The importance of bubble ring-up and pulse length in estimating the bubble distribution from acoustic propagation measurements

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Abstract

Consideration is given to the effect of pulse length upon inverse methods of determining bubble population. Standard techniques of inversion are founded upon several basic assumptions. Consideration is given to these assumptions and the degree to which they may be compromised in oceanic measurements. This is demonstrated using experimental data taken during the recent Hurst Spit 2000 experiment. Finally a first illustration is made of how modelling of an oceanic bubble population may be used to add confidence to, and to infer extra information from, such bubble population measurements.

1. Introduction

A number of different techniques exist for bubble sizing including optical, electrochemical and acoustical. Acoustical techniques have been particularly successful owing to the fact that bubbles present a high impedance mismatch to acoustical energy. Methods of acoustic bubble sizing include the combination frequency technique [1, 2], the resonator system [3] and inversion of acoustic propagation [4-6]. This paper considers the latter technique, which can be used to determine bubble populations in general over a larger volume than the former, by making simple measurements of the characteristics of an acoustic disturbance propagating through a bubbly medium. Existing inversion techniques all rely on the same basic formulation and make similar assumptions, namely, that the bubble oscillates linearly in a free field and is driven by a plane wave. These assumptions allow linear bubble theory, as described by Commander and Prosperetti [7], to be applied to the problem.

The validity of such methods of determining bubble population must be carefully considered prior to any experimental measurements in order to ensure that any violations of the basic assumption stated above do not invalidate the results. This is of particular importance when considering measurements to be made in the surf zone as in the case of the recent experiment at Hurst Spit as discussed in Leighton *et al.* [8]. The length of pulse used in this environment is critical. Section 2 will investigate whether it is possible to select a pulse that ensures that linear, steady state bubble oscillation is achieved, but not so long that multi-paths and reverberation compromise the plane wave and free field assumptions.

Testing of the linear, steady state assumption is necessary because, owing to inertial effects, there is a finite the bubble 'ring-up' time (the interval from the start of motion of the bubble wall until it attains 1/*e* of the amplitude it would attain in steady state). Prior to reaching steady state oscillation the bubble response will be greatly reduced [9]. The nonlinear time-dependent response bubble of a bubble can be predicted from the formulations of [10] and Herring [11]:

$$\left(1-\frac{\dot{R}(t)}{c_0}\right)R(t)\ddot{R}(t)+\frac{3\dot{R}^2}{2}\left(1-\frac{\dot{R}(t)}{3c_0}\right)=\left(1+\frac{\dot{R}(t)}{c_0}\right)\frac{1}{\rho}\left\{p_B(t)-p_o-P\left(t-\frac{R(t)}{c_0}\right)\right\}+\frac{R(t)}{\rho c_0}\frac{dp_B(t)}{dt} \tag{1}$$

$$p_B(t)=\left(p_0+\frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3\kappa}-\frac{2\sigma}{R}-\frac{4\eta\dot{R}}{R} \tag{2}$$

Here R is the bubble wall radius, R_0 the equilibrium bubble radius, c_0 is the (constant) speed of sound in the liquid, ρ the density of the liquid, σ is the surface tension and η is the shear viscosity coefficient of the liquid. Also κ is the polytropic index of the gas within the bubble and p_0 is the hydrostatic pressure to which the bubble is subjected.

The existence of a ring-up time (which can readily be demonstrated by this model [9]) suggests that if the pulses are of short enough duration, the use of a steady state model could introduce additional errors into the inversion process, as it will not correctly predict the bubble response and thus its effect on the acoustic propagation. Section 3 will describe how this model may be used in a different fashion in order to infer extra information about an estimated bubble population.

2. Pulse length dependence of the inversion technique

As outlined in Section 1, in order to use standard techniques for inverting acoustic propagation characteristics to estimate the bubble population, a number of criteria must be met. They are that the bubble must oscillate linearly in a free field and be driven by a plane wave. This section will analyse some of the data obtained in the Hurst Spit experiment and then consider the validity of each one of the assumptions in that case.

