

**Fatigue and Modal Analysis of Carbon Fibre Reinforced  
Plates with Embedded Optical Fibres**

**F. Garesci and N.S. Ferguson**

ISVR Technical Memorandum 830

December 1998



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UNIVERSITY OF SOUTHAMPTON  
INSTITUTE OF SOUND AND VIBRATION RESEARCH  
DYNAMICS GROUP

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Reinforced Plates with Embedded Optical Fibres**

by

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Authorized for issue by  
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Group Chairman

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# FATIGUE AND MODAL ANALYSIS OF CARBON FIBRE REINFORCED PLATES WITH EMBEDDED OPTICAL FIBRES

## Introduction

This work considers measuring the natural frequency and normal mode shape variation of carbon fibre reinforced plates (CFRP) with an embedded optical fibre, and correlating this with the damage caused by fatigue stresses. For traditional materials (homogenous and isotropic), such as steel and other metals, it is assumed that a stiffness decrease, due to fatigue damage, causes a decrease of the natural frequencies of vibration.

Different tests have been performed for determining experimental natural frequencies and mode shapes of undamaged and damaged composite specimens. The areas prone to damage are also identified. The samples (140mm×70mm×2mm) were investigated experimentally using modal analysis and fatigue tests in bending were undertaken. The aim of the tests was to correlate modal investigation with fatigue in order to detect and to predict damage in the composite structure.

Three specimens with  $((\pm 45^\circ/90^\circ/0^\circ)_2)_s$  lay-up, two with an optical fibre (OF) embedded between the two centre  $0^\circ$  orientation plies, and one with an optical fibre (OF) embedded between the last and penultimate layers is  $\pm 45^\circ$  orientation plies were used.

The first configuration has been shown to have high fatigue resistance [1], therefore it has been assumed to be the best solution. The second configuration was used to compare the modal test with the different location of the OF. Research studies in fatigue tests with a mechanical rig [2], designed to reproduce representative material fatigue, exhibited damage induced near the clamping area of the specimen.

On the basis of the results obtained from this study it will be possible, by a method of monitoring based on modal analysis and frequency response functions (FRF), to check for signs of structural modifications. It could be very difficult to make an experimental survey using traditional investigation systems, such as non destructive scanning techniques, whilst such a structure was in-service.

## **Experimental investigations**

The experiments were carried out using the following procedure:

1. the specimen is subjected to an initial modal test to identify the natural frequencies and mode shape;
2. the specimen is subjected to a fatigue test through a certain number of loading cycles;
3. the specimen is removed from the rig and an ultrasonic scan is made;
4. stages (1), (2) and (3) are then repeated until damage appeared.

The scan test showed the location of damage, but the type cannot be specified. It may be a combination of delamination, fibre breakage, etc. Fatigue test experiments are performed at large displacement amplitudes ( $\pm 20\text{mm}$ ) at one end of cantilevered specimen. A mechanical (flexural) fatigue rig, consisting of a half-sine clamp, was used as this concentrates the stresses due to bending in the centre of the composite near to the end of clamp.

The modal investigation was performed using an instrumented hammer applied at various positions and a fixed accelerometer. Nine acquisition points were used and the accelerometer was positioned at the point 1. Figures 1-a and 1-b show the mesh used to model the specimen and the nine acquisition points.

The first (fundamental) mode shape is a flexural mode, the second is primarily a torsional mode which would not be observed for anisotropic cantilevered specimen.

In the first mode shape the nine points move in phase; in the second mode shape there are points moving in opposite phase about a centre line parallel to the long edge of the specimen.

## **Experimental Results**

### **First Specimen**

In the first specimen the optical fibre (OF) was embedded between the two centre  $0^\circ$  orientation plies. The natural frequencies of the specimen before any loading cycles were 1028 Hz and 1599 Hz. After 5000 loading cycles, the first frequency decreased (1024 Hz) by 0.39% whereas the second frequency unexpectedly increased (1744 Hz) by 9.07%.

No damage is detected after 5000 loading cycles. Then the specimen is

subjected to other 15000 cycles.

After 20000 cycles the first frequency (992 Hz) decreased again by 3.50% compared to specimen before any loading and by 3.13% compared to the specimen after 5000 loading cycles. The second frequency (1559 Hz) decreased by 2.50% compared to specimen before any loading cycles and it decreased by 10.61% compared to specimen after 5000 loading cycles. After 20000 loading cycles no damage is detected by an ultrasonic scan test. The specimen is subjected to a further 10000 loading cycles.

After 30000 loading cycles, the first frequency (1014 Hz) and the second frequency (1670 Hz) increased by 2.22% and by 7.12% respectively compared to the specimen after 20000 loading cycles. This probably happened because delamination has started. A sign of damage was actually detected by the scan test. The specimen is then subjected to 5000 loading cycles to show the frequency change after damage was started.

After 35000 loading cycles, the first frequency (1021 Hz) increased by 0.69% compared to the specimen after 30000 loading cycles. The second frequency (1709 Hz) increased by 2.34% compared to the specimen after 30000 loading cycles. The scan test showed a slight increase in the damage.

After 40000 loading cycles the first frequency (996 Hz) decreased by 2.15% compared to the specimen after 35000 loading cycles. The second frequency (1611 Hz) decreased by 5.73% compared to the specimen after 35000 loading cycles. The propagation of the damage was noticeable.

The frequency percentage variation with respect to the specimen before any loading cycles are illustrated graphically in figure 2, whereas figure 3 shows the frequency percentage variation with respect to the specimen in the previous loading fatigue test. Figures 4 and 5 show respectively the sequences of the first and the second natural frequency actual values during the tests.

Figure 6 shows the ultrasonic scans of the specimen. After 35000 and 40000 cycles both scans show the delamination occurring at the edge and increasing in size and moving towards the centre of the composite as the number of loading cycles is increased. At 40000 cycles it was also noticeable that an area of damage was at the centre of the composite, the location where one would have expected initiation of damage to occur.

Figures 7 to 12 show respectively the amplitudes obtained at the nine measurement points at the same frequency near the resonances, for the specimen before any loading cycles, after 5000, 20000, 30000, 35000, 40000 loading cycles.

In the Y-axes is displayed the modulus of the receptance (displacement/force).

The comparison between these data shows that

- in all the tests the points 2 and 8 have the maximum relative amplitude,
- points 3 and 7 have the minimum relative amplitude for the specimen before any loading cycles and after 5000 cycles,
- points 3, 6, and 9 have the minimum relative amplitude for the specimen after the other fatigue tests.

This could be due to the start of the delamination phenomena. Points 3, 6, and 9 were actually near the clamp in the centre of the composite.

## **Second Specimen**

In the second specimen the optical fibre (OF) was embedded between the two centre 0° orientation plies as for the first specimen. The natural frequencies of the specimen before any loading cycles were 1005 Hz and 1974 Hz. After 5000 loading cycles, the first frequency decreased (968 Hz) by 3.82% and the second frequency decreased (1909 Hz) by 3.40%.

Figure 13 shows the ultrasonic scan tests of the specimen before any loading cycles, the scan shows a little damaged zone near the metal strip. It was probably the result of existing defects inside the composite.

After 5000 cycles the scan shows a large area of damage at the centre of the composite, in the previous zone near the metal strip, the defects increase in size.

Figures 14 and 15 show respectively the amplitudes obtained at the nine measurement points at the same frequency near the resonances, for the specimen before any loading cycles and after 5000 loading cycles.

The comparison between these data shows that

- in all the tests points 2 and 8 have the maximum relative amplitude,
- points 3 and 7 have the minimum relative amplitude for the specimen before any loading cycles,
- points 3, 6, and 9 have the minimum relative amplitude for the specimen after 5000 loading cycles.

This confirms the result achieved for the first specimen. Points 3, 6 and 9 were actually near the large area where the damage occurred.

### **Third Specimen**

In the third specimen the optical fibre (OF) was embedded in the penultimate layer, between the  $\pm 45^\circ$  orientation plies. The natural frequencies of the specimen before any loading cycles were 1036 Hz and 2004 Hz. After 5000 loading cycles, the first frequency decreased (1005 Hz) by 2.99% and the second frequency decreased (1956 Hz) by 2.40%.

After 20000 cycles the first frequency (986 Hz) decreased again by 4.83% compared to the specimen before any loading and by 1.89% compared to the specimen after 5000 loading cycles. The second frequency (1917 Hz) decreased by 4.34% compared to specimen before any loading cycles and it decreased by 1.99% compared to specimen after 5000 loading cycles. The scan shows a small area of damage at the centre of the plate, where one would have expected the damage to occur.

The specimen is then subjected to a further 5000 loading cycles to show the frequency change after damage has started. After 25000 loading cycles, the first frequency (1010 Hz) and the second frequency (1940 Hz) increased by 2.43% and by 1.20% compared to the specimen after 20000 loading cycles. The scan shows that the area of damage has increased in size. It was noticeable that there was an area darker near the area of damage; it is a sign of carbon fibre breakage in the plies where the OF is embedded.

The frequency percentage variation with respect to the specimen before any loading cycles are illustrated graphically in figure 16, whereas figure 17 shows the frequency percentage variation with respect to the specimen in the previous loading fatigue test. Figures 18 and 19 show respectively the sequences of the first and the second natural frequency values during the tests.

Figure 20 shows the ultrasonic scans of the specimen. After 20000 and 25000 cycles it is noticeable the area of damage at the centre of the composite, the location where one would have expected intention of damage to occur.

Figures 21 to 24 show respectively the amplitudes obtained at the nine measurement points at the same frequency near the resonances, for the specimen before any loading cycles, after 5000, 20000, 25000, loading cycles.

The comparison between the first frequency amplitudes shows that

- the position of the points that have the maximum relative amplitude was similar to the previous tests,
- in all the tests points 3, 6, and 9 have the minimum relative amplitude.

This confirms, in part, the results achieved for the first and second specimen.

The different behaviour probably happens because the OF was embedded in different layers.

## Comparison

The comparison between the first and the second specimen shows the first natural frequencies before any loading cycles (1028 Hz and 1005 Hz) were quite similar; whereas the second natural frequency of the second specimen (1974 Hz) was significantly increased compared to the second specimen (1599 Hz).

The first natural frequency before any loading cycles (1036 Hz) of the third specimen was quite similar; whereas the second natural frequency of the third specimen (2004 Hz) was significantly higher when compared to the first and second specimen.

After 5000 cycles the first natural frequency was only noticeably changed for the second specimen. The second natural frequency for the first composite unexpectedly increased compared to the specimen before any loading cycles, whereas for the second specimen decreased slight.

After 5000 cycles the first natural frequency of the third specimen (1004 Hz) was lower than the first frequency of the first and second specimen. The second natural frequency for the third specimen decreased slightly compared to the specimen before any loading cycles.

After 20000 cycles the comparison between the first and the third specimen showed the first natural frequencies (992 Hz and 986 Hz) are quite similar; whereas the second natural frequency of the third specimen (1917 Hz) was significantly higher compared to the first specimen (1559 Hz). In both cases, after this test, the minimum values for the first and second natural frequencies were recorded.

After the delamination began the first and the second natural frequencies of the first and third specimen increased with respect to the specimen in the previous loading

fatigue test.

Inbetween scans the specimen had to be replaced in the fatigue rig and slight changes in the position in fitting the composites may have resulted in changed boundary conditions and hence natural frequency characteristics. This is to be expected as for any structure there will be environmental factors, etc., which will result in natural frequency fluctuations.

