ACOUSTIC BUBBLE SIZING THROUGH NONLINEAR COMBINATIONS INVOLVING PARAMETRIC EXCITATIONS

A.J.HARDWICK, T.G.LEIGHTON*, A.J.WALTON and J.E.FIELD
Cavendish Laboratory,
University of Cambridge,
Madingley Road,
Cambridge,
CB3 0HE,
United Kingdom.
(*Inst.Sound and Vibration Res., Univ.Southampton, Hampshire, SO9 5NH, UK.)

SUMMARY

Acoustic methods of bubble sizing are useful in media where light is strongly attenuated, or where the bubble is deformed significantly from the spherical. The population is insonated by a fixed imaging frequency (ν_i) of megahertz order and a pumping beam, the frequency (ν_p) of which is swept. When the latter coincides with the breathing-mode resonance of a bubble, parametrically-excited subharmonics of the pump combine with the imaging beam. The technique described here is particularly suited to obtaining a measure of bubble volume for a small number of bubbles of arbitrary size. It has typically been possible to distinguish radii to ±1% by the detection of signals at $\nu_i \pm {}^1\!/_2\, \nu_p$.

1. INTRODUCTION

Bubble sizing can provide information on oceanic phenomena, for example about the transfer of soluble gases from the atmosphere entrapped in near-surface bubbles which may be carried down to depths of tens of metres[1]. Other off-shore applications can be found in the petroleum industry. Oil is extracted from deep underground where the hydrostatic pressure is high. When brought to nearer atmospheric pressure, gaseous components exsolve from the liquid phase. The ability to characterise this multiphase flow in pipelines without interruption would be of great value. Related to this is the study[2] of decompression sickness caused when bubbles similarly form in the human body. Another medical use is the monitoring of pressure changes in inaccessible places[3] – the size of a gas bubble is dependent on applied pressure.

Bubble sizing is therefore an important field of study; how is it to be done? The large differences in acoustic impedance an compressibility between gases and liquids makes acoustic methods viable. They are also preferable over the most obvious other non-interruptive set of methods, the optical[4] techniques, since the latter fail if the liquid, container or unavoidable surroundings are opaque. Acoustic bubble sizing can be broadly subdivided into the measurement of bulk acoustic properties of the mixture which are can be related to the bubble distribution[5] though the computational procedure may not be simple, stable and accurate[6], and techniques whereby acoustic features can be correlated with interactions with individual bubbles.

In addition to the basic scattering methods, advantage may be taken of the resonant frequency (ν_o) of the spherically symmetric mode of a bubble to determine its size. This is[7] $\nu_o = R_o^{-1} \sqrt{3\, \kappa\, p_o\, \rho^{-1}}$ for linear oscillations if R_o is the equilibrium radius, κ is the polytropic index (between unity and the ratio of specific heats), ρ is the liquid density, p_o is the ambient pressure and the surface tension is negligible. Thence $\nu_o R_o \approx 3$ Hz m for air bubbles in shallow water. Either sound at ν_o from bubbles excited during entrapment can be passively picked up[8], or suitable frequencies may be actively transduced into the medium. Sizing through the strong linear resonance scattering[3] is not ideal since the geometric scattering from non-resonant but larger bubbles and from other objects may give false signals. For this reason, the method reported here exploits non-linear behaviour.

Since the radius cannot contract beyond zero, the oscillation becomes less symmetrical between expansion and contraction as the amplitude rises and so linearity is lost. Quadratic and higher terms in the power-series expansion of the bubble's time-response function are manifested as second and higher harmonics appear in the bubble's reradiation. Advantage may be taken of the existence of harmonics to indicate that the incident insonation is at resonance so implying the size[9]. Because other aspects of the of the system – especially the transducers – can produce harmonics, alternative non-linear effects have been tried. Simultaneously insonating with sound at two frequencies (ν_p, 'pumping'; ν_i, 'imaging') produces[10] sidebands at the sum and difference frequencies ($\nu_i \pm \nu_p$) when non-linearity is induced by pumping near resonance ($\nu_p \approx \nu_o$). The pumping frequency is variable; the imaging frequency is higher and fixed. This can be considered as the scattering of the imaging ultrasound varying with the bubble area varying. The resultant amplitude modulation of a ν_i frequency sinusoid at the frequency ν_p is equivalent to the superposition of two sinusoids at $\nu_i - \nu_p$ and $\nu_i + \nu_p$. However, the spectrum around the imaging frequency contains additional reradiation[11,12] at $\nu_i \pm {}^1\!/_2 \nu_p$. This response is more highly dependent on the pumping frequency than are the basic sum and difference frequencies. The frequency of ${}^1\!/_2 \nu_p$ is the halfeth subharmonic of ν_p and the results suggest a period doubling in the response of the bubble may be responsible.