2.1 Experimental Data

As part of the recent Hurst Spit trial, an experiment was designed whereby bubble size distributions would be estimated, via inversion technique, using pulses consisting of different numbers of cycles. For each frequency two pulses were emitted, firstly a 5 cycle pulse then a 20 cycle pulse. Each pulse was separated by 10 ms, hence the bubble population can be considered constant between pulses and the entire pulse train lasted just under 180 ms. The frequencies ranged between 200 kHz and 360 kHz in order to investigate the smaller range of bubble radii specified as objective (i) of Section 3 of Leighton *et al.* [8]. The experimental set-up is shown in Figure 1. Here the source and receiver were both positioned at Tx. Single frequency pulses were emitted and the backscattered reflection from a 50" diameter buoy measured. The source was positioned 2.35 metres from the buoy, 0.5 metres from the sea floor and at a depth of approximately 1 m. Associated details are given by Leighton *et al.* [8].

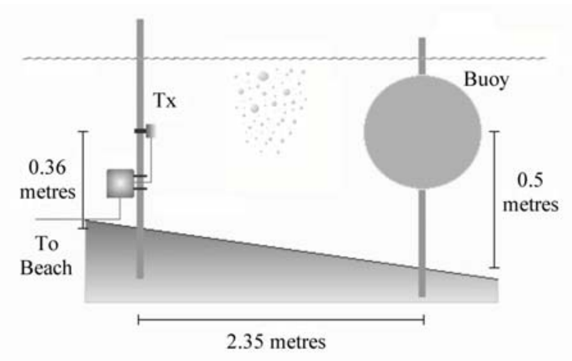


Figure 1. Experimental set-up, source and receiver positioned at Tx.

The attenuation caused by the presence of bubbles was measured by comparing experimental measurements to those made *in situ* under conditions of sustained calm and similar water depth where all other sensors [8] detected no measurable bubbles. Because this technique measures excess attenuation between bubble and bubble-free conditions, it is self-calibrated for any spherical spreading, absorption losses and the scattering function of the buoy itself.

Figure 2 shows the measured attenuation for both 5 and 20 cycle pulses and Figure 3 shows the resultant bubble populations calculated from these measurements.

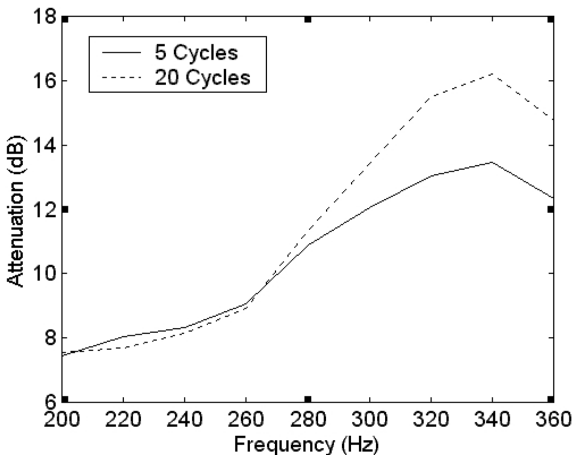


Figure 2. Measured attenuation for 5 and 20 cycle pulses. The pulse separation was 10 ms and the entire sweep of frequencies lasted 180 ms

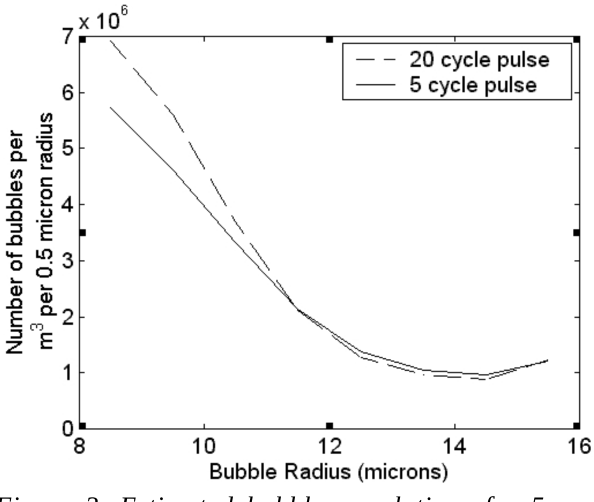


Figure 3. Estimated bubble populations for 5 and 20 cycles pulse. The measurement was made in a wind speed of 14-15 mph off shore breeze, with a water temperature of 11 °C and at a depth of ~1 m. The pH of the water was 6.12 and the dissolved oxygen content was 5.3 mg/litre. Estimation of bubble population was made by inversion of attenuation measurements following the method described in [4-6].