Hence a small change in frequency, either an increase or decrease, is not sensitive solely to damage as can be seen for the first specimen which, until 35000 cycles, or more remained effectively undamaged.

The comparison between the first frequency amplitudes shows that

- before any loading cycles the amplitudes for the first and third specimen are similar and few differences are observed compared to the second specimen,
- after 5000 loading cycles the amplitudes for the first, second and third specimen are similar; only the amplitude of the point 7 is different,
- after 20000 loading cycles the amplitudes for the first and third specimen are similar.

The comparison between the second frequency amplitudes show minimal differences.

The comparison between the first and the second mode shape for the first specimen before any loading cycles and after 5000 cycles (figure 25 to 28) shows unimportant differences; whereas there are significant changes for the second mode shape for the specimen after 5000 cycles compared to after 20000 cycles (figure 29). These change are located at the centre of the specimen where, probably, the delamination has started. Few differences were observed for the other cases.

The first and the second mode shape for the second specimen before any loading cycles and after 5000 cycles (figure 30 to 33) are quite identical.

The comparison between the first and the second mode shape for the third specimen before any loading cycles and after 5000 cycles (figure 34 to 37) showed negligible differences; whereas there were significant changes for the second mode shape between the specimen after 5000 cycles and after 20000 cycles (figure 38).

This confirms the result achieved for the first specimen.

## **Conclusion**

Three carbon fibre reinforced plates (CFRP) specimens with the same lay-up and with an optical fibre (OF) embedded between different orientation plies, were used in the experiments.

The natural frequency and normal mode shape variation were measured and correlation this with the damage caused by fatigue stresses was investigated. The damage due to fatigue occurred on the three specimens after different numbers of loading cycles. The third specimen was damaged before the first specimen, as one would expect, because the optical fibre (OF) is embedded between different layers [1]. The damage in the second specimen occurred early, probably because of defects inside the composite.

For the same type of fatigue damage, a good correlation between the results for the first and the second specimen were obtained whereas differences for the third specimen were observed.

The first and the second natural frequencies of the first and third specimen were reduced at their lowest value when delamination has started compared to the non-damage specimens.

Future investigations using the mode shape data, in conjunction with the frequencies, will aim to identify the change in flexibility or stiffness and locate where it has occurred.

Material Properties		
	CFRP	Optical fibre
Young's modulus $E_x$	106.5 GPa	70.7 GPa
Young's modulus $E_y = E_z$	8.6 GPa	
Shear modulus $G_{xy}$	4.59 GPa	26.18 GPa
Poisson's ratio $\nu_{xy} = \nu_{yz}$	2.23	0.35
Poisson's ratio $\nu_{xz}$	0.28	0.35
Density	1360 g/cm <sup>3</sup>	

### *References*

- [1] B.Benchekchou, N.S. Ferguson "The effect of embedded optical fibres on the fatigue behaviour of composite plate", Composite Structures, 41, 1998.
- [2] Drew, R.C. "An investigation into damage initiation and propagation in carbon fibre reinforced plastics", PhD Thesis, University of Southampton, 1988.

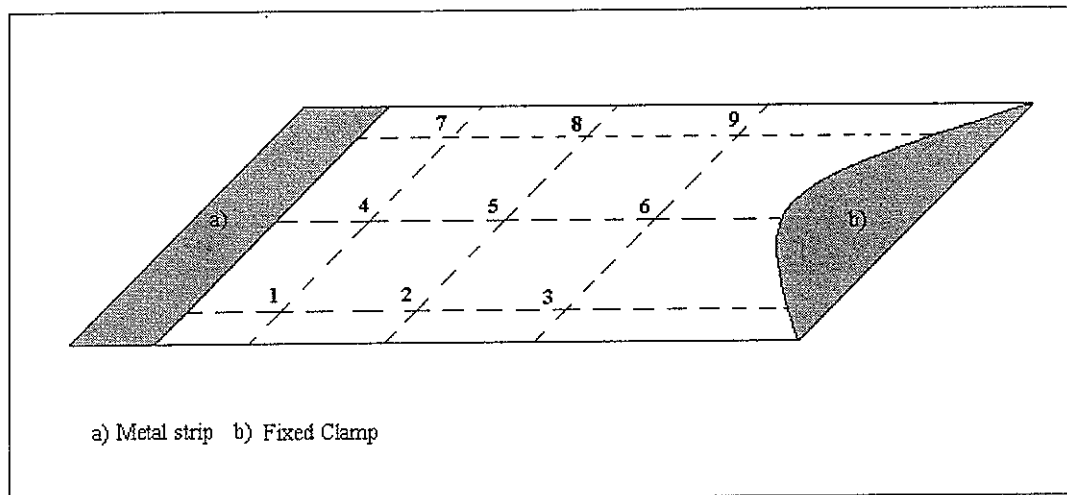


Figure 1-a. Mesh generated for the specimens.

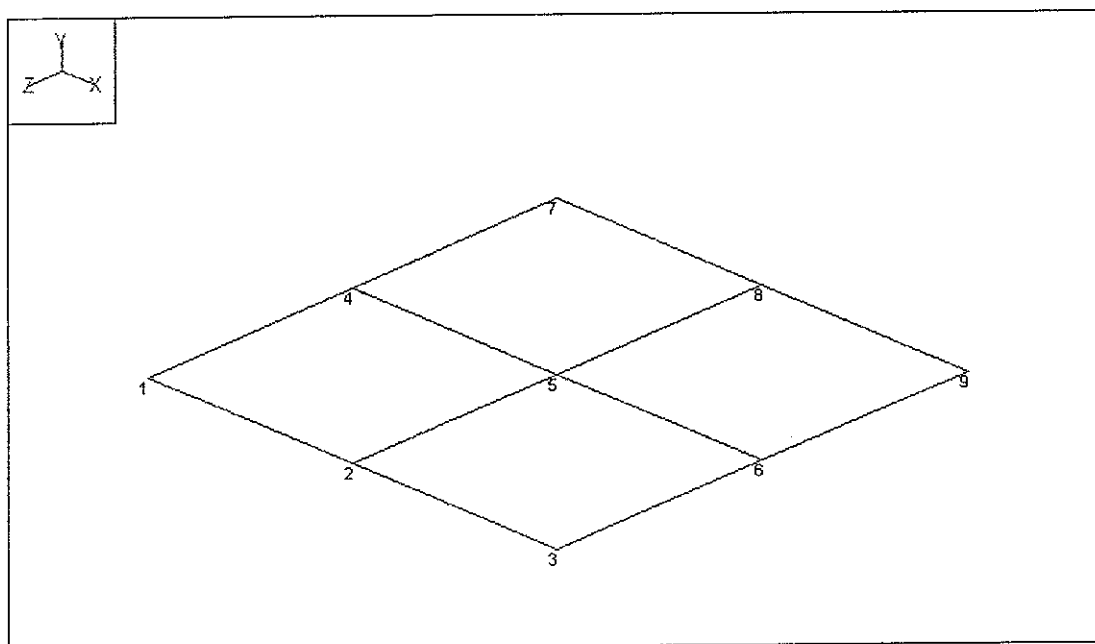


Figure 1-b. Mesh generated for the specimens.

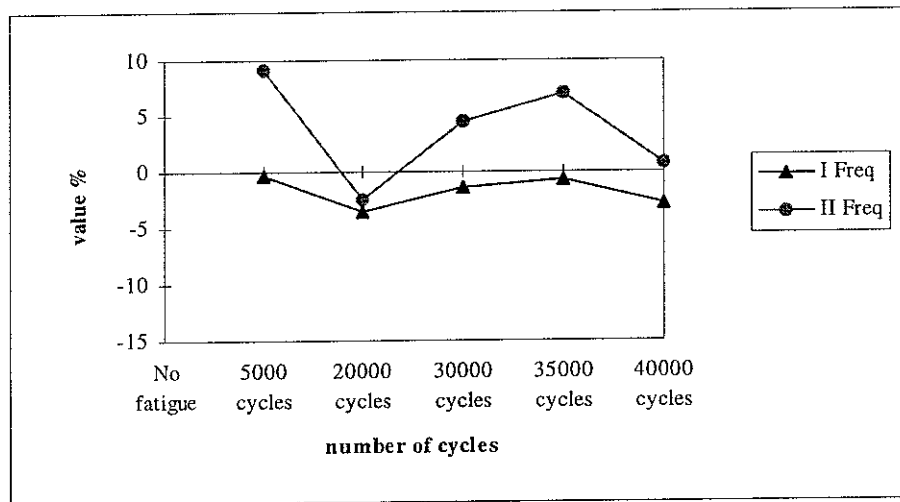


Figure 2. Frequency percentage variation with respect to the first specimen before any loading cycles.

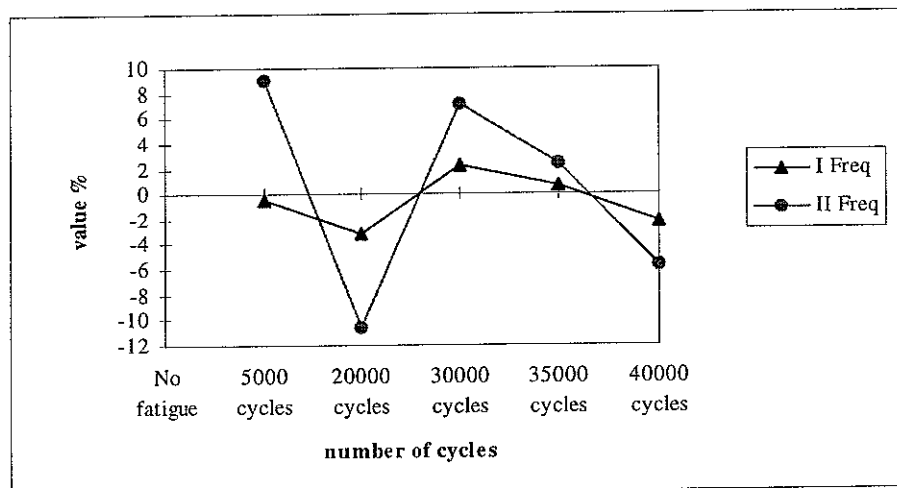
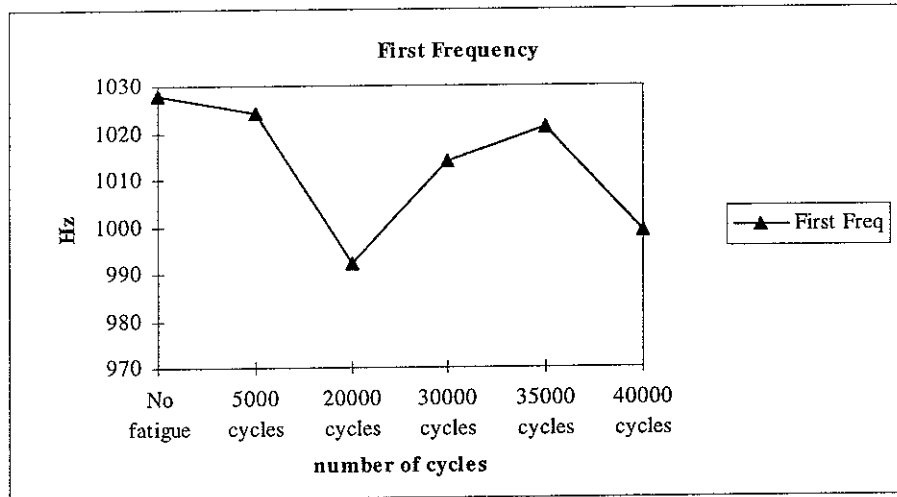
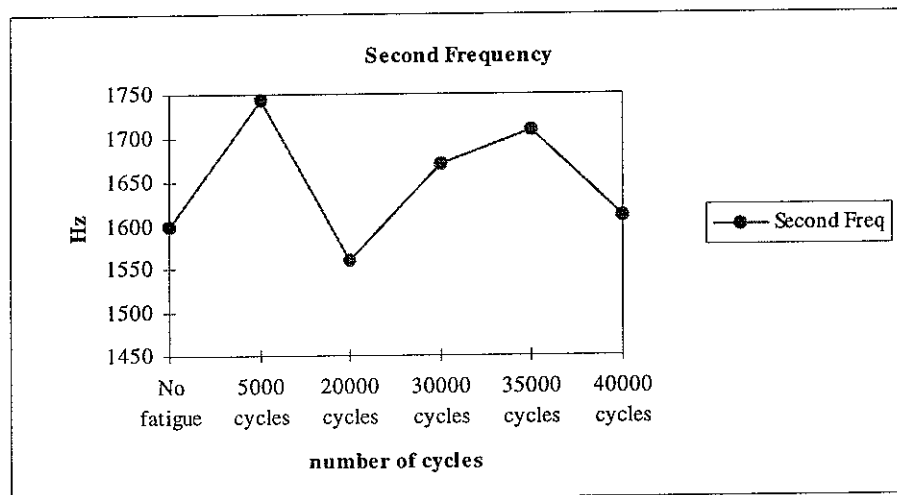


Figure 3. Frequency percentage variation with respect to the first specimen in the previous loading fatigue test.



