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FLUID DYNAMICS AND ACOUSTICS GROUP

A Review of Scattering Models for Ultrasonic Propagation in Trabecular Bone

by

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Abstract

Trabecular bone, found at the core of many bones in the human skeleton, is composed of a porous, marrow-filled lattice of calcified rods. This structure erodes with the onset of the debilitating bone disease osteoporosis. Osteoporosis can be treated effectively to reduce the risk of fracture, if diagnosed sufficiently early. In recent years, a wide range of ultrasonic bone assessment systems have become commercially available, collectively known as Quantitative Ultrasonography (QUS). QUS assessment measures Broadband Ultrasonic Attenuation and the speed of sound in bone. In the absence of a satisfactory model of the interaction of ultrasound and bone, it is unclear how this complementary information may be used to improve diagnosis. In particular, the main loss factors in bone at frequencies between 200 kHz and 1 MHz are poorly understood. This report is concerned with reviewing recent advances in understanding loss factors at these frequencies, and in particular, several theoretical models of scattering in bone. Models from five authors are considered, each with slightly differing assumptions in terms of ka region in which it is valid (for wavenumber, k, and scatterer size, a), scatterer distribution and the modelling of two compressional waves in trabecular bone.

1 Osteoporosis

Osteoporosis is a skeletal disease that contributes to causing over 60,000 hip fractures in the UK each year (National Osteoporosis Society 1998). It is characterised by a reduction in bone mass and the erosion of the microstructure of bone tissue. The tissue at the core of many bones in the human skeleton is composed of a spongy marrow-filled lattice of calcified rods (figure 1). These rods, known as 'trabeculae', become thinner and their spacing increases with the onset of osteoporosis.

If osteoporosis is diagnosed sufficiently early, it can be treated effectively to reduce the risk of fracture. The reduction in bone mass may be assessed by Dual Energy X-Ray Absorptiometry (DXA), but this provides no information on tissue distribution. The microstructure of trabecular bone lends itself to assessment with ultrasound.

A wide range of ultrasonic bone assessment systems are commercially available, collectively known as Quantitative Ultrasonography (QUS). The popularity of such systems has grown markedly in recent years, as they are less expensive and more portable than DXA. However, QUS methods are empirical, with questionable accuracy, and, in the absence of a satisfactory model of the interaction of ultrasound and bone, it is unclear how this complementary information may be used to improve diagnosis. In particular, the main loss factors in bone at frequencies between 200 kHz and 1 MHz are poorly understood. This report is concerned with reviewing recent advances in the understanding of loss factors at these frequencies, and in particular, several theoretical models of scattering in bone.

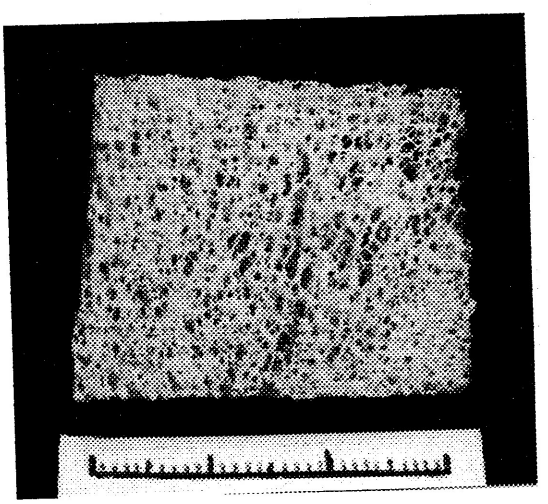

***Figure 1**: Photograph of trabecular bone from the bovine femur. Scale with thick lines at 10mm also shown.*

2 Broadband Ultrasonic Attenuation

QUS assessment is based on the measurement of attenuation and speed of sound of ultrasound transmitted through sites that contain trabecular bone, such as the heel bone. A transmission method is used owing to the high attenuation coefficient of bone. Langton *et al.* (1984) introduced the investigation of Broadband Ultrasonic Attenuation (BUA), which measures the frequency-dependence of the attenuation, $\alpha(f)$, found from,

