of domain 1 : 160	There are 160 elements on the whole BE domain 1.
Number of nodes of domain 1 : 321 ***************************************	There are 321 nodes on the whole BE domain 1.

***********Information for loads***********	
Load frequency: 40.000000000000	
Load speed: 100.00000000000	
Loads are symmetric	
Number of loads on domain 1 : 0	No loads are applied on the BE domain 1.
Number of loads on domain 2 : 2	The two loads on the FE domain are symmetric.
**	
A finite element domain is present	
Input path and directory for FE information d:\working	Specify where the matrices and other information of the FE domain are held.
Read information of the FE domain	
Read total dof, half band and number of elements	
Read local/global/mirror node numbers	
FE_information read successfully	
****Information for the finite element domain****	
Number of on-z-nodes of domain 2 : 6	On the FE domain there are 6 nodes on the *z*-axis.
Number of elements of domain 2 : 60	There are 60 elements on the whole FE domain.
Number of nodes of domain 2 : 300	There are 300 nodes on the whole FE domain.
Dof the FE domain: 900	The degrees of freedom of the whole FE domain are 900.
Half band of the FE domain: 474	The half-band width of the FE matrices of the whole FE domain is 474.
***	
Dimension of the global equation: 762	Dimension of the global coupled FE/BE equation is 762, which is for half the structure since it is symmetric.
***************************************	
Calculate matching matrix for the BE/FE interface	
*******information for the FE/BE interface*******	
Number of the BE elements on the FE/BE interface: 60	Information for the whole FE/BE interface.
Number of nodes on the FE/BE interface: 120	
Number of nodes on the z-axis: 2	
**	
For the 1th wavenumber = -8.035Rad/m	Indicating calculation is being carried out for the first wavenumber. This and what follows indicate the progress of the calculation.

7.2 OUTPUT DATA FILES

If the structure is not symmetric about the *xz* plane, or if the structure is symmetric about the *xz* plane and the loads applied are symmetric or anti-symmetric about the same plane, then the program FE_BE_2_5D.EXE creates six files named *u*.dsp, u*.pha, v*.dsp, v*.pha, w*.dsp* and **w.pha* in the user's output directory to store the amplitudes (*dsp*) and phase angles (*pha*) of the longitudinal (*u*), lateral (*v*) and vertical (*w*) nodal displacements. Here '*' is not a part of the names but stands for a number showing the frequency of the loads. From the first node to the last one, the displacements are stored consecutively for each of the x-positions given by $x = -(ngrid - 1) \times \Delta x, \cdots, 0, \Delta x, \cdots, ngrid \times \Delta x$, where, $2 \times ngrid$ is the number of the discrete wavenumbers, $\Delta x = 2\pi / (2 \times ngrid \times \Delta\beta)$ and $\Delta\beta$ is the wavenumber spacing in rad/m. If a track is present, its displacements are stored at the bottom of the files *w*.dsp* and *w*.pha*.

If the structure is symmetric about the *xz* plane but the loads applied are not symmetric or anti-symmetric about the same plane, then in addition to the six files explained above, six more files are also created in the user's output directory by the program. These are named as *un*.dsp, un*.pha, vn*.dsp, vn*.pha, wn*.dsp* and *wn*.pha*, and are used to store the displacement amplitudes (*dsp*) and phase angles (*pha*) of the mirrors of the nodes.

The files may be read using the following Matlab commands:

```
fid = fopen('d:\working\w20.dsp');  %Open a file;
a = fscanf(fid, '%f', [2×ngrid, N]);
```

Where 2×*ngrid* is the number of the discrete wavenumbers and *N* is the number of the nodes.

8. OUTPUTS FROM PROGRAM WHEEL_RAIL_FORCE.EXE

The program WHEEL_RAIL_FORCE.EXE calculates the dynamic wheel/rail forces due to a vertical rail irregularity with *constant amplitude of 0.1 mm* but different wavelengths (or different excitation frequencies. The excitation frequency equals the train speed divided by wavelength).

8.1 OUTPUTS ON THE SCREEN

The outputs on the screen from the program WHEEL_RAIL_FORCE.EXE are listed in Table 5.

TABLE 5. OUTPUTS ON THE SCREEN FROM THE PROGRAM WHEEL_RAIL_FORCE.EXE
(Sans Serif front indicates user input)

On the screen	Explanation
Input path and name of the vehicle/track data file d:\wheel_rail_force\vehicle_track.dat *Input path and directory for the outputs* d:\wheel_rail_force *Input train speed* 25.d0 ******Information for the vehicles and track****** *Number of unsprung masses:* 7 *Degree of freedom of the track:* 2	

** *For frequency = 5.0000000000000 Hz*	Calculations are performed for excitation frequencies from 5 Hz to 500 Hz with a frequency spacing of 5 Hz.