As can be seen from Figure 2 and Figure 3 the length of the insonification pulse affects the measured attenuation and, hence, the estimated bubble population. The difference in measured attenuation varies as a function of frequency and becomes small, <0.5 dB below 280 kHz. Unfortunately, owing to the statistical non-stationary characteristics of the system (i.e. in the surf zone bubble population is highly dependent upon wave breaking events), it has not been possible to quantify error bars for the data presented in Figures 2 and 3. However a similar trend was observed across a wide range of datasets. The assumptions inherent in the inversions used here will now be discussed.

2.2 Validation of Assumptions

2.2.1 Assumption 1: Plane Wave Propagation

In order for the plane wave condition to be met, the pulse must not suffer interference from either by multi-path reflections or in the limit, reverberation. Consideration is simplified if the bubble is in the far field of the source. The transition between near and far field for a plane transducer occurs at L^2/λ as discussed in [12], where L is the effective faceplate radius of the transducer and λ the wavelength of the radiated sound field. The source used in the Hurst Spit experiment had an effective faceplate radius of 50 mm. At 200 kHz this indicated that the transition to far field conditions occurred at 0.33 metres. Therefore the bubbles in over 93% of the direct beam path length are in the source's far field. However the presence of far field conditions does not guarantee planarity of the field which drive the bubbles into oscillation. The beam pattern of the source at 200 kHz is given in Figure 4. If a bubble is driven by multi-path reflections, these will reduce the validity of the plane wave assumption. Take, for example, the interference caused by reflection from the free surface. A first order estimate of the upper permissible bound of pulse length is determined by the difference in arrival times for both the direct path and the shortest non-direct path. Using the experimental set-up shown in Figure 1 the plane wave assumption is invalid for a pulse length greater than 110μs. Since the longest pulse duration used was 100 μs, the first non-direct reflection will arrive at the receiver after the duration of the direct pulse. Reflections from other entities (e.g. neighbouring bubbles) may reduce this upper limit.

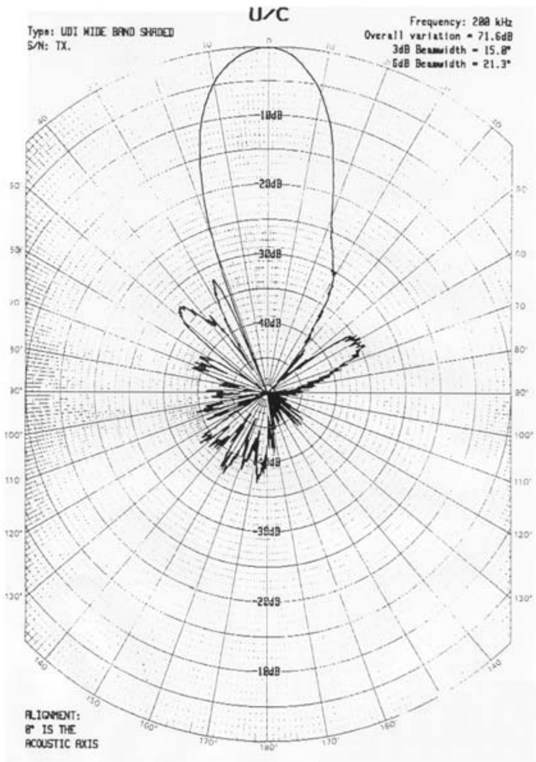


Figure 4. Beam Pattern at 200 kHz for source used in Hurst Spit sea trial: 3 dB beamwidth=15°; 6 dB beamwidth=21.3°.