$$\alpha(f) = -10\log\frac{H_{water}(f)^2}{H_{bone}(f)^2}, \tag{1}$$

where $H_{water}(f)$ and $H_{bone}(f)$ are the amplitude spectra of the signal through water (without bone present) and bone, respectively for frequency, f. The assumption is made that measured $\alpha(f)$ in dB versus frequency, is linear between 200 - 600 kHz, that is,

$$\alpha(f) = (BUA)f + C, \tag{2}$$

for constant, C. The term BUA is the gradient in dB/MHz, evaluated by linear regression and was found to be significantly lower in older women compared with young normals (figure 2). Although this relationship does not have a physical basis, it has been accepted as the principal index in ultrasonic bone assessment.

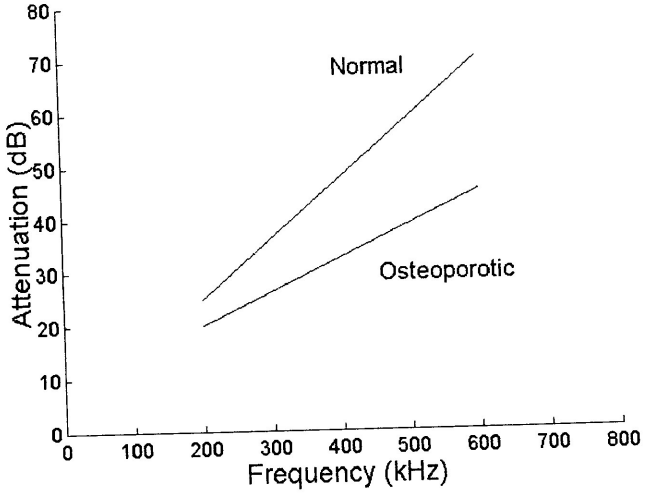

Figure 2: *Linear relationship between attenuation and frequency assumed for BUA.*

3 Modelling Loss Factors in Trabecular Bone

The propagation of direct ultrasonic waves through trabecular tissue has been considered by several authors using Biot's theory (1956a,b) for porous media (McKelvie and Palmer 1991; Williams 1992; Lauriks *et al.* 1994; Hosokawa and Otani 1997). However, success has been limited to the prediction of the phase velocities of two compressional waves ('fast' and 'slow' waves), since measured attenuation was a few orders of magnitude higher that predicted viscous absorption (Hosokawa and Otani 1997, Hughes *et al.* 1999). It is generally believed the discrepancy is due to the omission from Biot's theory of losses resulting from reflection, diffraction, phase cancellation and scattering.

Of these, particular importance is given to scattering (the re-radiation of wave energy from irregularities in a medium). Trabeculae have a cross section of about 0.1 mm and spacing of about 0.5 mm in healthy bone (Mellish *et al.* 1989), which are close to the wavelengths at QUS frequencies. Tavakoli and Evans (1992) showed that losses in bone were more dependent on microstructure than on density, thus highlighting the importance of scattering. Fry and Barger published their seminal work on scattering in bone in 1978, and since then, the problem has been tackled by several authors using differing approaches (Strelizki *et al.* 1998; Wear 1999; Kitamura *et al.* 1996; Kaczamrek *et al.* 1998). Some models have enjoyed more success than others, and they are considered below.

4 Scattering Models for Trabecular Bone

4a Fry and Barger (1978)

Fry and Barger (1978) presented a comprehensive study of ultrasonic attenuation in the skull. The skull is composed of a hard cortical shell, with a trabecular core. The authors studied insertion loss over 0.3 – 1.9 MHz, and a schematic of their findings is shown in figure 3. Scattering losses dominate at higher frequencies, with reflection and absorption in the core governing losses between 0.3 - 0.5 MHz and 0.5 - 0.9 MHz, respectively.