8.2 OUTPUTS AS DATA FILES

The program WHEEL_RAIL_FORCE.EXE creates two files named *load* and *load.pha* in the user's output directory to store the amplitudes and phase angles (*pha*) of the wheel/rail forces. From the first axle to the last one, the wheel/rail forces are stored consecutively for each of the excitation frequencies from 0 Hz to 500 Hz with a spacing of 5 Hz. The wheel/rail forces at zero Hz correspond to the axle loads. Therefore there are $M \times 101$ data items in each of the files, where M denotes the number of the axles.

9. OUTPUTS FROM PROGRAM SPECTRUM.EXE

The program SPECTRUM.EXE calculates the Fourier transformed displacements of the structure due to a single unit vertical stationary harmonic load applied on the rail. The frequencies of the load are specified in the **frequency* block in the main input data file. For each of these frequencies, the program calculates the Fourier transformed displacements at 100 wavenumbers: 0, 0.05, 0.1, …, 4.95 rad/m. For wavenumber greater than 4.95 rad/m, the Fourier transformed displacements are approximated as an exponentially decaying function of wavenumber.

9.1 OUTPUTS ON THE SCREEN

The outputs on the screen from the program SPECTRUM.EXE are almost identical to those from the program FE_BE_2_5D.EXE, therefore no further explanation is given here.

9.2 OUTPUT DATA FILES

If the structure is not symmetric about the *xz* plane, or if the structure is symmetric about the *xz* plane and the loads applied are symmetric or anti-symmetric about the same plane, then the program SPECTRUM.EXE creates six files named *ubar.dsp, ubar.pha, vbar.dsp, vbar.pha, wbar.dsp* and *wbar.pha* in the user's output directory to store the amplitudes (*dsp*) and phase angles (*pha*) of the Fourier transformed longitudinal (*ubar*), lateral (*vbar*) and vertical (*wbar*) nodal displacements. From the first node to the last one, the displacements are stored consecutively for each of the wavenumbers of 0, 0.05, 0.1, …, 4.95 rad/m. The displacements of the track are stored at the bottom of the files *wbar.dsp* and *wbar.pha*.

If the structure is symmetric about the *xz* plane but the loads applied are not symmetric or anti-symmetric about the same plane, then in addition to the six files explained above, six more files are also created in the user's output directory by the program. These are named *ubarn.dsp, ubarn.pha, vbarn.dsp, vbarn.pha, wbarn.dsp* and *wbarn.pha*, and are used to store the displacement amplitudes (*dsp*) and phase angles (*pha*) of the mirrors of the nodes.

The files may be read using the following Matlab commands:

```
fid = fopen('d:\working\wbar.dsp');  %Open a file;
for j = 1:numfre
        a = fscanf(fid, '%f', [100, N]);
        b(:, :, j) = a(:, :);
end;
clear a;
```

Here N is the number of the nodes, *numfre* is the number of the frequencies. These data, together with the wheel/rail dynamic forces calculated by the program WHEEL_RAIL_FORCE.EXE, will be processed to give the vibration spectra of the track/tunnel/ground system. This will be explained in Section 10.2.

10. RESULTS FOR A HOMOGENEOUS HALF-SPACE WITH A LINED CIRCULAR TUNNEL

To demonstrate the use of the programs, this section presents an analysis for a homogeneous half-space with a lined circular tunnel. The parameters for the half-space are listed in Table 6 and those of the concrete tunnel lining in Table 7. The lined tunnel has an outer radius of 3.6 m and its axis is at 16.5 m below the ground surface. Firstly in Sub-section 10.1, responses of the tunnel/ground structure (without a track and vehicles) are calculated for a unit vertical harmonic load of 40 Hz moving at 100 m/s along the tunnel invert. Secondly in Sub-section 10.2, a slab track structure is coupled with the tunnel lining and vibration levels of the ground/tunnel/track structure are calculated for a train running along the track.

TABLE 6. PARAMETERS FOR A HALF-SPACE

Young's modulus ($\times 10^6$ Nm^{-2})	Possion's ratio	Density (kg/m^3)	Loss factor	P-wave speed (m/s)	S-wave speed (m/s)
1770	0.4	1700	0.15	1500	610

TABLE 7. PARAMETERS FOR A CIRCULAR TUNNEL LINING

Young's modulus ($\times 10^9$ Nm^{-2})	Possion's ratio	Density (kg/m^3)	Loss factor	Inner radius (m)	Outer radius (m)
37.6	0.15	2400	0.05	3.4	3.6

10.1 DISPLACEMENTS DUE TO A MOVING HAMONIC LOAD

Responses of the ground/tunnel structure due to a vertical harmonic load of 40 Hz moving on the tunnel invert are calculated using the programs FE_2_5D.EXE and FE_BE_2_5D.EXE. Since the excitation is well below the ground surface, the fineness of the boundary elements on the ground surface may be determined according to the Rayleigh wave length. The Rayleigh wavelength at 200 Hz is about 3 m for the soil. Boundary elements of about one-third of this wavelength are suitable for sufficient accuracy at the ground surface. However, the fineness of the elements on the tunnel lining and on the lining/soil interface must be determined by test. The test may be performed quickly just for a single wavenumber, *e.g.*, the zero wavenumber which corresponds to the plane strain problem. Figure 5 shows such a test. There are 50 three-noded boundary elements (see Figure 6) representing the ground surface to a distance of 50 m from the x-axis on the ground surface, and a different number of elements for half the tunnel ring. The test indicates that about 30 eight-noded finite elements are enough to capture the response of half the tunnel lining even at 200 Hz.