*Figure 4. First frequency variation of the first specimen.*



*Figure 5. Second frequency variation of the first specimen.*

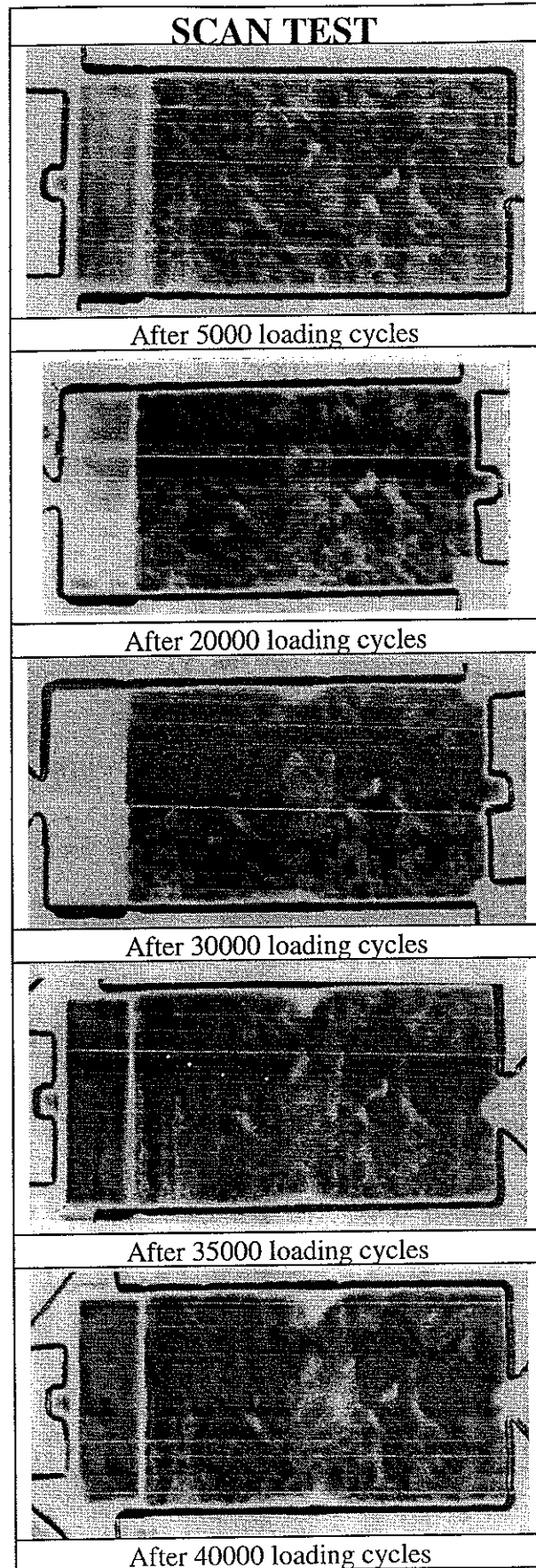


Figure 6. Ultrasonic scans of the first specimen.

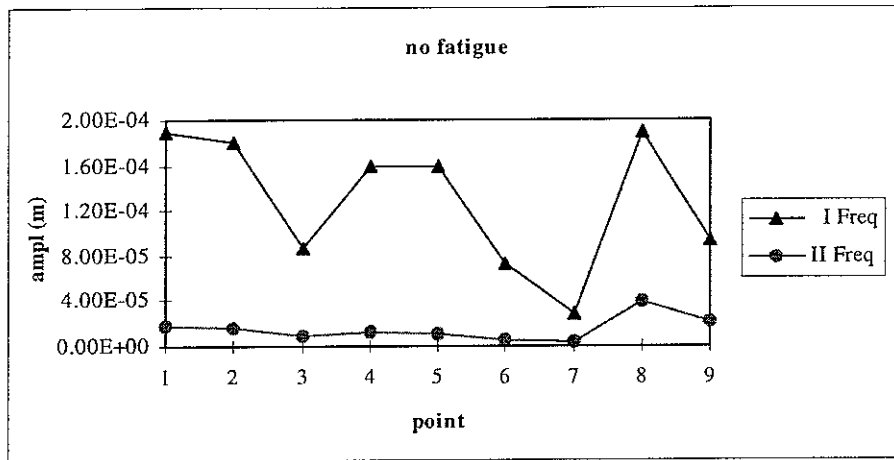


Figure 7. Amplitude variation for the 9 points before any loading cycles of the first specimen.

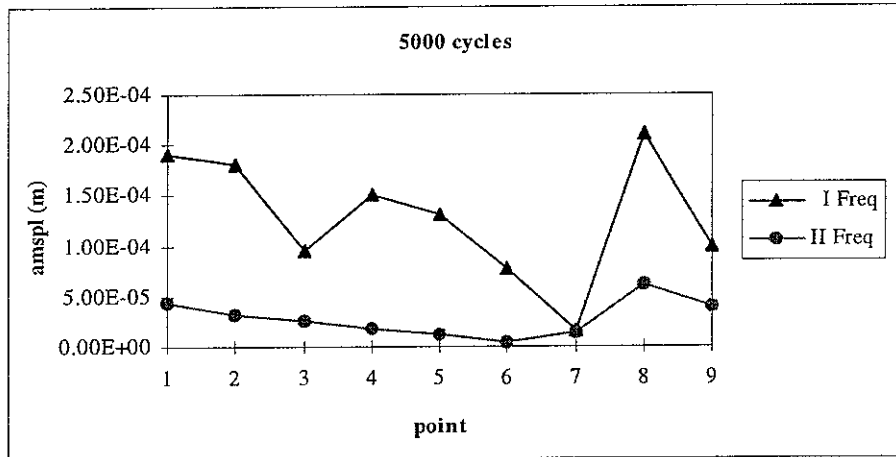


Figure 8. Amplitude variation for the 9 points after 5000 loading cycles of the first specimen.

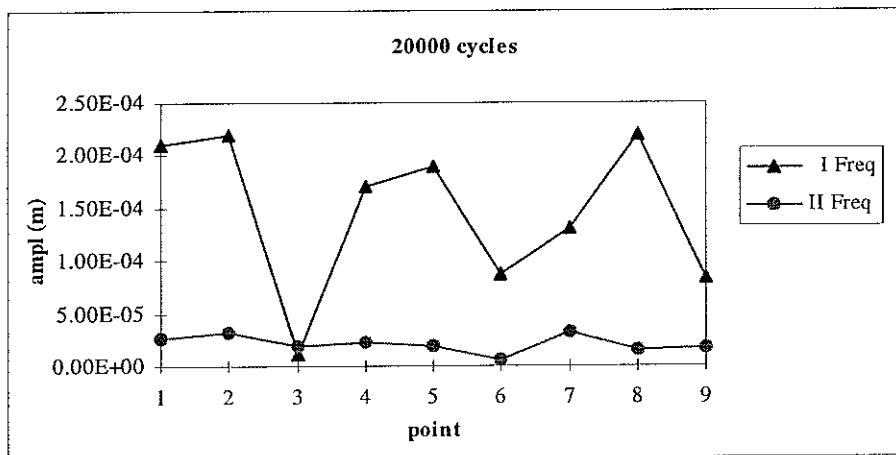


Figure 9. Amplitude variation for the 9 points after 20000 loading cycles of the first specimen.

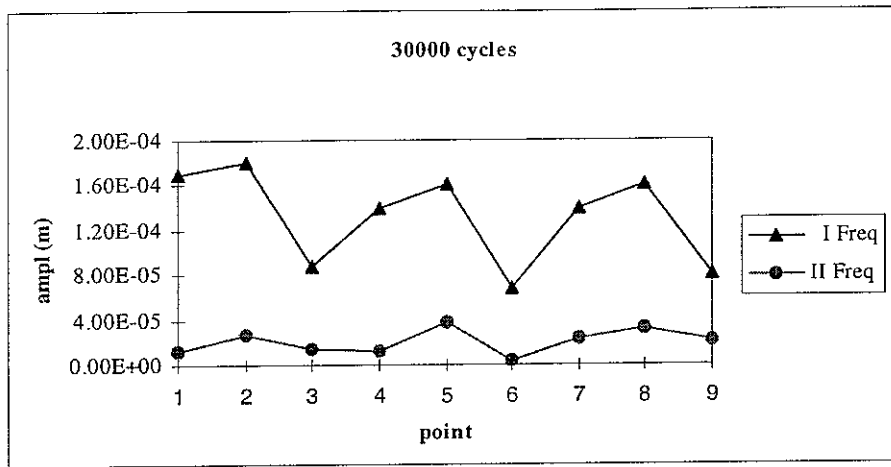


Figure 10. Amplitude variation for the 9 points after 30000 loading cycles of the first specimen.

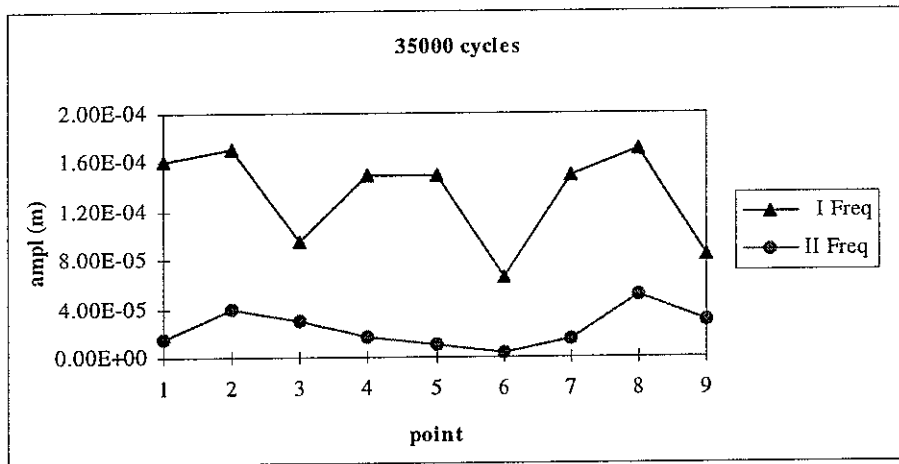


Figure 11. Amplitude variation for the 9 points after 35000 loading cycles of the first specimen.