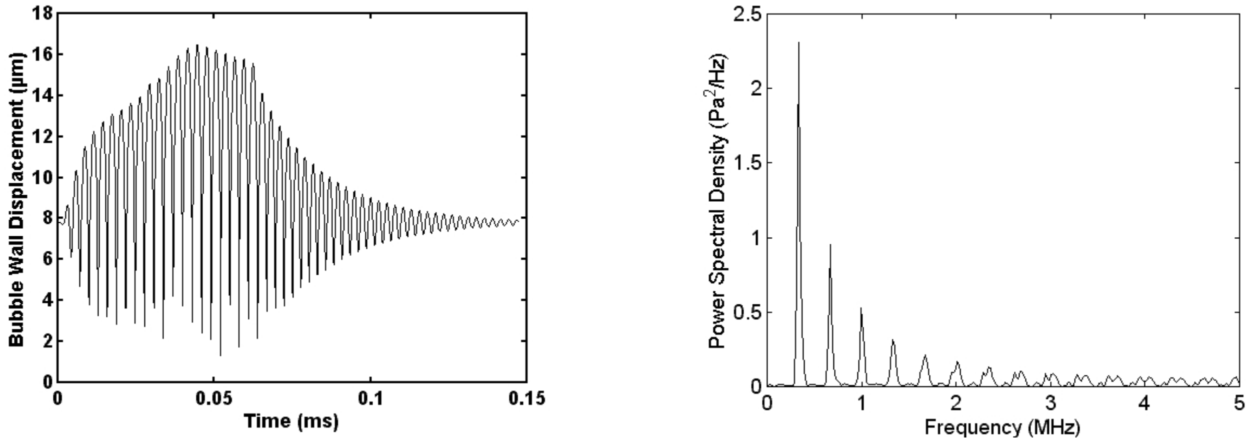
2.2.2 Assumption 2: Free Field Conditions

Whilst the assumption of planarity relates to the field which drives the bubbles into pulsation, that that of free field conditions relates to the sound field emitted by those pulsations. This has two effects. First, if the bubble is not in free field, application of the free field assumptions which are ubiquitous in current oceanic bubble acoustics, can lead to errors: both the natural frequency and damping of bubbles in reverberant environments differ from the free-field values [13]. A bubble at a depth of 1 m will, for example, receive reverberation of its own emissions from the atmosphere/ocean surface just over 1 ms after it beings to emit. The 100 μs limit described above precludes this particular source of reverberation affecting the bubble's resonance characteristics [13], but it should be recalled that there are closer sources of reverberation (such as neighbouring bubbles). Second, since the emission from each bubble is finite in time (it rings up, may attain steady state, then rings down) then the reverberation from a bubble at a given range will affect the time-history of the pressure field at any point in the liquid. Two specific points of interest are at the receiver transducer (since from this the population is estimated, using a formulation which neglects reverberation), and at any given bubble (since the reverberant component will contribute, if only in a small way, to the driving field on the bubble).

2.2.3 Assumption 3: Linear Bubble Oscillation

As stated in Section 1 at the heart of the procedure for estimating bubble populations though inversion of propagation characteristics (attenuation and phase speed) is the assumption of linear, steady state bubble oscillations in the free field. However since these assumptions may be violated when short pulses of high amplitude are used, as may be the case in the surf zone, it is necessary to develop a model based on the non-linear time-dependent cross-section as described by Leighton *et al.* [8].

In the Hurst Spit experiment, attenuation along the ~4.7 m two-way direct propagation path was as much as 40 dB. Clearly propagation through the dense bubble populations which can be encountered in the surf zone may require high source amplitude (in this case a maximum of 28 kPa at the faceplate). Hence the degree to which bubble non-linearity can occur must be considered. For example, the solution of the Herring-Keller equation for an example bubble in the surf zone is illustrated in Figure 5. The rise time of the bubble can be clearly seen in Figure 5(a) and for this case it can be seen that steady state oscillations are not achieved within 20 cycles (see Figure 3 of Leighton *et al.* [8] for a discussion of some implications of this). Whilst any pulse contains more than a single frequency, Figure 5(b) clearly shows that a considerable amount of energy is invested in the harmonics. This, and the strong asymmetry in the expansion/collapse of Figure 5a, indicate that this bubble cannot be considered to be undergoing linear oscillations in these insonation conditions.