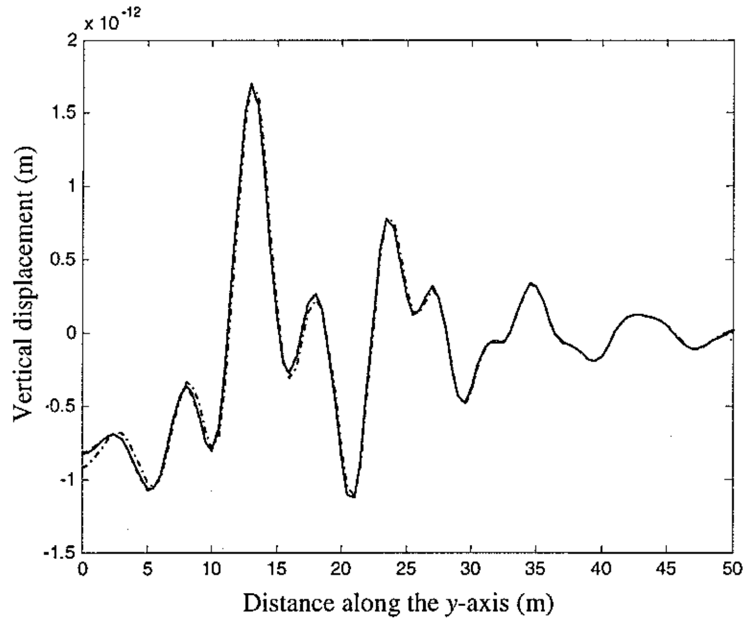


Figure 5. Zero-wavenumber vertical displacement along the y-axis on the ground surface due to a unit vertical harmonic load of 200 Hz applied at the tunnel invert. ——, 60 elements for half the tunnel; – – –, 30 elements for half the tunnel, – · –, 15 elements for half the tunnel.

The mesh of the ground/tunnel system used in the analysis is shown in Figure 6 in which there are 30 eight-noded finite elements distributed along half the tunnel lining, and the load (half) is applied at the 254^{th} node. For the reverse FFT, 512 samples of wavenumber β are used with a spacing of $0.005 \times 2\pi$ rad/m. The input data file is presented below:

```
*title
Calculate the response of a half-space including a lined tunnel
*material
 1, 1.77d9,    0.40, 1700.0,  0.150
 2, 3.76d10,  0.15, 2400.0,  0.050
*global node coordinates
    50.000000,       0.000000
    49.500000,       0.000000
    49.000000,       0.000000
       ...              ...
     1.000000,       0.000000
     0.500000,       0.000000
     0.000000,       0.000000
     0.000000,     -12.900000
     0.000000,     -13.000000
     0.000000,     -13.100000
     0.177942,     -13.104660
     0.188409,     -12.904934
     0.376302,     -12.919721
     0.365850,     -13.019173
     0.355397,     -13.118626
       ...              ...
*boundary element topology (domain 1)
   1,   2,   3
       ...
 102, 106, 107
       ...
*finite element topology
   5,   2, 102, 103, 104, 105, 109, 108, 107, 106
       ...
*edge element
 1, -1, 1, 2, 3, 4, 5
*frequency
40.0
*load speed
100.0
*structure symmetry
 1
*load symmetry
 1
*load position and amplitude on FE
 2, 254, 0.00, 0.00, 0.00, 0.00,-0.50, 0.00
*end
```

(Note: for the calculation of the power spectra presented in Section 10.2, the *frequency* block is

```
*frequency
8.
10.
12.5
16.
20.
25.
```

31.5
40.
50.
63.
80.
100.
125.
160.
200.
These are part of the one-third octave band centre frequencies).