Above 0.9 MHz, scattering becomes the dominant loss factor. Fry and Barger used a model based on Rayleigh scattering, for a random distribution of scatterers with diameters smaller that incident wavelengths (i.e., the product $ka << 1$, for wavenumber k, and scatterer size, a). Scattering arises from differences between the mechanical properties of the

calcified matrix and interstitial marrow. The difference in bulk moduli is greater than in densities, and consequently, scattering resulting from the latter is neglected. The intensity absorption coefficient is proportional to the squared relative difference in bulk moduli in the two media, ($\Delta B/B$), for wavelength, λ, and average inclusion volume, $\overline{V}$, as,

$$\alpha = 4\pi^3 \overline{V} \beta_{ray} \lambda^{-4} (\Delta B / B)^2, \tag{3}$$

for a Rayleigh scatterer distribution, $\beta_{ray} = {}^4/\pi$. Hence, scattered loss increased with the fourth power of frequency. The upper frequency limit for Rayleigh scattering is $f_o = c/3a$, which was evaluated for bone as 1.3 MHz, for trabecular thickness, a = 0.6 mm, and sound speed, c = 2500 m/s. Above 1.3 MHz, stocastic scattering losses develop, increasing with the second power of frequency. Hence, in decibels per unit length, insertion loss is,

$$\mathrm{IL}/l \begin{matrix} = 11.5 f^4 dB/cm, f < f_o \\ = 11.5 f_o^2 f^2 dB/cm, f > f_o \end{matrix} \tag{4}$$

Fry and Barger found agreement between their model and data from skull bone to within 2 dB. This study is probably the most comprehensive performed on trabecular bone. Other researchers have focused on specific effects, such as the reported non-linear relationship between attenuation and porosity (Williams *et al.* 1996; Hodgskinson *et al.* 1996). Strelitzki achieved some success in modelling this response.

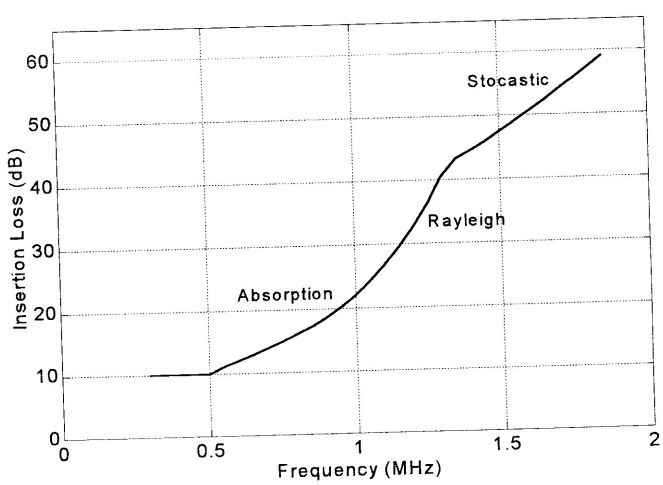

***Figure 3**: Schematic graph of frequency-dependent insertion loss from the skull bone.*

4b Strelitzki (1998)

Strelitzki's model (1998) is based on quantitative laws proposed by Sehgal (1993). The medium is regarded as a continuum whose material properties vary about a mean value. Fluctuations in velocity, μ^2, may be used to calculate scattered pressure and the resulting attenuation (density fluctuations are neglected, as in Fry and Barger's model). A correlation function is used that represents the organisation of the tissue. This is generally represented as an exponential or Gaussian function but choosing an appropriate correlation function for biological tissue is critical, as it will vary from one organ, and state, to another.