(a) (b)

Figure 5. (a) Solution of equation (1) for bubble wall displacement of a 7.8 µm bubble insonified by a 20 cycle pulse at 339 kHz at 28 kPa acoustic pressure amplitude; (b) Power Spectral Density of the radiated pressure from the bubble at a distance of 0.01 m.

Having established that linear, steady state oscillations are not occurring in the bubble cloud when insonified by short pulses for at least some of the propagation path, it is desirable to be able to quantify the effect this has. Figure 6 shows the results predicted by the non-linear, time-dependent and range-dependent forward model [8] for the bubble population estimated by the measurements made during the Hurst Spit experiment. It shows the predicted levels of attenuation, for insonification of the bubble population by pulses of duration 5, 20, 50 and 100 cycles. It demonstrates a pulse length dependence in the attenuation of the cloud. The small change in attenuation between the 50 and 100 cycle pulses suggest that the cloud has reached steady state for the 50 cycle pulse. The 20 cycles pulse shows a small drop in attenuation of the order 1 dB above approximately 300 kHz while the 5 cycles pulse shows stronger degradation in attenuation across the whole frequency range. As shown in Section 2.2.1, the planarity condition prohibits the use of pulses of duration longer than 100 µs i.e. 20 cycles at 200 kHz.

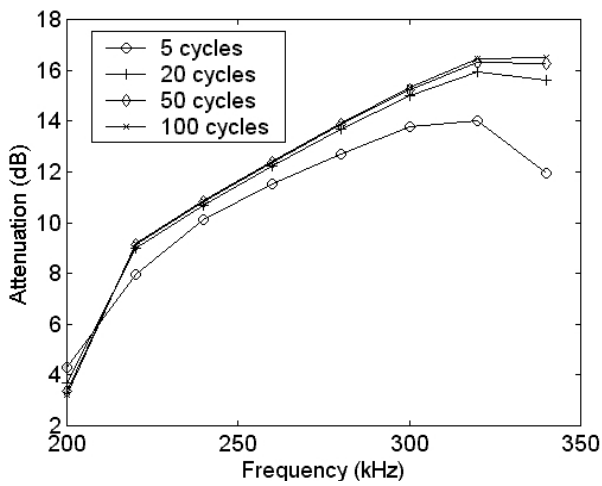


Figure 6. Theoretical calculations of attenuation for 5, 20, 50 and 100 cycle pulses based upon a bubble population estimated during the Hurst Spit experiment.

3. Extrapolation of bubble population

The data processed to date from the Hurst Spit experiment has concentrated on answering objective 1 from Section 3 of Leighton *et al.* [8] Hence the priority has been to invert the attenuation measurements to give the bubble population for an equilibrium bubble radius of between 9 μm and 15 μm. The only other measurements to such small bubble sizes given in Figure 6 were taken, not in the surf zone, but in oceanographically deep water (as explained in [1]). These were by Farmer and Vagle 1989 [14], and show a peak at R_0 = 20 microns. That such a peak exists, and is expected from oceanographic considerations, was confirmed Phelps and Leighton [1]. The evidence from the Hurst Spit 2000 data (Figure 3) suggests that such a peak does not seem to be present in that surf zone trial. This suggests that this population of small bubbles is newly entrained, and dissolution effects have not has sufficient time to reduce their number. This is reasonable given the conditions and video data.

Having satisfied objective 1 from [8], a more speculative test is undertaken. As can be seen from Figure 7, the Hurst Spit bubble population has a gradient similar to that of 'deep' water datasets and hence a hypothetical extrapolation of the bubble population may be made where the gradient of the surf zone population follows that of the deep water measurements ('Extrapolation 1'). The equation used for this first extrapolation is:

$$n_b=3*10^{11}\,R_0^{-5} \tag{3}$$

where n_b is the number of bubbles per cubic metre per micrometer increment in radius.