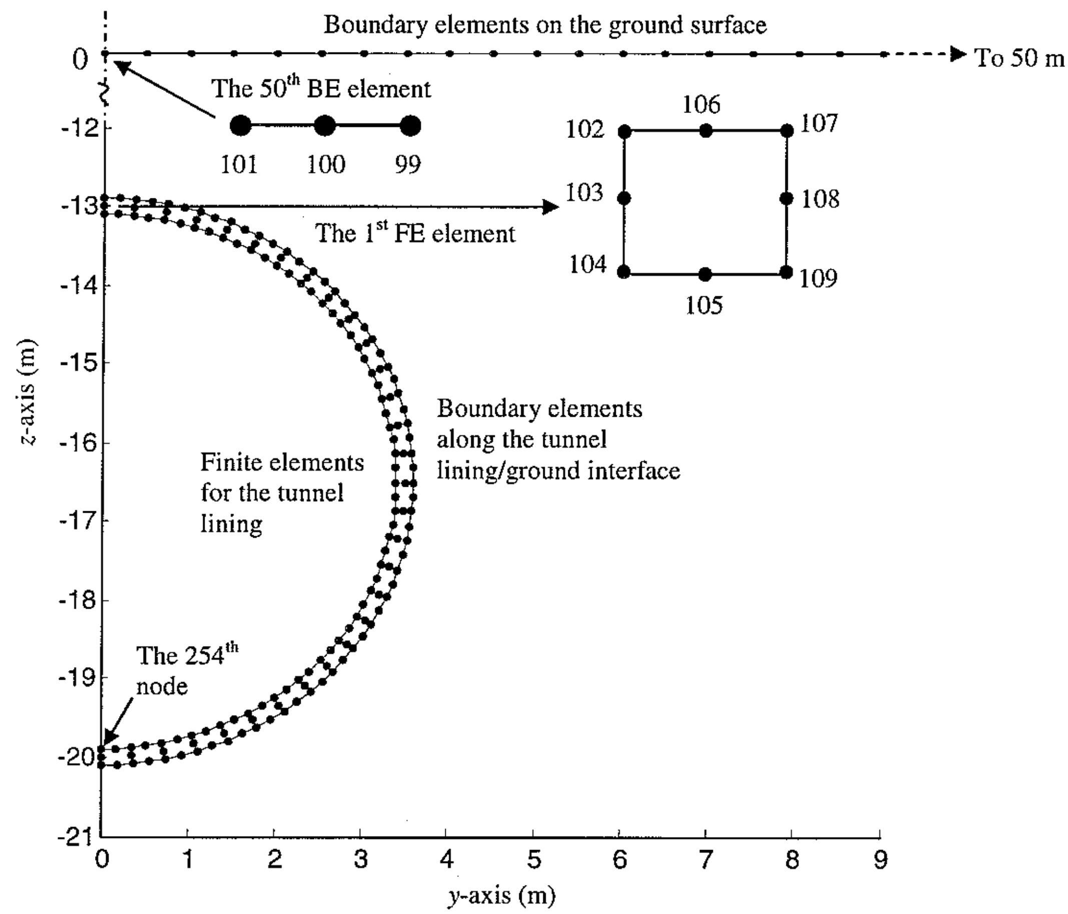


Figure 6. The mesh for a homogeneous half-space with a circular lined tunnel.

Figures 7 and 8 show the vertical displacements along the x-and y-axes on the ground surface. In these figures results are also shown of an earlier developed model [9], *the fictitious forces model*. That model is limited to the case of a circular tunnel ring and is based on the Green's functions for a complete layered half-space, a circular solid cylinder and a circular hollow cylinder. Though the Green's functions are calculated analytically, there are certain approximations which bring some errors in the prediction from the fictitious forces model. Especially the number of 'nodes' on the tunnel/soil interface has a great effect. Here in Figures 7 and 8, 64 'nodes' along the whole lining/soil interface have been used in the fictitious force model. The comparison is very good even at the edge of the FE/BE model.

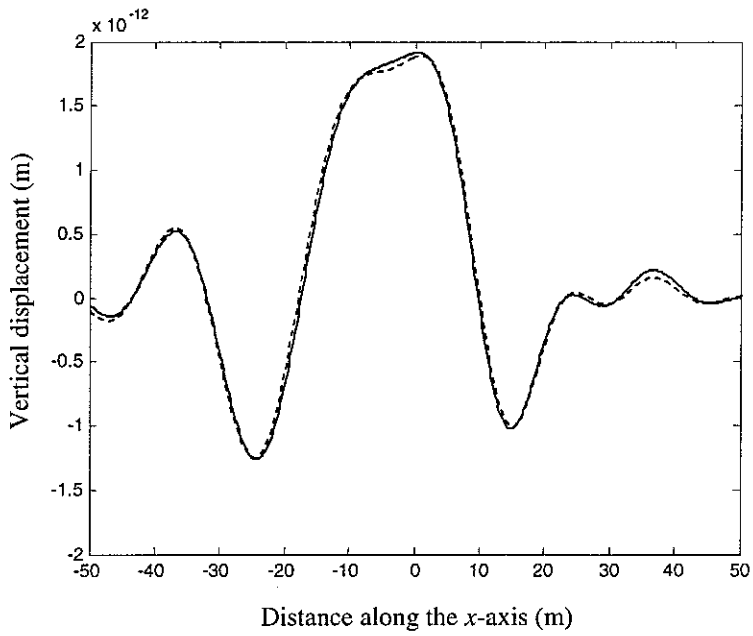


Figure 7. Vertical displacement along the x-axis on the ground surface due to a unit vertical harmonic load of 40 Hz moving at 100 m/s in the x-direction. The load is applied on the invert of the lined tunnel. ——, from the fictitious forces model; – – –, from the FE/BE model.