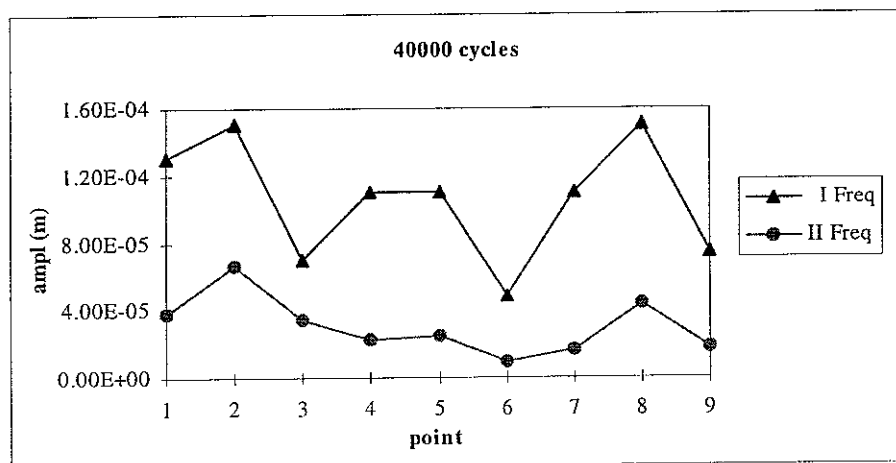
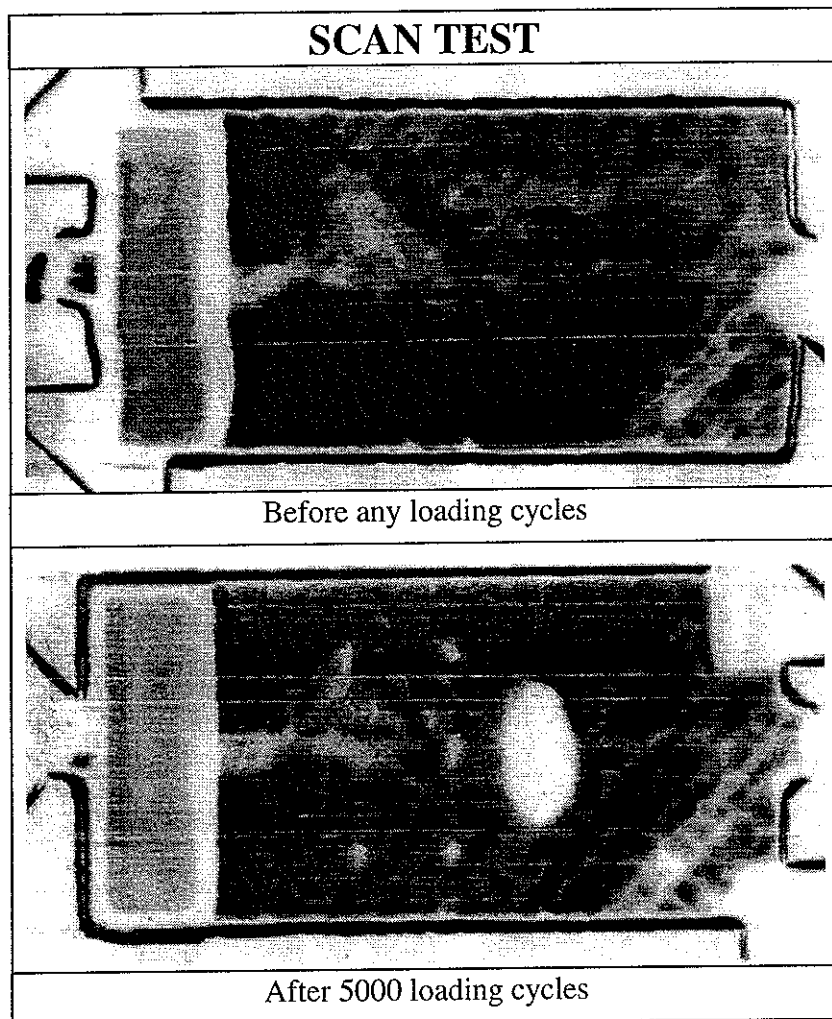


Figure 12. Amplitude variation for the 9 points after 40000 loading cycles of the first specimen.



*Figure 13. Ultrasonic scans of the second specimen.*

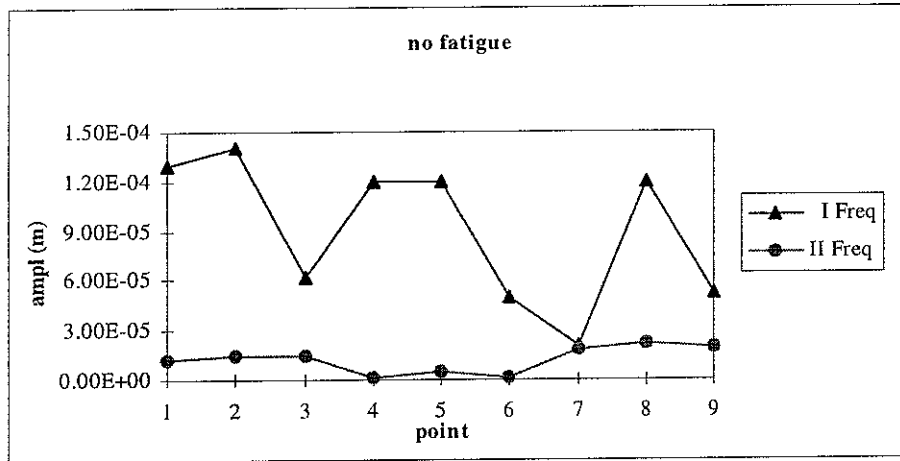


Figure 14. Amplitude variation for the 9 points before any loading cycles of the second specimen.

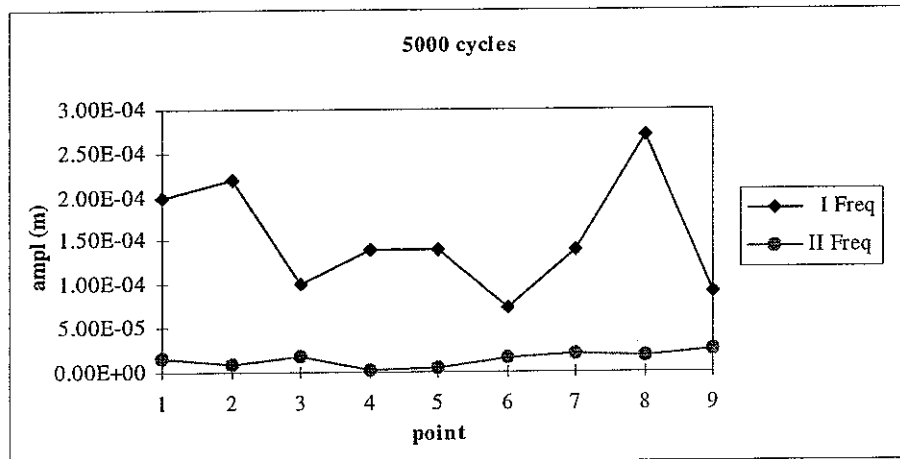


Figure 15. Amplitude variation for the 9 points after 5000 loading cycles of the second specimen.

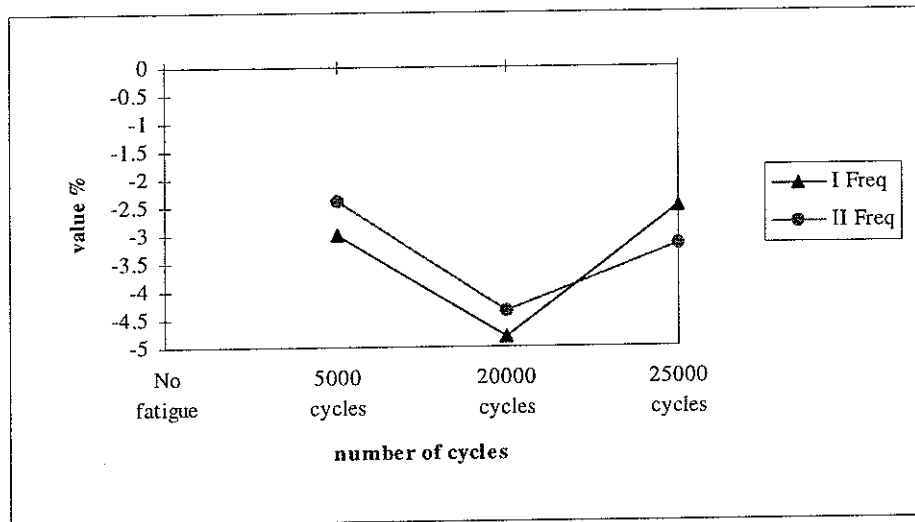


Figure 16. Frequency percentage variation with respect to the third specimen before any loading cycles.

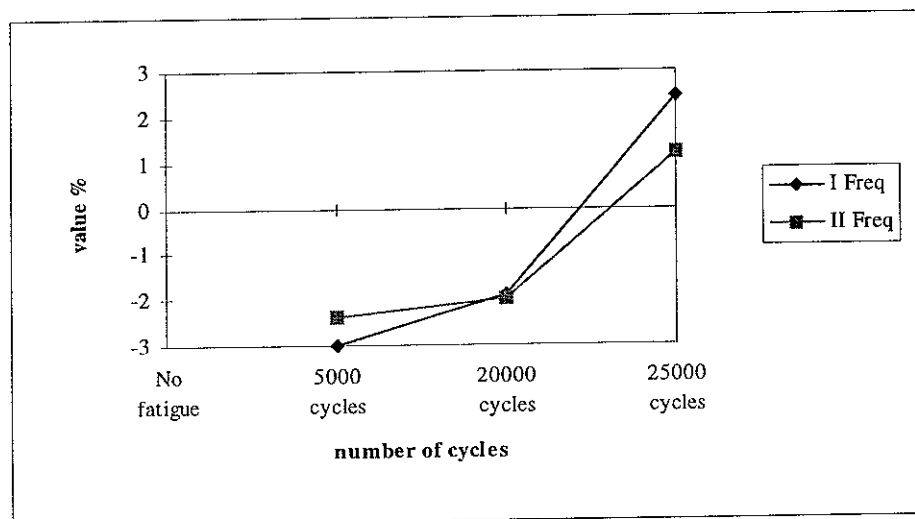
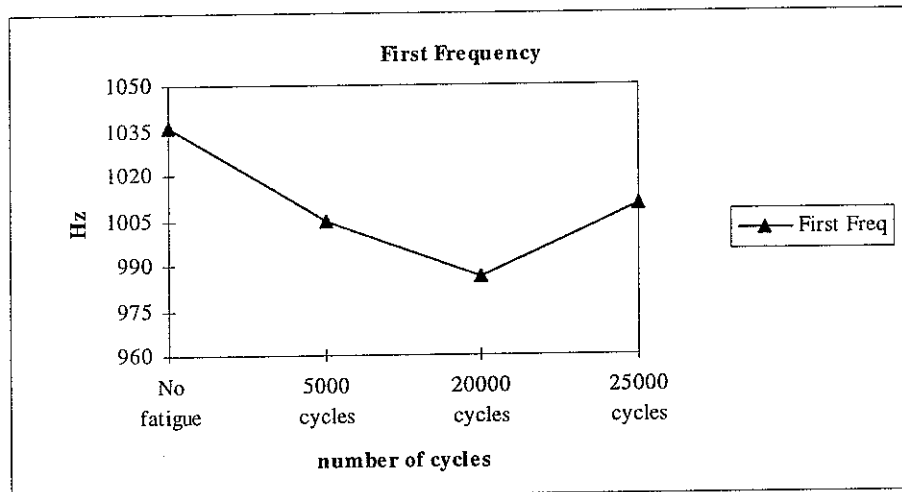
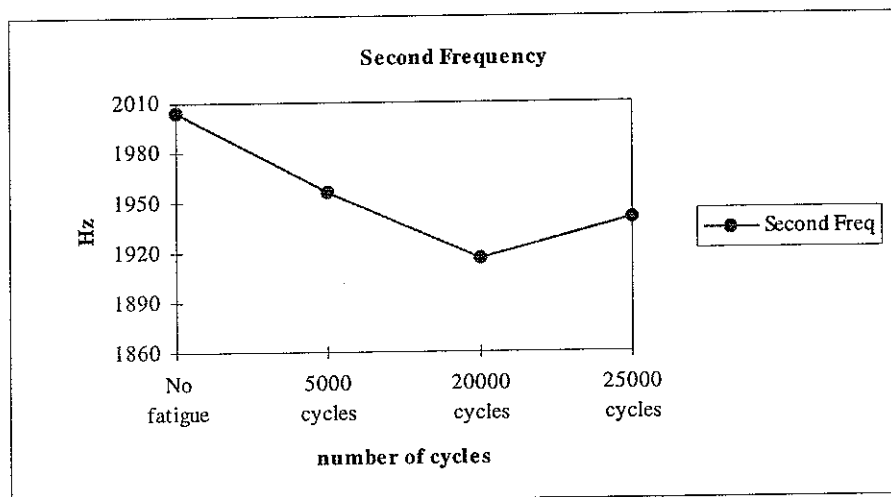