However, as can also be seen from Figure 7, the Hurst Spit data points have an absolute level similar to that obtained for larger bubbles in the surf zone, as obtained by Leighton and the generation of student previous to those involved in the Hurst Spit 2000 trial [2, 15]. (A direct comparison for surf zone bubbles as small those of the Hurst Spit data is not possible, as no previous data exists – see objective (i) of Leighton *et al.* [8]). Hence it is not unreasonable to suggest and test Extrapolation 2, which intersect the Hurst Spit data and the previous surf zone data. Is it possible to use the time-dependent model to say whether Extrapolation 2, or Extrapolation 1 (which by contrast intersect the Hurst Spit data and the previous surf zone data), is a more likely fit to the surf zone population which provide the attenuation data at Hurst Spit? The two extrapolations are shown in Figure 7, the population of Extrapolation 2 being given by:

$$n_b= 6*10^6\,e^{-0.02\,R_0} \tag{4}$$

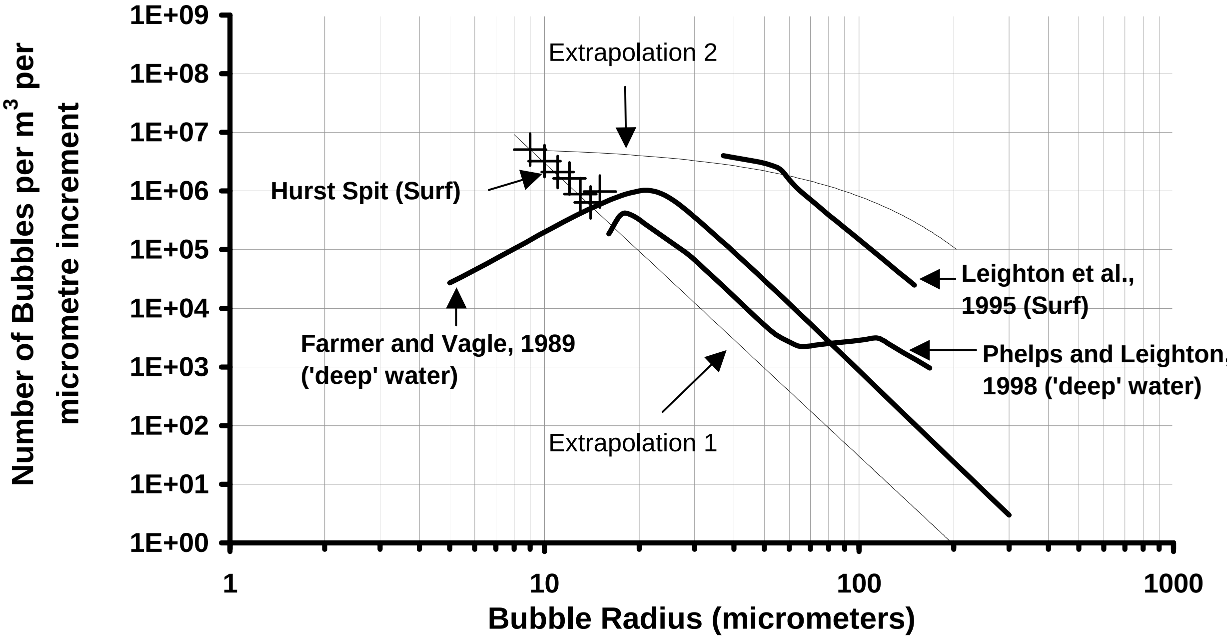


Figure 7. A comparison of several experimental measurements of bubble populations. Phelps and Leighton 1998 [1] and Farmer and Vagle 1989 [14] are two measurements of 'deep water' oceanic bubble population. A measurement of surf-zone bubble populations was taken by Leighton and the previous generation of students [2, 15] using the combination frequency technique. The population measurement marked Hurst Spit is the population measured during the sea trial by inversion. The crosses represent measured data points and the two thin lines are the two proposed extrapolations.