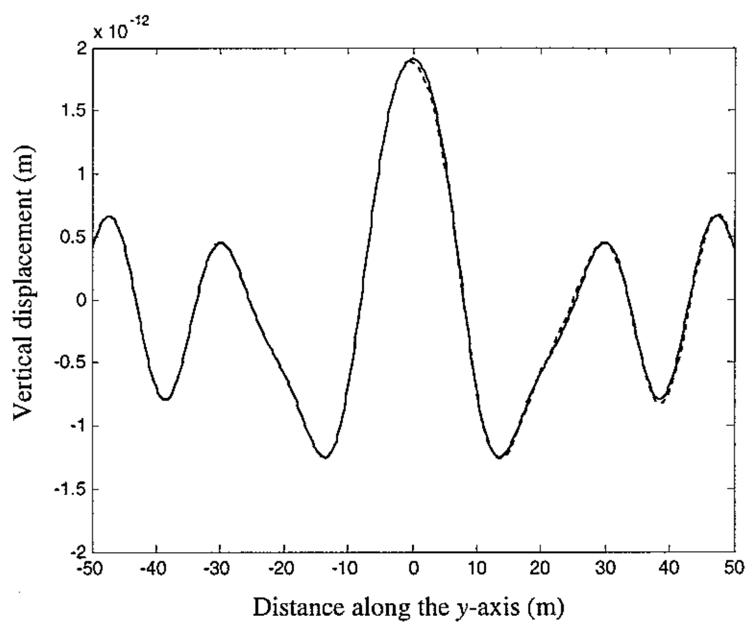


Figure 8. Vertical displacement along the y-axis on the ground surface due to a unit vertical harmonic load of 40 Hz moving at 100 m/s in the x-direction. The load is applied on the invert of the lined tunnel. ——, from the fictitious force model; – – –, from the FE/BE model.

10.2 VIBRATION POWER SPECTRA DUE TO A MOVING TRAIN

In this sub-section, vibration spectra are calculated for a train running along a track in the tunnel using the programs FE_2_5D.EXE, WHEEL_RAIL_FORCE.EXE and SPECTRUM.EXE, and equation (24). To do so, parameters for the train and track as well as the power spectral density of the track irregularities are required. Parameters for the track and vehicles, adopted from reference [10], are listed below.

TRAIN: Axle load = 1×10^5 N, Unsprung mass = 500 kg, distance between axles = 20 m, number of axles = 10. The train speed is 25 m/s.

TRACK: Concrete slab (EI = 1430 MNm^2, m = 3500 kgm^{-1}); two rails (EI = 10 MNm^2, m = 100 kgm^{-1}), railpads (k = 400 MNm^{-2}, loss factor = 0.3); springs between slab and tunnel invert (k = 1262 MNm^{-2}, loss factor = 0.5).

Figure 9 presents the track irregularity spectrum (versus wavelength) used in the calculations. This has been taken from measurements on a UK mainline.

The vehicle/track data file is shown below. The vertical displacement of the rail is chosen to be the locally first degree of freedom while that of the slab is the second. Notice that, since the whole system is symmetric about the xz-plane, the data in this file are taken for half the vehicle/track system. The slab is connected with the tunnel at nodes 249, 250 and 254. The total stiffness of the three connecting springs equals half the stiffness between the slab and the tunnel.

```
*vehicle parameter
50000.d0, 250.d0, 60.d0
50000.d0, 250.d0, 40.d0
50000.d0, 250.d0, 20.d0
50000.d0, 250.d0, 0.d0
50000.d0, 250.d0, -20.d0
50000.d0, 250.d0, -40.d0
50000.d0, 250.d0, -60.d0
50000.d0, 250.d0, -80.d0
50000.d0, 250.d0, -100.d0
50000.d0, 250.d0, -120.d0
*element in matrix [m]
50.d0, 0.d0
0.d0, 1750.d0
*element in matrix [k]0
2.0d8, -2.0d8
-2.0d8, 2.0d8
*element in matrix [c]0
0.6d8, -0.6d8
-0.6d8, 0.6d8
*element in matrix [k]2
5.0d6, 0.d0
0.d0, 715.d6
```

**element in matrix [c]2*
0.d0, 0.d0
0.d0, 0.d0
**track/tunnel position in connection, stiffness and damping*
2, 249, 2.10d8, 1.05d8
2, 250, 2.10d8, 1.05d8
2, 254, 2.10d8, 1.05d8
**load position and amplitude on track*
1, 0.5d0, 0.d0
**end*