The attenuation due to scattering cross-section, α_{sc}, in dB/cm, for $ka >> 1$, is,

$$\alpha_{sc} = 8.686 \frac{4\mu^2 (2\pi f / c)^4 a^3}{\left[1 + (2\pi f / c)^2 a^2\right]\left[1 + 9(2\pi f / c)^2 a^2\right]}, \tag{5}$$

for sound speed, c; correlation length (equivalent to the mean scatterer diameter), a; and where the mean velocity fluctuations for a mixture of two components are,

$$\mu^2 = \mu_s^{\,2}(1-\beta) + \mu_e^{\,2}\beta, \tag{6}$$

for porosity, β, and mean velocity fluctuations of suspended marrow and embedding bone, μ_s^2 and μ_e^2, respectively.

Attenuation, α_{sc} from equation (5), when plotted again porosity, β, strongly resembled the non-linear relationship observed by previous researchers. Nicholson *et al.* (2000) later showed the same for BUA and backscatter versus porosity. Such agreement supports the idea of scattering as a major contribution to losses in bone.

4c Wear (1999)

Probably the most successful bone scattering model to date, has been Wear's investigation of the specific case of backscatter (Wear, 1999). Wear applied Faran's theory (1951) of scattering from a single cylinder to bone, by considering the trabecular rods as scatterers with diameters small relative to the wavelength (i.e. $ka << 1$).

The acoustic intensity of a wave scattered from an inelastic cylinder, I_s, is,

$$I_s \cong \frac{\pi k^3 r^4}{8d} I(1-2\cos\phi), \tag{7}$$

where I is the intensity of the incident plane wave; r is the cylinder radius, and d is the distance from the cylinder. The term ϕ is the angle between incident and scattered directions. This scattering from a single cylinder may be used to approximate the frequency dependence of the superposition of scattered waves from many cylinders, if: first, that cylinders are randomly distributed such that the phases of the scattered waves are uniformly distributed over 0 to 2π; and, second, that multiple scattering effects are neglected.

Backscatter from equation (7) increases with the cube of frequency. Wear reported such a response in backscatter from both trabecular samples and wire test objects. These findings were supported by Chaffai *et al.* (2000), who also provided evidence that Faran's second theory for spherical scatterers gave reasonable agreement with data.

Several authors are beginning to treat backscatter from bone as a valuable clinical parameter in its own right (Njeh *et al.*, 1999). Wear (2000) recently noted that backscatter might contain additional information to that provided by BUA or Speed of Sound measurements. Furthermore, a clinical parameter called Broadband Ultrasonic Backscatter (BUB) has been proposed to provide greater information on bone structure (Roux 1999). However, although Wear's approach to backscatter has certainly enjoyed some success, cubic dependency of backscatter does not provide an explanation for the approximately linear relationship assumed by BUA. Wear hypothesized that the presence of another frequency dependent loss, such as absorption, may contribute a greater component of attenuation than scattering, thus maintaining a linear dependency.

4d Kitamura (1996)

Like Wear, Kitamura *et al.* (1996) also utilise Faran's theory of scattering from a cylinder. However, in this case the trabecular cylinders are periodically distributed in space, forming a lattice that behaves as a diffraction grating. The properties of the scattered field may thus be estimated using diffraction theory.

The Born and Fraunhofer diffraction models describe properties of the scattered wave in the x-direction. At a certain far-field distance from the origin, the scattered wave, as a function of angle to the principal axes of propagation, θ.

$$S(f,\theta,\varphi,a,b) = A(f,\theta,a)\cdot B(f,\theta,\varphi,b), \tag{8}$$

for frequency, f. The function A is the scattering from a single cylinder of radius a, and the function B represents the effect due to the grating of cylinders with spacing, b.

Interference between diffracted waves gives rise to a spatial diffraction pattern. The periodicity of the fringes reflect the dimensions of the grating itself. The cylinder radius may be estimated from the backscatter envelope versus angle, whilst smaller fluctuations give information on cylinder spacing. Lattice dimensions may also be obtained from the spectrum of the scattering intensity.