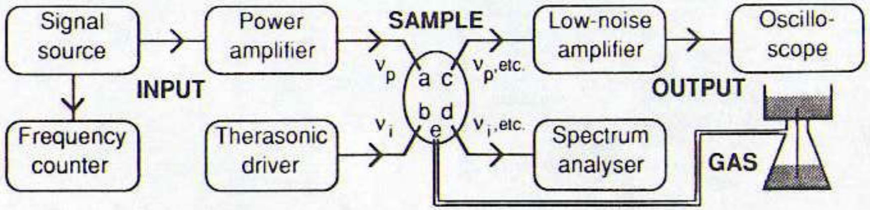


Fig.1; Schematic of the experimental setup: (a) speaker, (b) Therasonic transducer, (c) miniature hydrophone, (d) needle hydrophone & (e) drawn pipette nozzle.

This text reports a verification on the earlier[12] findings and covers some important additional aspects that were not investigated at that time with a view to developing the method into a useful system. In particular: the nature of the subharmonic emission from the bubble in addition to the subharmonic sum and difference emissions; and the dependence of the emission on the amplitude of the pumping sound.

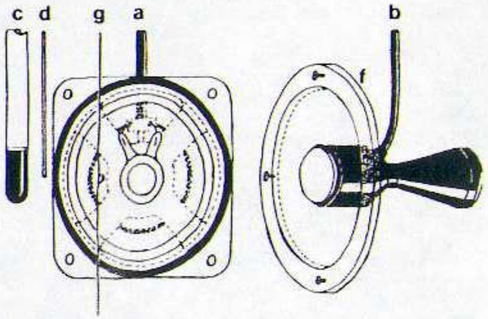


Fig.2; Detail of the tank contents: (a)–(d) as above, (f) acoustic window & (g) wire holding the bubble.

2. EXPERIMENTAL DETAILS

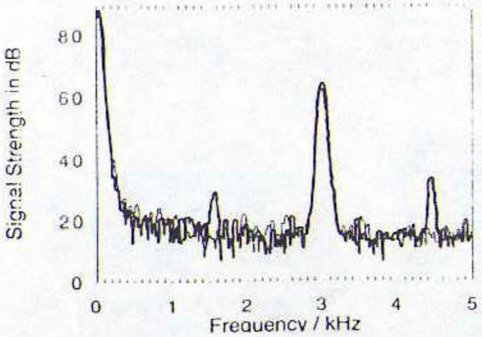


Fig.3; The spectrum in the kHz region showing subharmonic. The thinner line is modification in the absence of the bubble. The 0 dB level is 0.74 mPa.

The liquid and gaseous phases were represented in this study by tap-water and air respectively. The liquid was contained in an acoustic absorber lined 25 by 25 by 44 cm^3 poly(methylmethacrylate) tank in which the hydrophones, bubble or bubbles under study and pumping frequency source were immersed. The imaging insonation at $\nu_i = 3.35$ MHz was provided by an Electo-Medical Supplies Therasonic 1030 medical ultrasound unit operating at 15 W with its transducer outside the tank and the ultrasound coupled through an acoustic window. The pumping signal source ideally has a flat frequency response in the kilohertz region, is durable and is moderately powerful; it is more difficult to provide. A

85 mm Ø Mylar® RS-248-325 cone speaker bonded into a watertight container and driven by a Brookdeal 471 Signal Source via a Quad 405-2 Power Amplifier served as a prototype for 2 kHz $\lesssim \nu_p \lesssim$ 5 kHz. Reflections of this sound did not cause significant amplitude variations over the volume of interest. The pumping voltage was monitored in frequency and the resulting pressure amplitude measured by a Brüel & Kjær 8103 Miniature Hydrophone linked to an oscilloscope. The spectrum around ν_i was interrogated with a Dapco NP10-3 Needle Hydrophone and a Marconi Instruments TF 2370 Spectrum Analyser. This apparatus is schematically shown in Fig.1. Fig.2 details the acoustic interaction area.