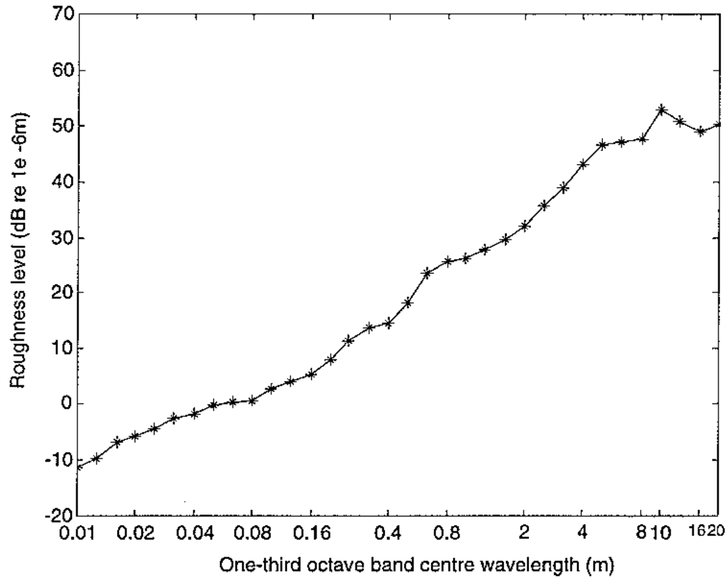


Figure 9. Vertical roughness levels in the wavelength range of 0.01 to 20 m of the rail top profile.

Figure 10 shows the wheel/rail dynamic force at the first axle calculated by the program WHEEL_RAIL_FORCE.EXE when the train runs at 25 m/s along the track on which a top profile of amplitude 0.1 mm at all wavelengths is imposed. It shows that the first peak occurs at about 150 Hz (wavelength being 0.17 m). This is the resonance frequency of a single unsprung mass and rails on the rail pad stiffness on the track.

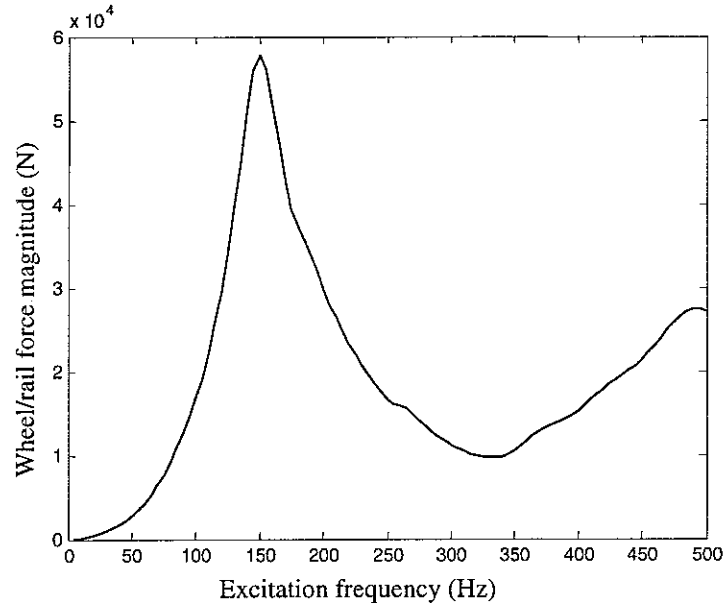


Figure 10. Wheel/rail dynamic force at the 1st axle for the train running at 25 m/s along the track. The amplitude of the rail irregularity is 0.1 mm.

Figure 11 shows the Fourier transformed vertical displacements ($\tilde{w}^0(\beta, y, z)$ in equation (20)) of five positions due to a unit stationary vertical load of 8 Hz applied on the rail. Those at 200 Hz are presented in Figure 12. It can be seen that beyond a particular wavenumber, the transformed displacements behave as a function decaying exponentially with increasing wavenumber. The displacement of the tunnel invert is not significantly different from that of the slab, indicating that a strong coupling exists between the track and the tunnel/ground.

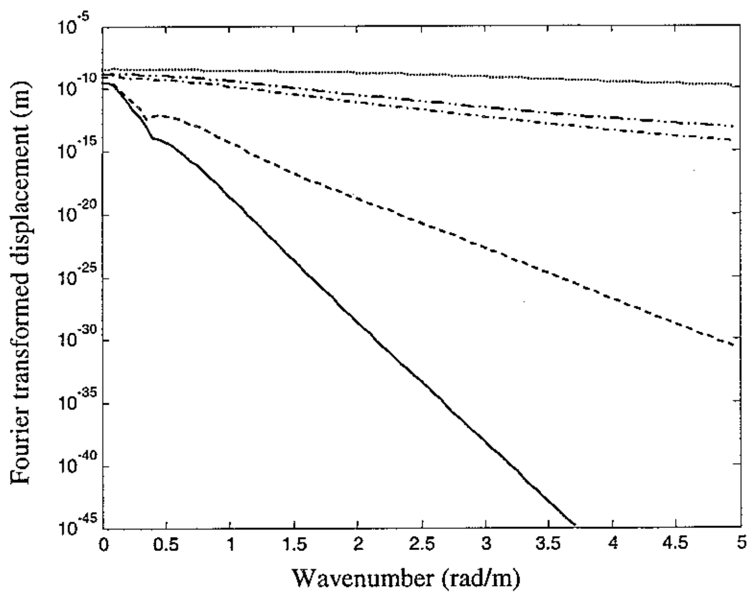


Figure 11. Magnitudes of the Fourier transformed vertical displacements at 8 Hz. —, $y = 0$ m on the ground surface; – – –, for the tunnel crown; – · –, for the tunnel invert (node 254); – ·· –, for the slab; ……, for the rails.