Figure 17. Frequency percentage variation with respect to the third specimen in the previous loading fatigue test.



*Figure18. First frequency variation of the third specimen.*



*Figure 19. Second frequency variation of the third specimen.*

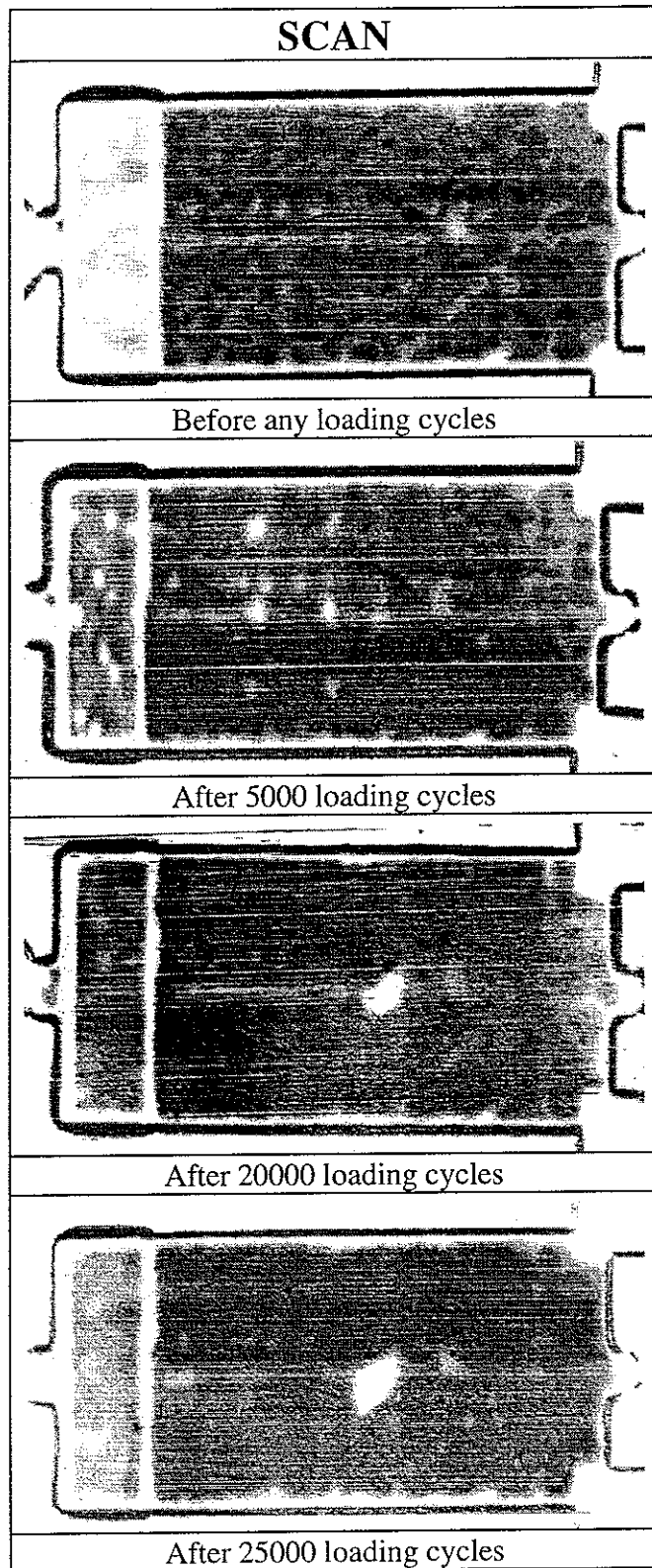


Figure 20. Ultrasonic scans of the third specimen.

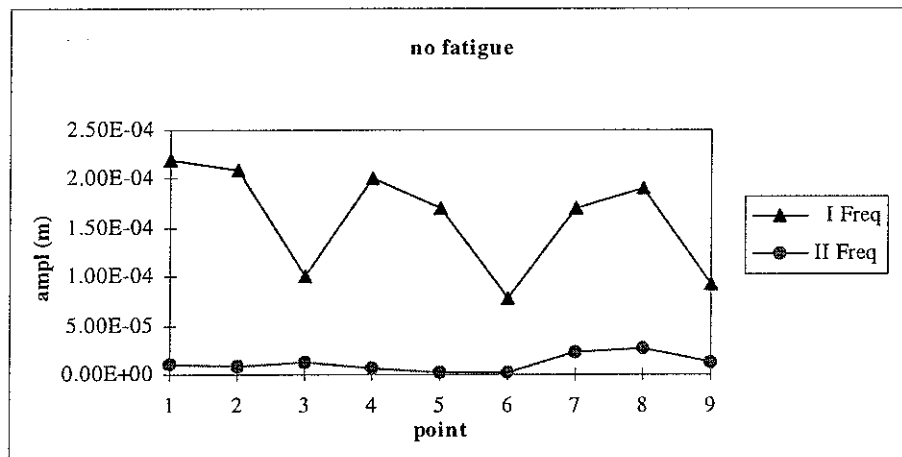


Figure 21. Amplitude variation for the 9 points before any loading cycles of the third specimen.

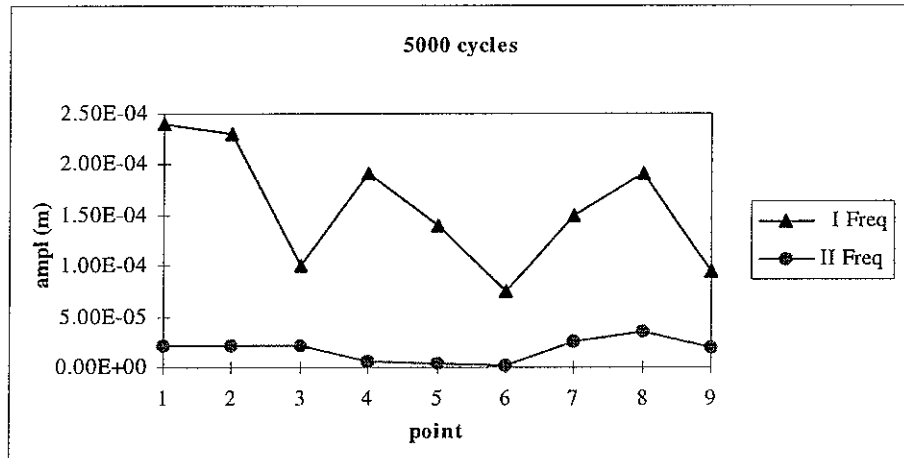


Figure 22. Amplitude variation for the 9 points after 5000 loading cycles of the third specimen.

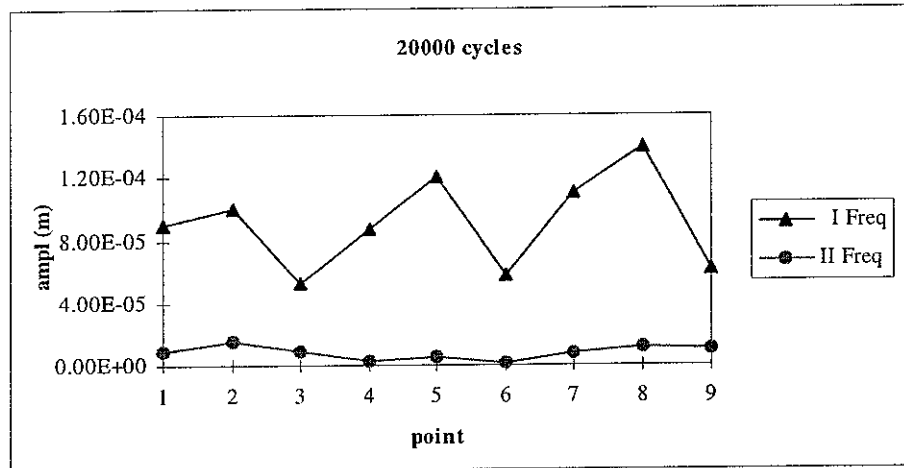


Figure 23. Amplitude variation for the 9 points after 20000 loading cycles of the third specimen.