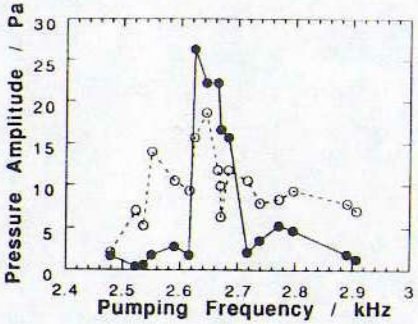
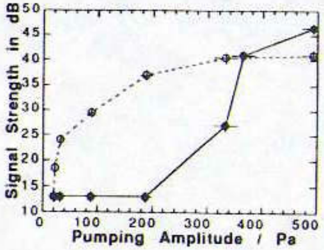


Fig.4; Amplitudes of the $\nu_i \pm \nu_p$ (dashed line) & $\nu_i \pm {}^1/_2\nu_p$ (continuous) signals verses ν_p.

Bubbles were either produced as a stream by forcing air through a 0.30 mm internal diameter drawn glass pipette by ~15 kPa overpressure or created individually with a 0.04 mm internal diameter pipette and tethered by low wettability to a vertical wire coated in petroleum jelly (drawn in Fig.1 and Fig.2 respectively). Tethered bubbles which survived experimentation were pipetted off for measurement by optical microscopy.

Fig.6 Signals verses pumping amplitude (curve assignations as in Fig.4, 0 dB is 50 mPa).



3. RESULTS

Fig.3 shows a typical variation of the amplitudes of the signals at $\nu_i \pm \nu_p$ and $\nu_i \pm {}^1/_2\nu_p$, verses ν_p, as measured by the hydrophone approximately 1 cm from a bubble (R_0 = 1.04 ± 0.1 mm) on a 0.5 mm Ø wire. Random amplitude errors are ± 15 %. The geometric average of the pair of sidebands was taken in each case. The pumping amplitude was about 43 Pa and the bubble was tethered.

The stream of free bubbles also gave $\nu_i \pm {}^1/_2\nu_p$ signals for suitably tuned ν_p albeit of the intermittent nature as was to be expected when sampling about 30 nominally dentical bubbles per second.

By connecting the miniature hydrophone to the Spectrum Analyser, low frequency phenomena were recorded. Fig.4 shows a ${}^1/_2\nu_p$ signal which was existent only when a bubble was present. An ultraharmonic at ${}^3/_2\nu_p$ is also visible.

Fig.5 demonstrates the pumping amplitude dependence with first the $\nu_i \pm \nu_p$ then the $\nu_i \pm {}^1/_2\nu_p$ signal appearing as that amplitude increases. This is graphed for a tethered bubble, resonant at ν_p = 4.05 kHz, in Fig.6. The durability of the source limited the pumping amplitude obtainable.

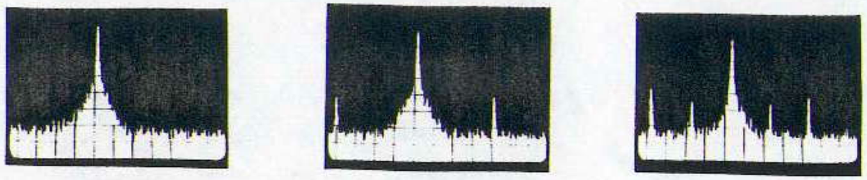


Fig.5; spectra showing ν_p, $\nu_i \pm \nu_p$ & $\nu_i \pm {}^1/_2\nu_p$ signals. Major divisions are 1 kHz apart horizontally & 1 B apart vertically. The 0 B level is 50 mPa. Insonation at 4.050 kHz and (left to right) 20 ± 1 Pa, 19 ± 1 daPa & 32 ± 1 daPa amplitude.