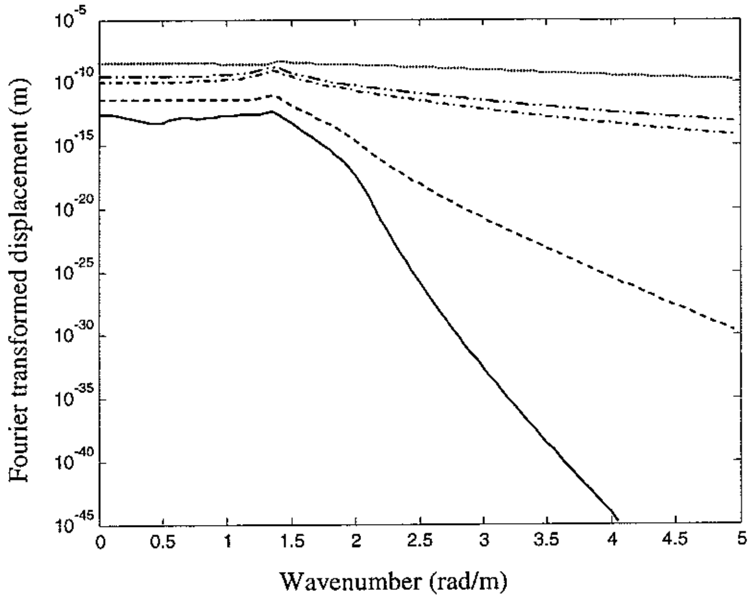


Figure 12. Magnitudes of the Fourier transformed vertical displacements at 200 Hz. —, $y = 0$ m on the ground surface; – – –, for the tunnel crown; – · –, for the tunnel invert (node 254); – ·· –, for the slab; ……, for the rails.

Having worked out the wheel/rail dynamic forces and the Fourier transformed displacements, the displacement spectra of the structure are then calculated using formulae ((16), (19) and (20). In the calculation, linear interpolation is performed for the evaluation of the wheel/rail dynamic forces and the Fourier transformed displacements from those previously calculated. Figures 13 and 14 show the vertical velocity levels at different positions on the track/tunnel/ground structure. On the rail, a strong harmonic feature is observed with obvious dips at 8, 16 and 32 Hz. This is caused by the passage of the axle loads which are identically separated by a distance of 20 m and which are the dominating sources for vibration of the rail at low frequencies. The passing frequency of the axles is 25/20 = 12.5 Hz at which the rail has peak responses. For frequencies higher than 40 Hz, the harmonic feature is overcome by the effect of the dynamic wheel/rail forces. As indicated in Figure 13, the track has a broad peak response at about 150 Hz where, as identified above, a single unsprung mass resonates on the track. It can be seen from Figure 14 that, for frequencies below 20 Hz, the vibration level of the tunnel crown is almost the same as that of the ground surface where $y = 0$ m. At 20 Hz, the shear wavelength in the soil is 31 m, about twice the depth of the tunnel axis (16.5 m). For higher frequencies, the vibration levels of the tunnel crown are much higher than those on the ground surface, and reach a maximum near the unsprung mass-on-track resonance frequency. Attenuation in the lateral direction on the ground surface is obvious at frequencies below 20 Hz. At higher frequencies however, due to the modes of the tunnel lining being excited, interferences effects are observed on the ground surface.

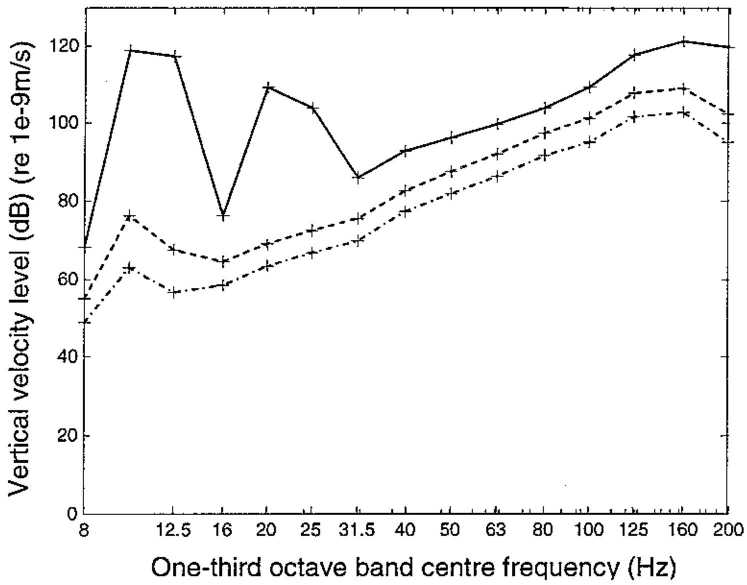


Figure 13. Vertical velocity levels for the track and tunnel. —, rail; – – –, slab; – · –, the 254th node at the tunnel invert.