Using data from steel lattices and bone samples, the authors achieved some success was obtained in estimating the envelope of fluctuations and the position of larger maxima. However, the detail of smaller oscillations could not be well resolved. Greater correspondence between data and calculated values was found in the variation in intensity with frequency. Kitamura later improved agreement by devising a method to determine the axis in which the sample was being tested (Kitamura *et al.* 1998), crucial for determining the midpoint of the pattern in the presence of an angular offset.

Basic *in vitro* studies by the present authors and co-workers may support the idea of trabecular bone as a diffraction grating (Edwards 1999; Hubbuck 2000). Whilst further evidence is clearly required for this research to continue, this novel treatment of scattering in bone does raise exciting possibilities. Measuring trabecular spacing and thickness from the diffraction pattern would be valuable in the monitoring of bone health. However, such patterns would need to be observable *in vivo*, given the presence of the cortical shell.

4e Kaczmarek (1998)

Following the inability of Biot's theory to model attenuation in bone, a recent encouraging development has been Kaczmarek's incorporation of scattering in the theory (Kaczmarek *et al.* 1998). Biot's theory is modified to describe scattering from fast and slow

compressional waves using complex elastic moduli. The elastic moduli P and R relate stress in the solid frame to its dilatation, and stress in the fluid to its dilatation, respectively, as,

$$P = \frac{\beta(K_s/K_f - 1)K_b + \beta^2 K_s + (1-2\beta)(K_s - K_b)}{1 - \beta - K_b/K_s + \beta K_s/K_f} + \frac{4N}{3},$$

$$R = \frac{K_s \beta^2}{1 - \beta - K_b/K_s + \beta K_s/K_f}, \tag{9}$$

where β is the porosity, and K_s, K_f, and K_b, are the bulk moduli of the solid material, the solid frame and the fluid, respectively (Biot 1956). In Kaczmarek's adaptation, equations (9) express the real part of the elastic moduli. The imaginary parts, are based on Rayleigh scattering in inhomogeneous media, giving a fourth power dependency on frequency,

$$P_i = 2\rho_s(1 - f_v)A_1\omega^3 v_d,$$
$$R_i = 2\rho_f f_v A_2 \omega^3 v_f, \tag{10}$$

where ρ_s and ρ_f are the densities of solid and fluid, respectively, and v_d and v_f are velocities of compressional waves in the frame and in the fluid, respectively. The terms A_1 and A_2 are found experimentally. Thus, absorption and scattering loss may be found the imaginary part of the complex wavenumber, $q_{fast,\ slow} = \omega / V_{fast,\ slow}$ where,

$$V_{fast,\ slow} = \sqrt{\frac{\Delta \pm \left[\Delta^2 - 4\left(PR - Q^2\right)\left(\rho_{11}\rho_{22} - \rho_{12}^2\right)\right]^{1/2}}{2\left(\rho_{11}\rho_{22} - \rho_{12}^2\right)}}, \tag{11}$$

for $\Delta = P\rho_{22} + R\rho_{11} - 2Q\rho_{12}$, with terms ρ_{ij} and Q being defined in Biot's 1956 papers.

Kaczmarek's predictions compared favourably with data from glass specimens at 1MHz. However, a fourth power of frequency dependence was not observed for trabecular samples at this frequency, and instead an f^2 variation was observed, in keeping with Fry and Barger's work (figure 3). However, the addition of scattering in Biot's theory appears to rectify the large discrepancy between measured and predicted attenuation from previous work. Kaczmarek's innovation, combining both propagation and scattering, represents a valuable advance.

5 Discussion

The diversity of models reviewed here lucidly demonstrates the complexity of scattering in trabecular bone and highlights issues that should be resolved in a unified model.

5a Consideration of *ka* Regions

First, all the above models are valid for the $ka << 1$ region, with the exception of Strelitzki's model, which covers the $ka >> 1$ regime. It is interesting that models with both conditions have enjoyed some success at the frequencies of interest. This may suggest, as proposed by Nicholson *et al.* (2000), that the BUA bandwidth contains the condition $ka = 1$.