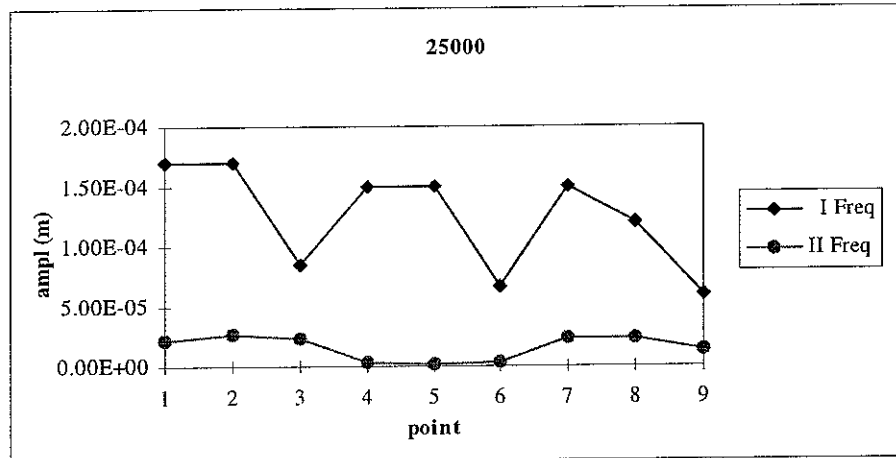
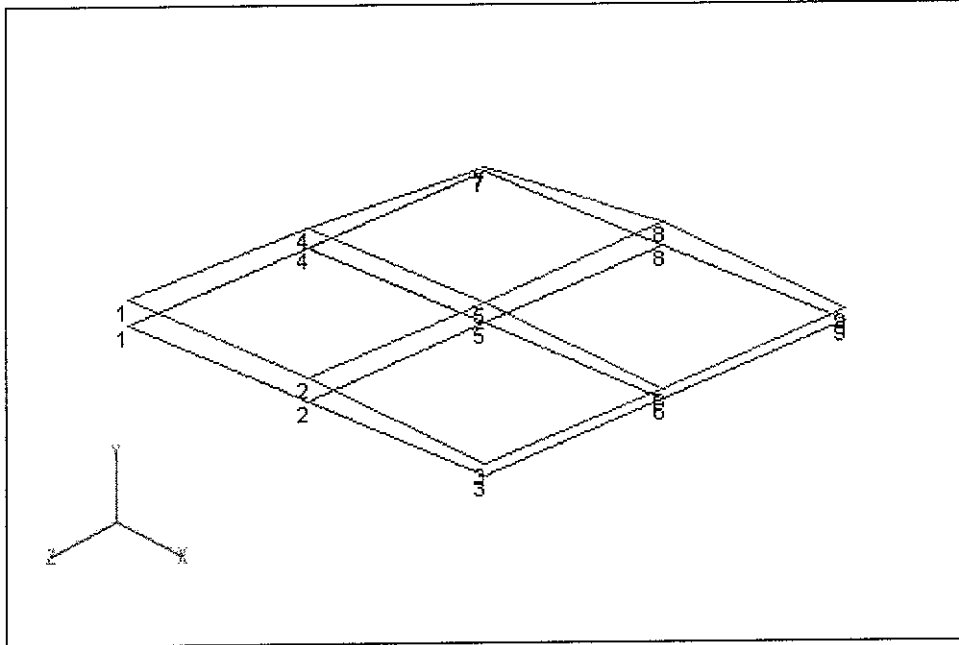
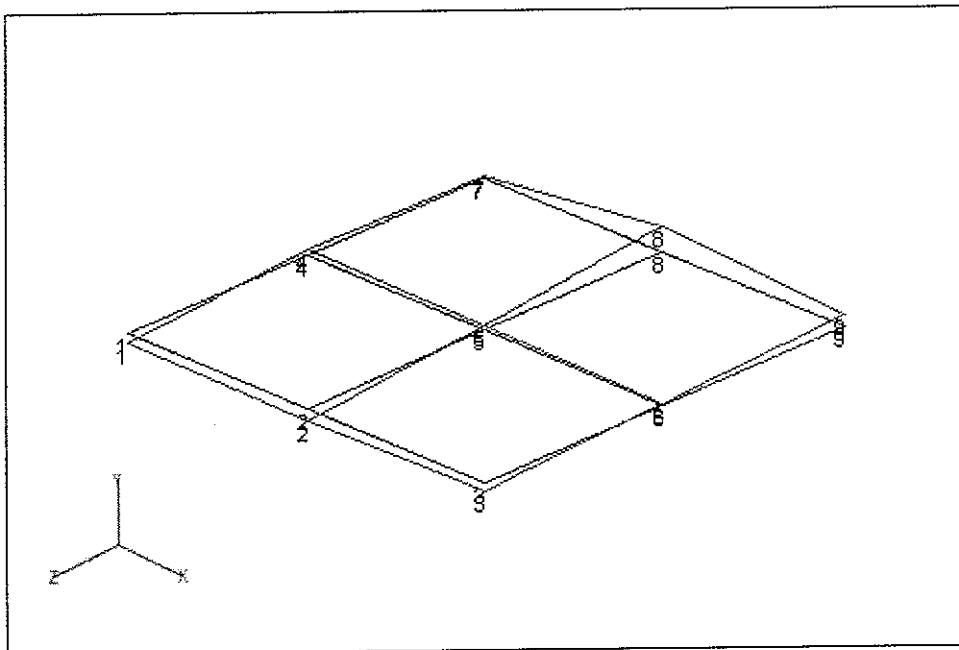


Figure 24. Amplitude variation for the 9 points after 25000 loading cycles of the third specimen.



*Figure 25. First mode shape for the first specimen before any loading cycles*



*Figure 26. Second mode shape for the first specimen before any loading cycles*

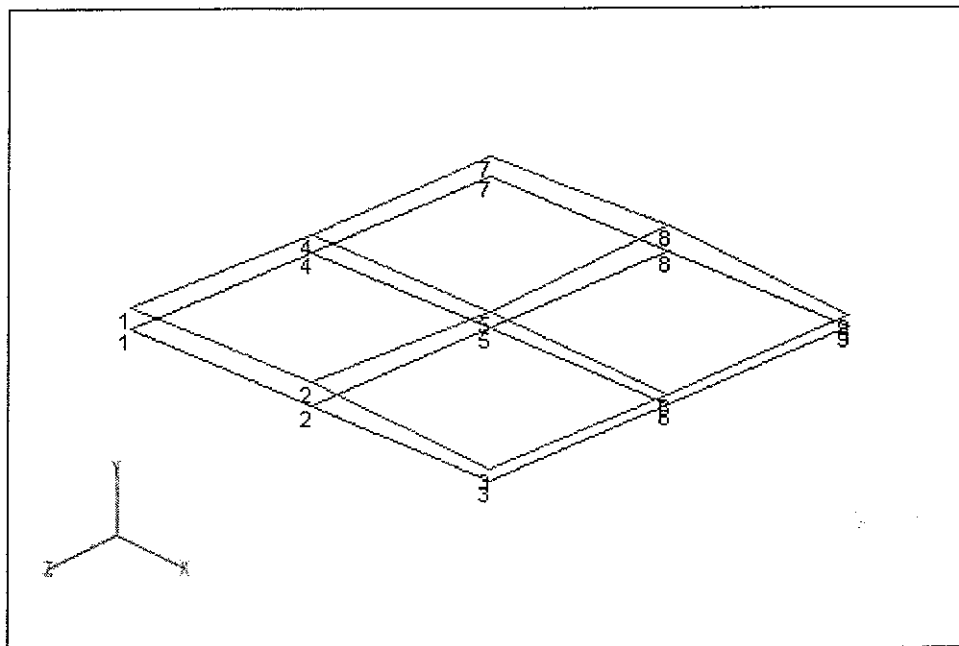


Figure 27. First mode shape for the first specimen after 5000 loading cycles

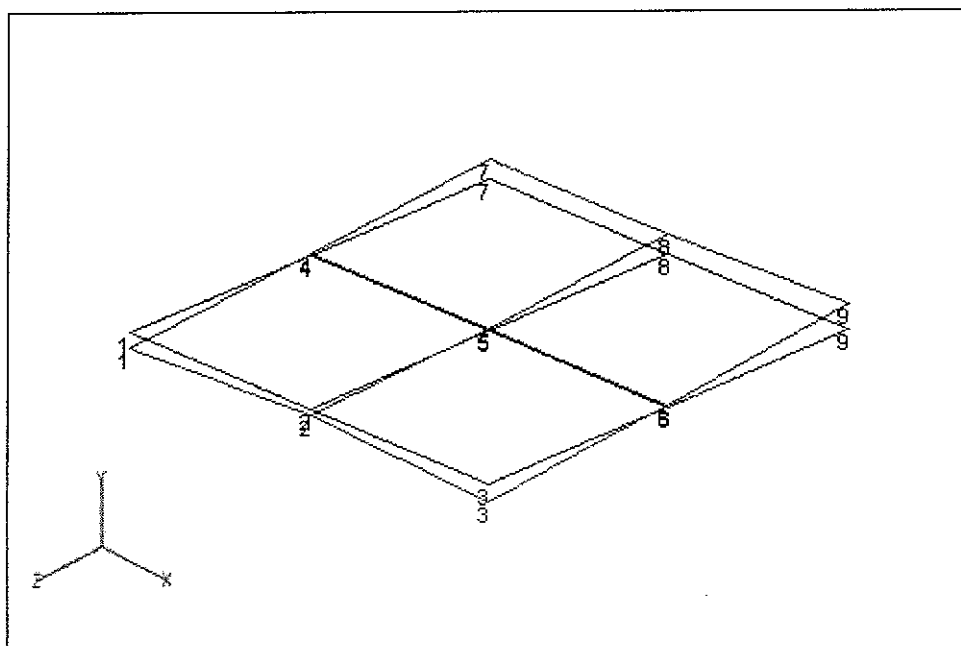
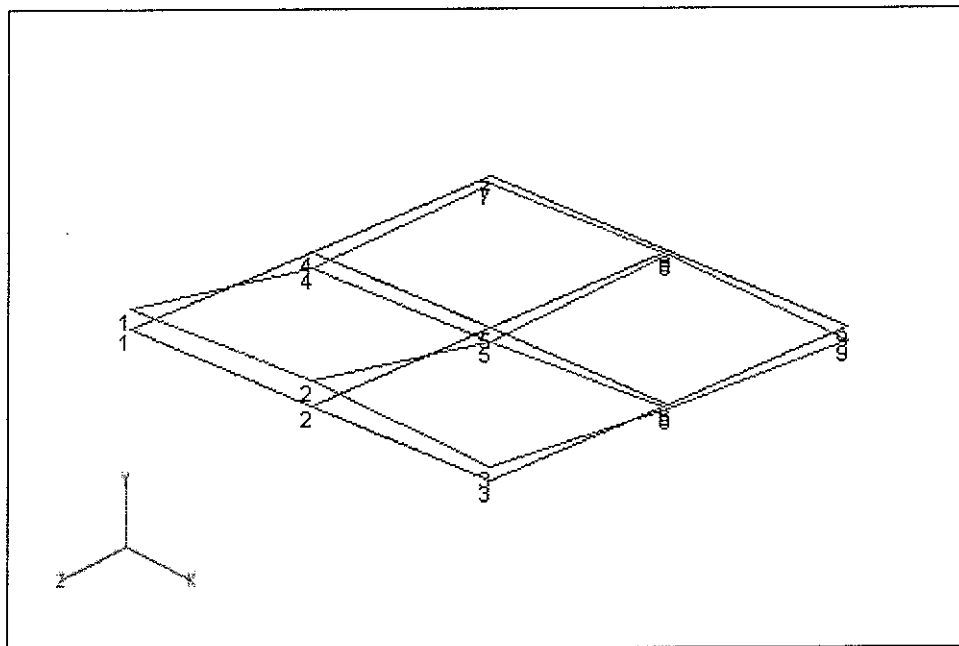
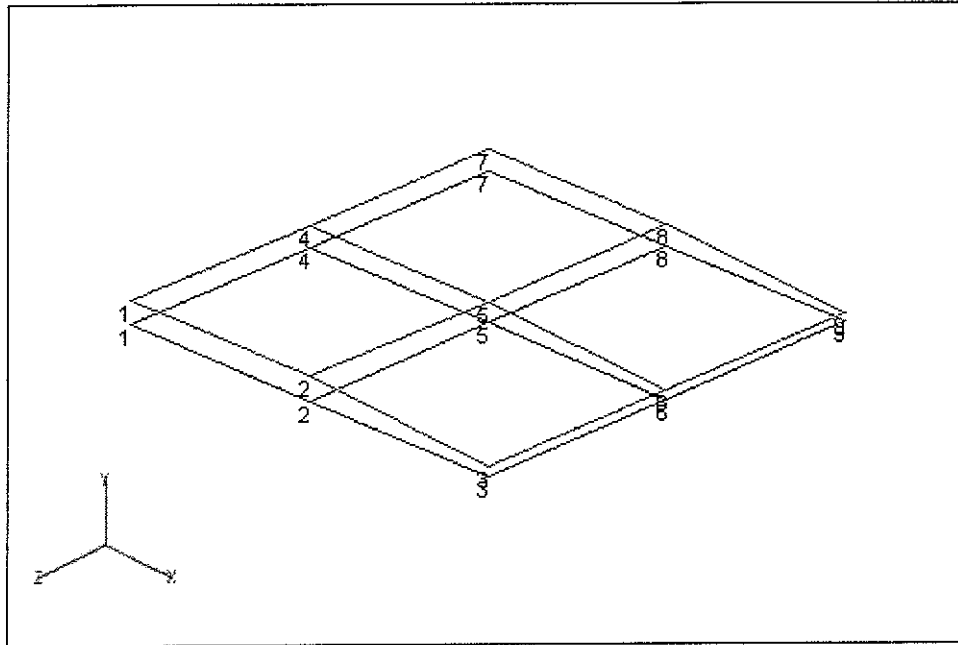