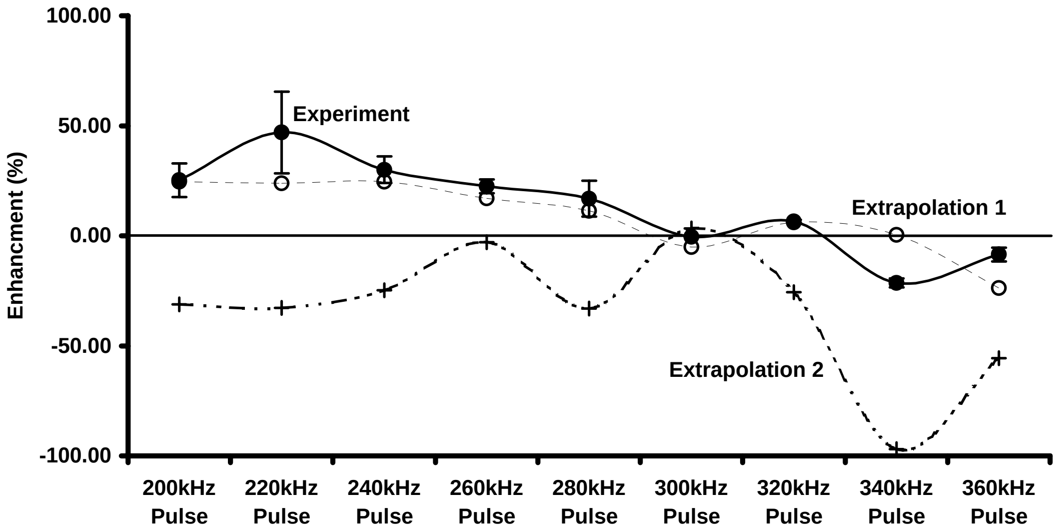


Figure 8. Experimental (solid line) and theoretical measurement (dashed lines) of the relative attenuation between short and long pulses. The error bars represent one standard deviation of the experimental results. Two sets of results for theoretical populations are shown. The first is Extrapolation 1 (Equation 3) of the Hurst Spit surf-zone population and the second is Extrapolation 2 (Equation 4).

This question can be answered by calculating from the model of [8] the differences in the levels of attenuation for 5 and 20 cycle pulses. The relative attenuations Extrapolations 1 and 2 are shown in Figure 8. It is apparent that the theoretical results for Extrapolation 1 show much better agreement with the experimental measurements than do those of Extrapolation 2.

It is important to note that the numbers of bubbles present were linearly scaled for the purposes of modelling the cloud so as to keep processing time within reasonable limits. However, because of the steep gradient of the extrapolated population the large bubbles, above ~50 μm in radius, are removed from the calculation. Thus the technique described above confirms the population distribution used up to a radius of 50 μm. Beyond this radius the model results confirm that the assumption, implicit in the scaling, that the contribution of bubbles greater than 50 μm is insignificant. This does not confirm the shape of the extrapolated population other than that the numbers of large bubbles must be small.

4. Conclusions

This paper has highlighted several considerations associated with estimation of bubble populations via inversion of acoustic propagation in the surf zone. The necessity of using short pulses in this environment to avoid interference with the acoustic propagation by multi-path reflections or reverberation casts a shadow of doubt over many of the basic assumptions made by the model of linear bubble oscillation used in standard inversion techniques. Comparison was made between modelled attenuation for a number of pulses containing different numbers of cycles. The discussion highlighted the problem that it was necessary to use more cycles to achieve linear oscillation than was possible without experiencing multi-path reflections. This problem will be exacerbated at lower frequencies when the pulse length necessary to contain a certain number of cycles (hence achieving steady-state oscillations) will be considerably longer. In the case where this circumstance makes use of the model by Commander and Prosperetti questionable, it may be possible to use a non-linear time-dependent equation of motion to describe the bubble response, and to invert propagation characteristic for bubble populations.