Figure 4 shows the product, ka ($= \{2\pi f/c\}.a$) plotted versus sound speed, c (over the range for *in vivo* and *in vitro* fast and slow waves), for (a) 200 kHz and (b) 600 kHz, the limits of the clinical bandwidth. Strictly, scattering may occur from both the trabecular strands *and* the pores, since both constitute material discontinuities, so curves of typical pore and trabeculae sizes are shown. The condition $ka = 1$ is also indicated. At 0.2 MHz, ka is less than unity for all speeds and structural dimensions. However at 0.6 MHz, the value of ka is greater than unity for larger discontinuities of 1.0 mm (e.g. pore size in diseased bone (Mellish *et al.* 1989), and for pores of 0.5 mm ka passes through unity with the change in sound speed. These figures illustrate two situations where a transition through $ka = 1$ may occur: first, for large-pored diseased bone at some frequency between 0.2 – 0.6 MHz; or, second, for waves of high frequencies in the range with a certain speed (speed being dependent on other factors, such a propagation direction).

In the event of the condition $ka = 1$ lying within the clinical bandwidth, the situation may arise that the dominant attenuation mechanism at 0.2 MHz is a different loss factor from that at 0.6 MHz. This is supported by Fry and Barger's analysis (figure 3). These losses may exhibit differing frequency dependencies, which may converge at some 'transitional' frequency. This transition may depend on the status of the bone tissue, since, for example, the trabecular thickness, a, will affect the limit $ka = 1$, which will alter with structural erosion owing to osteoporosis (figure 4a, b). BUA analysis is concerned with finding the frequency dependence of insertion loss over the bandwidth 0.2 – 0.6 MHz, but this analysis may average the slopes of different losses into a single BUA gradient. Furthermore, since the proportion of the bandwidth with differing dependencies vary with

tissue condition, the average gradients will alter the progression of osteoporosis. This process not only masks the true response but could also obscure the interpretation of BUA.