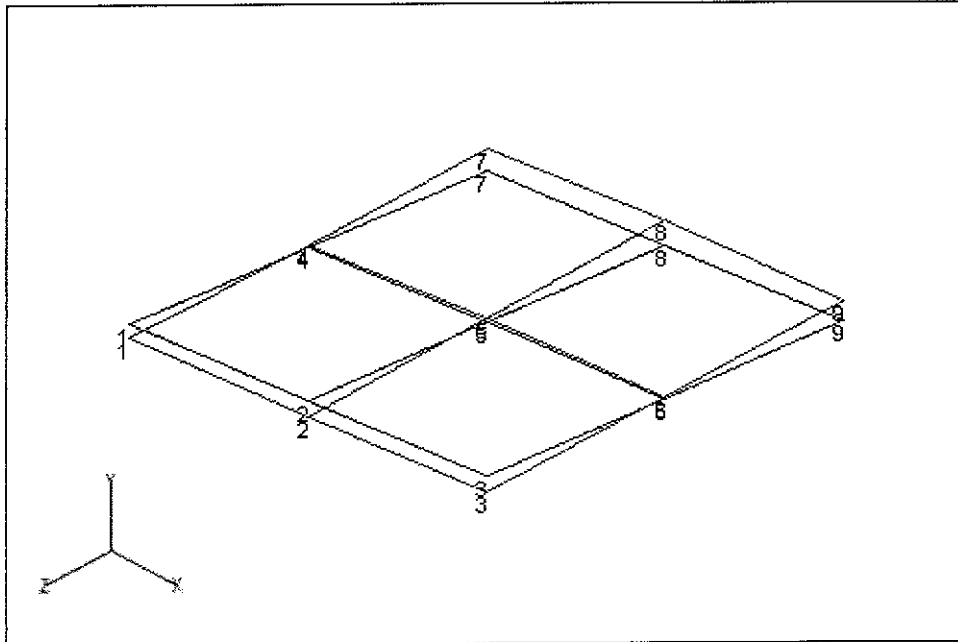
Figure 28. Second mode shape for the first specimen after 5000 loading cycles



*Figure 29. Second mode shape for the first specimen after 20000 loading cycles*



*Figure 30. First mode shape for the second specimen before any loading cycles*



*Figure 31. Second mode shape for the second specimen before any loading cycles*

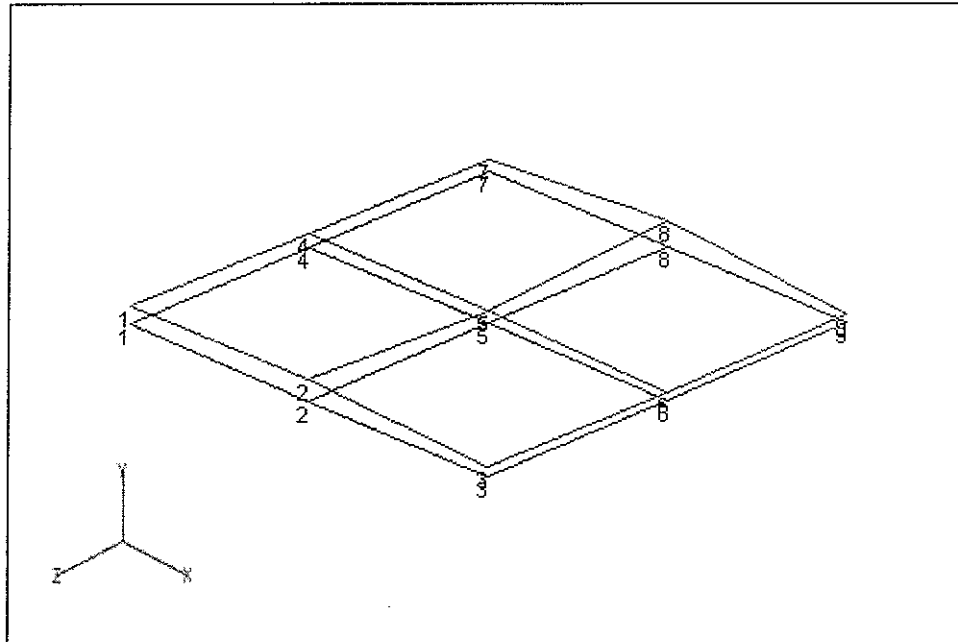


Figure 32. First mode shape for the second specimen after 5000 loading cycles

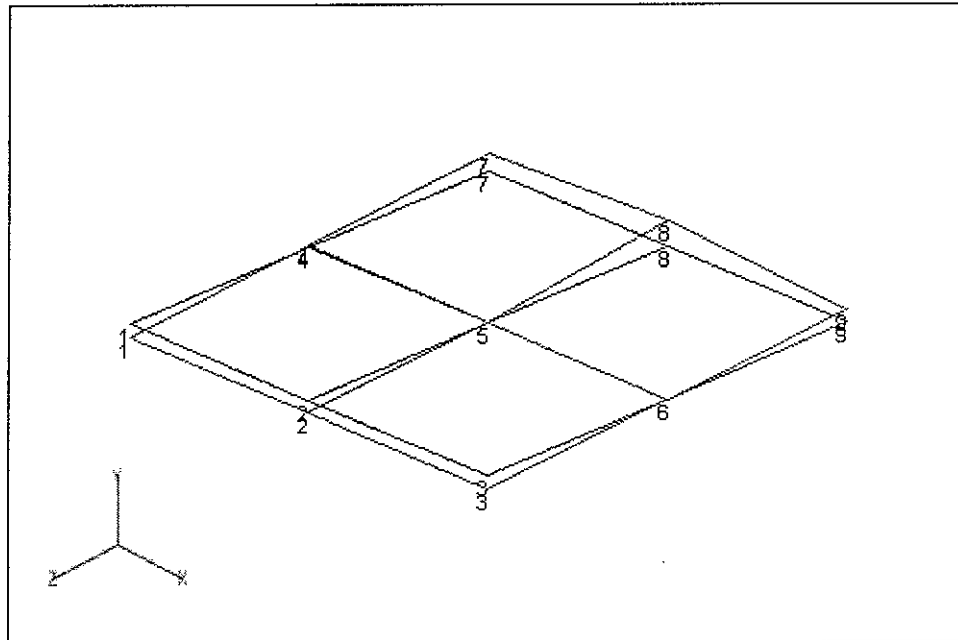
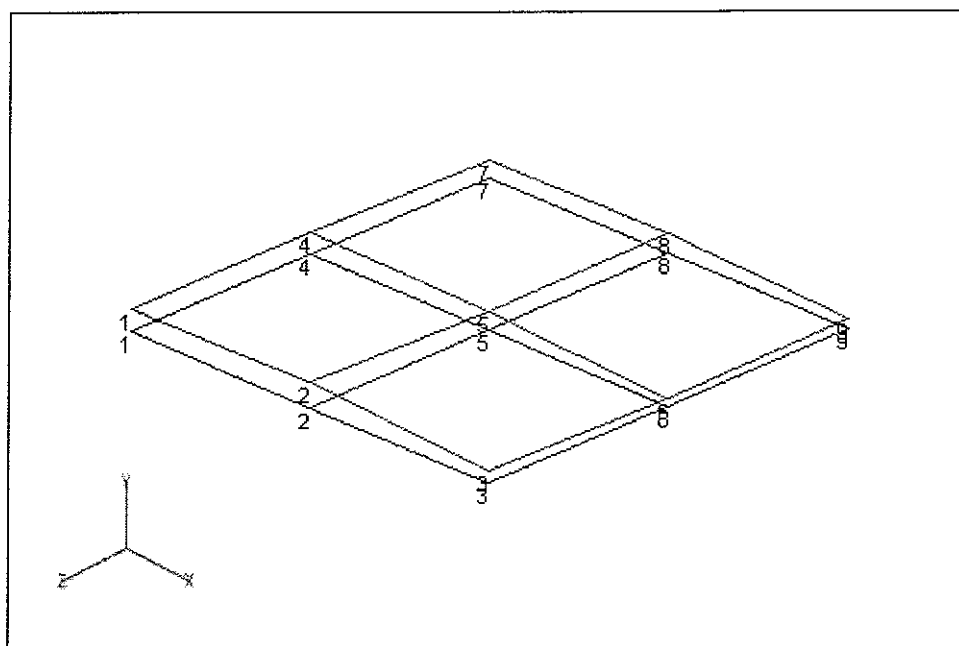
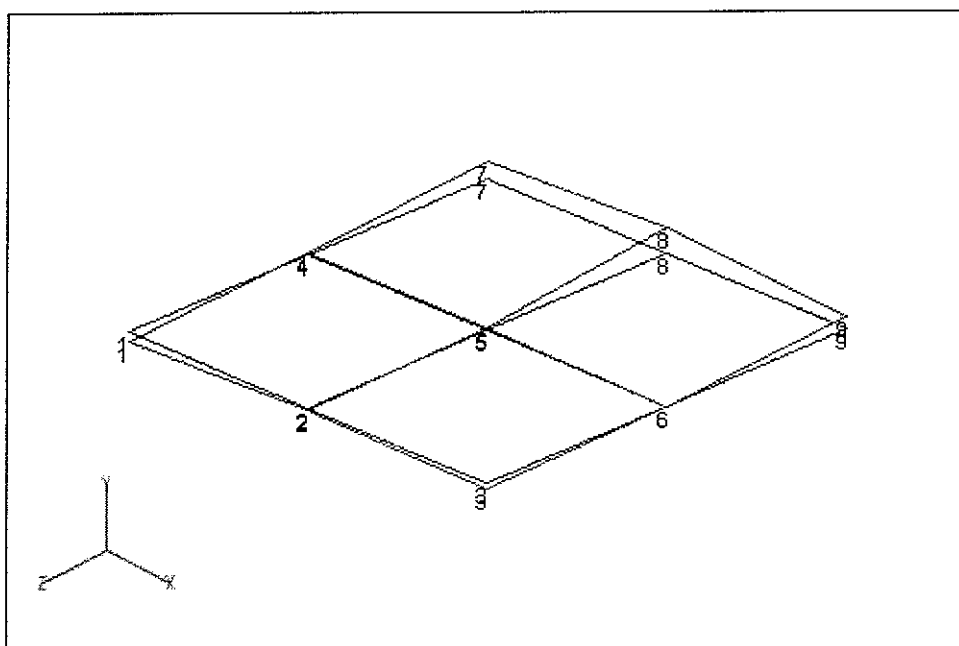