Objective (i) from Section 3 of Leighton *et al.* [17] has been accomplished, in that bubble population measurements were made in a critical size region where data for the surf zone was not previously available. The differences between this preliminary data and 'deep water' data in the same size range can be attributed to specific oceanographic features. In Section 3 it was shown how models of an extrapolated bubble population can be used to estimate the general trend of a bubble population, if measurements are only available over a limited range of bubble radii.

Acknowledgements

The authors would like to acknowledge the technical assistance of Rob Stainsbridge, Keith Sims, Dave Edwards Mark Bampton and particularly John Taylor and Tony Edgeley during the Hurst Spit trial. We are also very grateful to the New Forest District Council, particularly Matt Hosey, for allowing us access to the beach and EPSRC grant reference GR/M38094 for providing the funding to make this work possible.

References

[1] Phelps AD and Leighton TG. Oceanic Bubble Population Measurements using a buoy-deployed combination frequency technique. *IEEE J. Oceanic Eng*, 1998; **23**: 400-410.

[2] Phelps AD, Ramble DG, Leighton TG. The use of a combination frequency technique to measure the surf zone bubble population. *J. Acoust. Soc. Am.*, 1997; **101**: 1981-1989.

[3] Farmer DM and Vagle S. Bubble Measurements using a resonator system, in *Natural physical processes associated with sea surface sound.* T.G. Leighton (ed.) University of Southampton, UK, 155-162 (1997).

[4] Commander KW and McDonald RJ. Finite-element solution of the inverse problem in bubble swarm acoustics. *J. Acoust. Soc. Am.*, 1991; **89**: 592-597.

[5] Duraswami R, Prabhukumar S and Chahine GL. Bubble counting using an inverse acoustic scattering method. *J. Acoust. Soc. Am.*, 1998; **104**: 2699-2717.

[6] Terrill EJ, Lada G and Melville WK. Surf-Zone Bubble Populations, in *'Acoustical Oceanography', Proceedings of the Institute of Acoustics Vol. 23 Part 2, 2001*, T G Leighton, G J Heald , H Griffiths and G Griffiths, (eds.), Institute of Acoustics, (this volume), pp. 212-219.

[7] Commander KW and Prosperetti A. Linear pressure waves in bubbly liquids: Comparison between theory and experiments. *J. Acoust. Soc. Am.*, 1989; **85**: 732-746.

[8] Leighton TG, Meers SD, Simpson MD, Clarke JWL, Yim GT, Birkin PR, Watson Y, White PR, Heald GJ, Dumbrell HA, Culver RL and Richards SD. The Hurst Spit experiment: The characterization of bubbles in the surf zone using multiple acoustic techniques, in *'Acoustical Oceanography', Proceedings of the Institute of Acoustics Vol. 23 Part 2, 2001*, T G Leighton, G J Heald , H Griffiths and G Griffiths, (eds.), Institute of Acoustics, (this volume), pp. 227-234.

[9] Clarke JWL and Leighton TG. A method for estimating time-dependent acoustic cross-sections of bubbles and bubble clouds prior to the steady state. *J. Acoust. Soc. Am.*,2000, **107**: 1922-1929.

[10] Keller JB, Miksis M. Bubble oscillations of large amplitude, *J. Acoust. Soc. Am.*,1980, **68**: 628-633.

[11] Herring C, Theory of the pulsation of the gas bubble produced by an underwater explosion, OSRD, Rep. No. 236 (1941).

[12] Leighton TG. The Acoustic Bubble, Academic Press, London, 1994, p. 30.

[13] Leighton TG, Ramble DG, Phelps AD, Morfey CL and Harris PP. Acoustic detection of gas bubbles in a pipe. *Acta Acustica* 1998; **84**: 801-814.

[14] Farmer DM and Vagle S. Waveguide propagation of ambient sound in the ocean-surface bubble layer. *J. Acoust. Soc. Am*. 1989; 86: 1897-1908.

[15] Leighton TG, Phelps AD and Ramble DG. Acoustic bubble sizing: from laboratory to the surf zone trials, *Acoustic Bulletin,* **21**, 1996, 5-12.