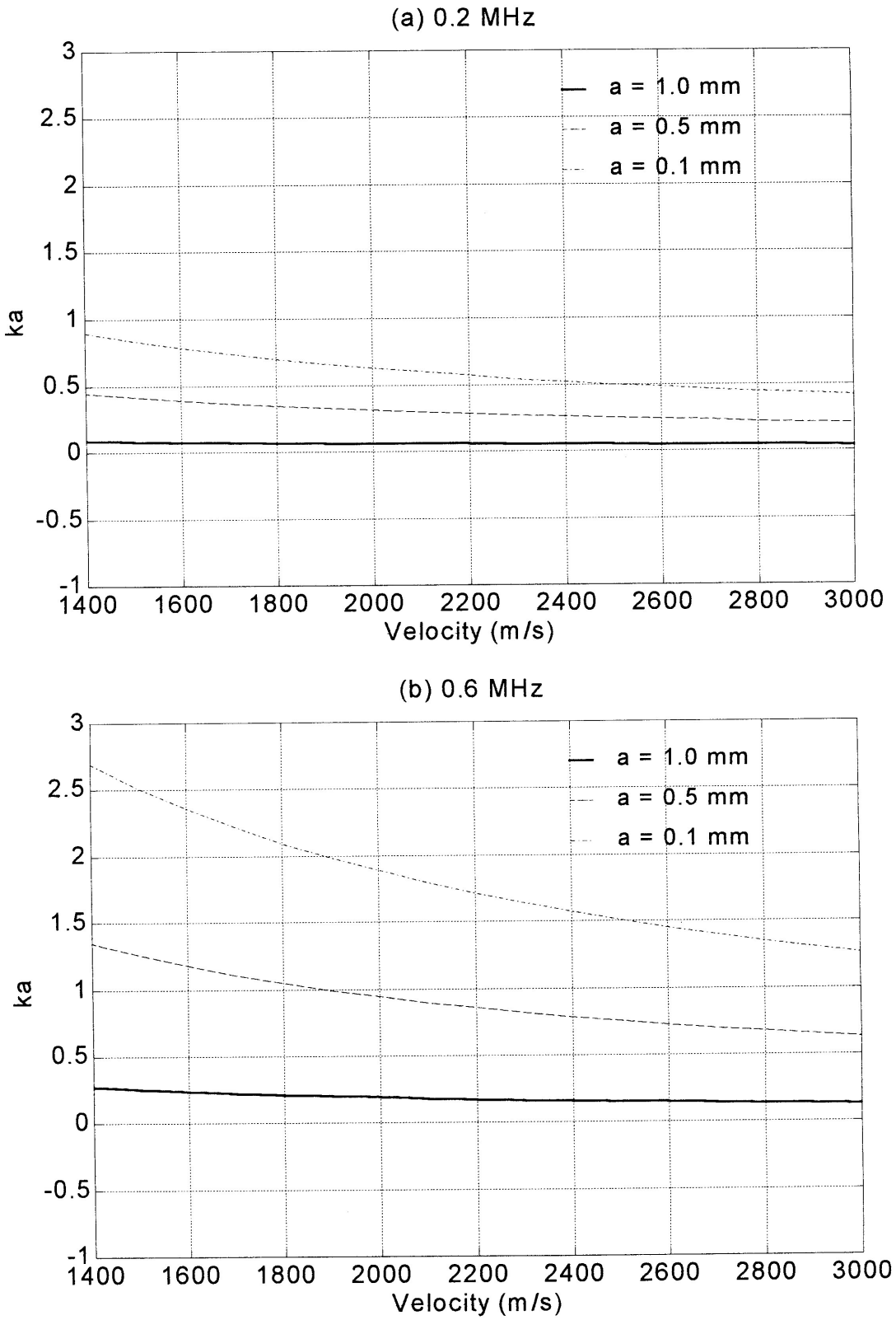

Figure 4: *The product ka versus sound speed in bone for scatterer sizes 0.1 mm, 0.5 mm and 1 mm at (a) 0.2 MHz and (b) 0.6 MHz.*

5b Spatial Distribution of Scatterers

Most scattering models considered here assume a random spatial distribution of scatterers. Interestingly, in the field of biomechanics, models of the strength of trabecular bone tend to use repetitive and regular structures (Gibson and Ashby, 1988). Furthermore, the present authors achieved some success in modelling ultrasonic propagation in trabecular bone using a periodic layered structure (Hughes *et al.*, 1999). In fact, the structure of trabecular bone is neither completely random nor regular. Rather, the architecture grows in a pattern that reflects the mechanical loading to which it is exposed (Currey, 1984) that may be considered 'regular' over roughly 8 mm, the width of typical ultrasonic beam. Whilst further work should establish the usefulness of Kitamura's diffraction grating approach to scattering, it nevertheless challenges basic assumption of the remaining models.

A novel solution to the problem of the scatterer organisation has recently been tackled by Chaffai *et al.* (2000). Samples of real bone were scanned using micro-CT, and the information was used to construct a three-dimensional architecture. An autocorrelation function was computed from this architecture, which was used in a model of a weakly scattering medium to predict backscatter. This innovation could prove valuable in researching the interaction between structure and scattering.

5c Two-wave Models

Finally, most of the reviewed models consider scattering from a single wave. Indeed, several appear to consider the specific interaction of a wave in the fluid (marrow, or water) with the solid trabecular scatterers. This approach not only neglects the hypothetical contribution of scattering from pores, but also ignores the propagation of both fast and slow compressional waves (and shear waves), an omission rectified by Kaczmarek's model. However, the omission of these details from theories such as Wear's, has not prevented them from being successful.

Although the field has made much progress recently in understanding loss factors in trabecular bone, it is important that the work continues in order to provide an interpretation of clinical ultrasonic bone assessment results.

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