4. DISCUSSION

The peak of the $\nu_i \pm {}^1/_2\nu_p$ effect in Fig.3 is clearly sharper than that of $\nu_i \pm \nu_p$ so enabling more precise sizing to be done. Its 3 dB width (full-width half-maximum) is ~0.06 kHz so the resonance (and therefore radius if the other relevant system parameters are known) is can be found to within ±1%. A considerable advantage to this method is that the likelihood of spurious $\nu_i \pm {}^1/_2\nu_p$ signals is lower than for ν_p harmonics (which may be produced by the transducers) and $\nu_i \pm \nu_p$ signals (which turbulence[13] can produce).

The bubble stream results confirm that $\nu_i \pm {}^1/_2\nu_p$ emission is not an artifact of tethering the bubble therefore increasing the technique's range of application. The low frequency results confirm that $\nu_i \pm {}^1/_2\nu_p$ emission, rather than something peculiar to bifrequency insonation, involves a period doubling in the bubble's action.

The dependence shown in Fig.6 is as expected for this system with linear behaviour at small amplitudes with higher-order effects consistent with the use of progressively stronger driving. The existence of definite thresholds is uncertain because of the 13 ± 1 dB noise background.

5. CONCLUSIONS

The $\nu_i \pm {}^1/_2\nu_p$ effect is caused by a period doubling in the breathing mode bubble oscillation at sufficiently high pumping amplitudes and it can discriminate bubble radii differences to ±1%.

6. REFERENCES

(1) S.A.THORPE, 1984, The effect of Langmuir circulation on the distribution of submerged bubbles caused by breaking wind waves; JFM 142, 151-70.

(2) R.S.MACKAY, G.RUBISSOV, 1978, Decompression studies using ultrasonic imaging of bubbles; IEEE Trans.Biomed.Eng. BME-25(6), 537-44.

(3) W.M.FAIRBANK, M.O.SCULLY, 1977, A new non-invasive technique for cardiac pressure measurement: resonant scattering of ultrasound from bubble; IEEE Trans on Biomed. Eng., BME-24(2), 107-10.

(4) P.A.HWANG, Y-H.L.HSU, J.WU, 1990, Air bubble prod. by breaking wind waves: a laboratory study; J. of Phys. Oceanography 20, 19-28.

(5) N.BREITZ, H.MERMIN, 1989, Instrumentation for *in situ* acoustical measurement of bubble spectra under breaking waves; JASA 86(2), 739-43.

(6) K.COMMANDER, E.MORITZ, 1989, Off-resonance contribution to acoustical bubble spectra; JASA 85(6), 2665-9.

(7) M.MINNAERT, 1933, On the musical air-bubbles and the sounds of running water; Phil.Mag. 16, 235-48.

(8) T.G.LEIGHTON, A.J.WALTON, 1987, An experimental study of the sound emitted from gas bubbles in a liquid; Eur.J.Phys. 8, 98-104.

(8) D.L.MILLER, 1981, Ultrasonic detection of resonant cavitation bubbles in a flow tube by their 2nd harmonic emissions; Ultrasonics 19, 217-224.

(9) V.L.NEWHOUSE, P.M.SHANKER, 1984, Bubble size measurement using the nonlinear mixing of two frequencies; JASA 75(5), 1473-7.

(11) T.G.LEIGHTON, R.J.LINGARD, A.J.WALTON, J.E.FIELD, 1990, Bubble sizing by the nonlinear scattering of two acoustic frequencies; Dordreckt (The Netherlands), Kluwer Academic Pub., Natural Physical Sources of Underwater Sound (Ed. B.Kerman).

(12) T.G.LEIGHTON, R.J.LINGARD, A.J.WALTON, J.E.FIELD, 1991, Acoustic bubble sizing by combination of subharmonic emissions with the imaging frequency; Ultrasonics 29, 31-24.

(13) M.S.KORMAN, R.T.BAYER, 1988, Nonlinear scattering of crossed ultrasonic beams in the presence of turbulence in water: I.Experiment; JASA 84(1), 339-49.

ACKNOWLEDGEMENT

This work was funded by the Petroleum Science and Technology Institute, Edinburgh, UK.