Figure 33. Second mode shape for the second specimen after 5000 loading cycles



*Figure 34. First mode shape for the third specimen before any loading cycles*



*Figure 35. Second mode shape for the third specimen before any loading cycles*

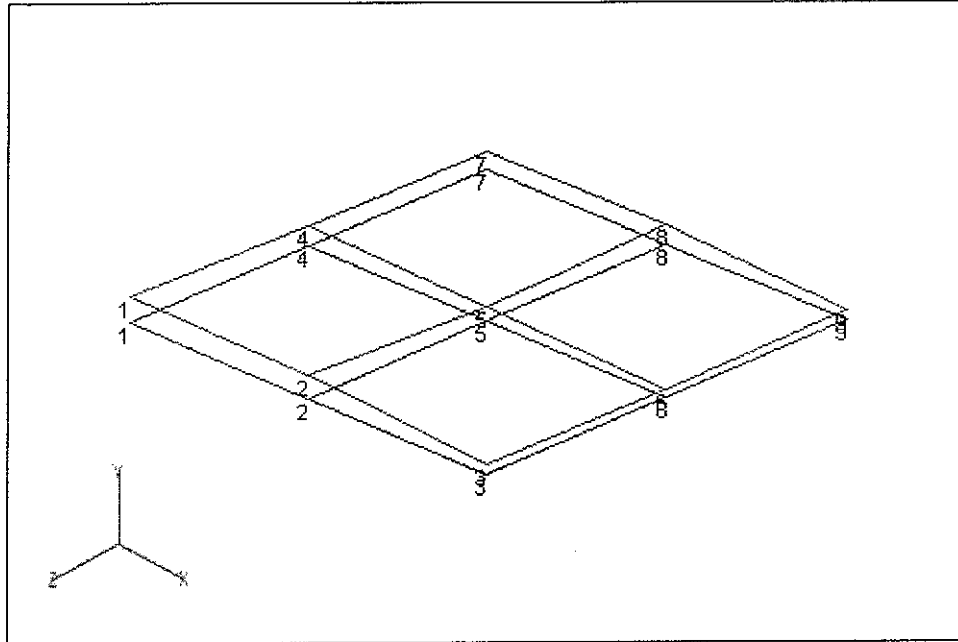


Figure 36. First mode shape for the third specimen after 5000 loading cycles

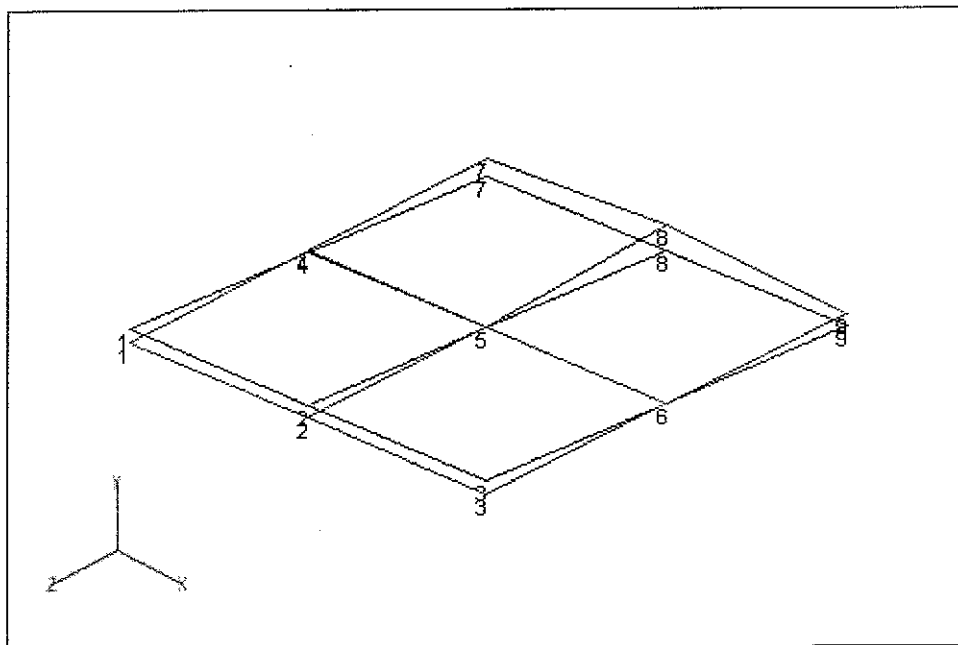
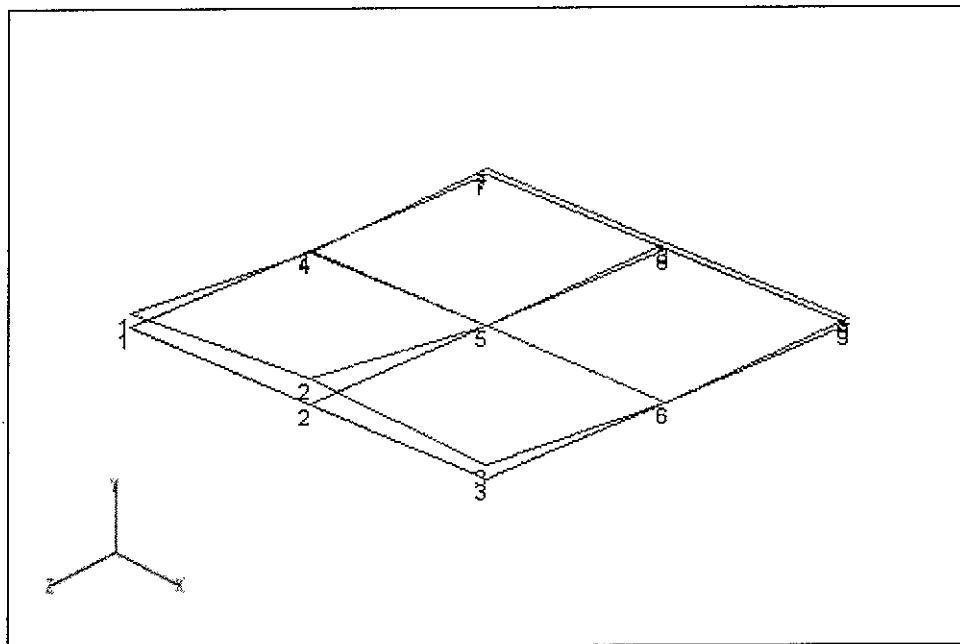


Figure 37. Second mode shape for the third specimen after 5000 loading cycles



*Figure 38. Second mode shape for the third specimen after 20000 loading cycles*

no fatigue	I Freq		II Freq
1	1.90E-04		1.70E-05
2	1.80E-04		1.60E-05
3	8.60E-05		8.80E-06
4	1.60E-04		1.20E-05
5	1.60E-04		1.00E-05
6	7.20E-05		5.60E-06
7	2.90E-05		4.40E-06
8	1.90E-04		3.90E-05
9	9.30E-05		2.10E-05

30000	I Freq		II Freq
1	1.70E-04		1.30E-05
2	1.80E-04		2.70E-05
3	8.67E-05		1.50E-05
4	1.40E-04		1.20E-05
5	1.60E-04		3.80E-05
6	6.80E-05		2.70E-06
7	1.40E-04		2.30E-05
8	1.60E-04		3.30E-05
9	8.10E-05		2.20E-05

5000	I Freq		II Freq
1	1.90E-04		4.30E-05
2	1.80E-04		3.10E-05
3	9.60E-05		2.60E-05
4	1.50E-04		1.70E-05
5	1.30E-04		1.10E-05
6	7.70E-05		4.30E-06
7	1.60E-05		1.30E-05
8	2.10E-04		6.20E-05
9	1.00E-04		4.00E-05

35000	I Freq		II Freq
1	1.60E-04		1.40E-05
2	1.70E-04		4.00E-05
3	9.50E-05		3.00E-05
4	1.50E-04		1.60E-05
5	1.50E-04		9.80E-06
6	6.60E-05		2.80E-06
7	1.50E-04		1.40E-05
8	1.70E-04		5.10E-05
9	8.40E-05		2.90E-05

20000	I Freq		II Freq
1	2.10E-04		2.60E-05
2	2.20E-04		3.30E-05
3	1.10E-05		1.80E-05
4	1.70E-04		2.20E-05
5	1.90E-04		1.80E-05
6	8.80E-05		5.90E-06
7	1.30E-04		3.30E-05
8	2.20E-04		1.60E-05
9	8.30E-05		1.70E-05

40000	I Freq		II Freq
1	1.30E-04		3.80E-05
2	1.50E-04		6.60E-05
3	7.00E-05		3.40E-05
4	1.10E-04		2.30E-05
5	1.10E-04		2.50E-05
6	4.90E-05		9.40E-06
7	1.10E-04		1.70E-05
8	1.50E-04		4.40E-05
9	7.50E-05		1.80E-05

Table 1. Data displacements for the first specimen.

no fatigue	I Freq		II Freq
1	1.30E-04		1.20E-05
2	1.40E-04		1.40E-05
3	6.20E-05		1.40E-05
4	1.20E-04		8.90E-07
5	1.20E-04		4.70E-06
6	5.00E-05		8.90E-07
7	2.10E-05		1.80E-05
8	1.20E-04		2.20E-05
9	5.20E-05		1.90E-05

5000	I Freq		II Freq
1	2.00E-04		1.70E-05
2	2.20E-04		1.00E-05
3	1.00E-04		1.80E-05
4	1.40E-04		2.00E-06
5	1.40E-04		4.40E-06
6	7.40E-05		1.70E-05
7	1.40E-04		2.00E-05
8	2.70E-04		1.90E-05
9	9.20E-05		2.60E-05

Table 2. Data displacements for the second specimen.

no fatigue	I Freq		II Freq
1	2.20E-04		1.00E-05
2	2.10E-04		7.40E-06
3	1.00E-04		1.20E-05
4	2.00E-04		5.20E-06
5	1.70E-04		1.20E-06
6	7.80E-05		1.30E-06
7	1.70E-04		2.20E-05
8	1.90E-04		2.70E-05
9	9.20E-05		1.30E-05

20000	I Freq		II Freq
1	9.00E-05		8.90E-06
2	1.00E-04		1.60E-05
3	5.20E-05		9.00E-06
4	8.70E-05		2.00E-06
5	1.20E-04		4.80E-06
6	5.80E-05		9.30E-07
7	1.10E-04		7.40E-06
8	1.40E-04		1.10E-05
9	6.20E-05		1.00E-05

5000	I Freq		II Freq
1	2.40E-04		2.20E-05
2	2.30E-04		2.10E-05
3	1.00E-04		2.20E-05
4	1.90E-04		6.20E-06
5	1.40E-04		3.80E-06
6	7.50E-05		2.60E-06
7	1.50E-04		2.60E-05
8	1.90E-04		3.60E-05
9	9.40E-05		2.00E-05

25000	I Freq		II Freq
1	1.70E-04		2.20E-05
2	1.70E-04		2.70E-05
3	8.50E-05		2.30E-05
4	1.50E-04		3.90E-06
5	1.50E-04		2.20E-06
6	6.70E-05		3.50E-06
7	1.50E-04		2.40E-05
8	1.20E-04		2.40E-05
9	6.00E-05		1.40E-05

Table 3. Data displacements for the third specimen.

## **ACKNOWLEDGEMENTS**

The author is very gratefully to Dr N.S. Ferguson for the guidance, help and support during the course of this work.

The technical staff, Messrs. Jim Baker, Dave Edwards and A. Sanger for the technical support during the experimental investigations.