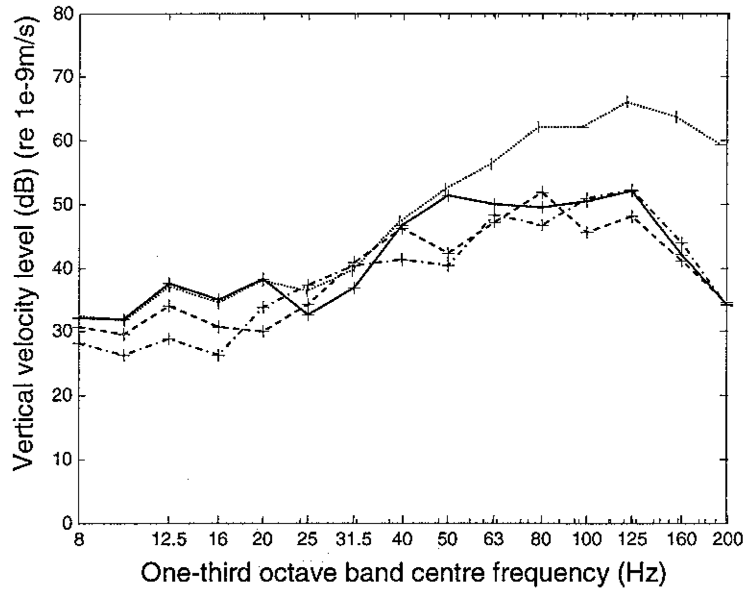


Figure 14. Vertical velocity levels at four positions in the ground. —, $y = 0$ m on the ground surface; – – –, $y = 10$ m on the ground surface; – · –, $y = 20$ m on the ground surface ; ......, the tunnel crown.

11. CONCLUSIONS

Based on the discrete wavenumber finite/boundary element methods developed by the authors, programs have been developed for prediction of ground vibration from tunnels and other engineering vibration problems. Structures under analysis are assumed to be infinitely long and homogeneous in one direction (called the longitudinal direction), however its cross-section normal to that direction can be arbitrarily shaped, with either closed or open boundaries. The FE program

FE_2_5D.EXE produces FE matrices which can be utilised not only for prediction of responses of a long built-structure to stationary or moving harmonic excitations, but also for the investigation of the propagation properties of the structure. The coupled FE/BE programs, FE_BE_2_5D.EXE and SPECTRUM.EXE, allow multiple BE domains, a single FE domain and a track model. The former is designed to calculate displacements due to point stationary harmonic loads or point harmonic loads moving uniformly in the longitudinal direction. The latter is to compute the Fourier transformed displacements at many wavenumbers due to harmonic loads of different frequencies. Using a simplified wheel/rail force model to produce the wheel/rail dynamic forces, vibration spectra of a ground/tunnel/track structure may be evaluated from the Fourier transformed displacements, the wheel/rail forces and the power spectral density of the rail irregularities. The use of the programs has been demonstrated by analysing a homogeneous half-space with a circular lined tunnel and a slab track on which a train is running.

ACKNOWLEDGEMENT

This work is supported by the EPSRC under research grant GR/R67309/01.

REFERENCES

[1] Sheng, X., *Ground vibrations generated from trains*, PhD Dissertation, University of Southampton, 2001.

[2] Petyt, M., *Introduction to Finite Element Vibration Analysis*, Cambridge University Press, 1990.

[3] Dominguez, J., *Boundary Elements in Dynamics*, Elsevier Applied Science, 1993.

[4] Jones, C.J.C., Thompson, D.J. and Petyt, M., A model for ground vibration from railway tunnels, *Proc. Instn. Civ. Engs, Transp.*, 2002, **153**, 121 – 129.

[5] Andersen, L. and Jones, C.J.C., Vibration from a railway tunnel predicted by coupled finite element and boundary element analysis in two and three dimensions, *Structural Dynamics-EURODYN'2002*, ed. Grundman and Schuëller, Balkema, Lisse, 2002, 1131–1136.

[6] Takemiya, H. and Yuasa, S., Lineside ground vibration induced by high-speed trains and mitigation measure WIB, *Structural Dynamics-EURODYN'99*, Balkema, Rotterdam, 1999, 821-826.

[7] Sheng, X., Jones, C. J. C. and Thompson, D. J., Modelling ground vibration from rail traffic using the discrete wavenumber finite and boundary element methods, *ISVR Technical Memorandum No. 899*, 2002.

[8] Lei, X., New methods in railroad track mechanics & technology. China Railway Press, Beijing, 2002.

[9] Sheng, X., Jones, C. J. C. and Thompson, D. J., Ground vibration generated by a harmonic load moving in a circular tunnel in a layered ground, *ISVR Technical Memorandum No. 890*, 2002.

[10] Forrest, J. A. and Hunt, E. M., Ground vibration due to trains in underground railway tunnels. *The Sixth International Congress on Sound and Vibration*, Copenhagen, Denmark, 1999, 1483-1940.