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NONLINEAR PROPAGATION IN BUBBLY WATER: THEORY AND MEASUREMENT IN THE SURF ZONE

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Abstract: *Nonlinear bubble dynamics are often viewed as the unfortunate consequence of having to use high acoustic pressure amplitudes when the void fraction in the near-surface oceanic bubble layer is great enough to cause severe attenuation (e.g. >50 dB/m). This is seen as unfortunate since existing models for acoustic propagation in bubbly liquids are based on linear bubble dynamics. However, the development of nonlinear models does more than just allow quantification of the errors associated with the use of linear models. It also offers the possibility of propagation modeling and acoustic inversions which appropriately incorporate the bubble nonlinearity. Furthermore, it allows exploration and quantification of possible nonlinear effects which may be exploited. As a result, high acoustic pressure amplitudes may be desirable even in low void fractions, because they offer opportunities to gain information about the bubble cloud from the nonlinearities, and options to exploit the nonlinearities to enhance communication and sonar in bubbly waters. This paper presents a method for calculating the nonlinear acoustic cross-sections, scatter, attenuations and sound speeds from bubble clouds which may be inhomogeneous. The method allows prediction of the time dependency of these quantities, both because the cloud may vary and because the incident acoustic pulse may have finite and arbitrary time history. The method can be readily adapted for bubbles in other environments (e.g. clouds of interacting bubbles, sediments, structures, in vivo, reverberant conditions etc.). The implications for signal propagation and inversion in oceanic bubbly waters are explored.*

Keywords: *Nonlinear propagation, bubbles, surf zone.*

1. NONLINEAR THEORY FOR PROPAGATION THROUGH BUBBLY WATER

In 1989, Commander and Prosperetti [1] produced the most widely-used formulation for predicting the propagation characteristics of an acoustic wave through bubbly liquids. It assumes linear steady-state bubble pulsations in response to a monochromatic driving field. Leighton *et al.* [2,3] developed a theoretical framework into which any single-bubble model could be input, to provide propagation characteristics (*e.g.* attenuation and sound speed) for a polydisperse bubble cloud (which may be inhomogeneous) incorporating whatever features (*e.g.* bubble-bubble interactions) are included in the bubble dynamics model. Because of the inherent nonlinearity, such a model cannot make use of many familiar mathematical tools of linear acoustics, such as Green's functions, complex representation of waves, superposition, addition of solutions, Fourier transforms, small-amplitude expansions *etc*. The crux of this model is in the summation of the volume responses of the individual bubbles to the driving pressures. If the bubble cloud is divided into volume elements, let dP_l be the change in the pressure applied to the *l*th volume element as a result on an incident ultrasonic field. Divide the polydisperse bubble population into radius bins, such that every individual bubble in the *j*th bin is replaced by another bubble which oscillates with radius $R_j(t)$ and volume $V_j(t)$ (about equilibrium values of R_{0_j} and V_{0_j}), such that the total numbers of bubbles N_j and total volume of gas $N_j V_j(t)$ in the bin remain unchanged by the replacement. If the bin width increment is sufficiently small, the time history of every bubble in that bin should closely resemble $V_j(t) = V\left(R_{0_j}, t\right)$ (the sensitivity being greatest around resonance). Hence the total volume of gas in the *l*th volume element of bubbly water is $V_{g_l}(t) = \sum_{j=1}^{J} N_j\left(R_{0_j}, t\right) V_j(t) = V_{c_l} \sum_{j=1}^{J} n_j\left(R_{0_j}, t\right) V_j(t)$, where $n_j\left(R_{0_l}, t\right)$ is the number of bubbles per unit volume of bubbly water within the *j*th bin. From this scheme Leighton *et al.* [3] identified a parameter: $\xi_{c_l} \approx c_w \Big/ \sqrt{1 - \rho_w c_w^2 \sum_{j=1}^{J} n_j\left(R_{0_j}\right)\left(dV_j / dP_l\right)}$. Crucially this ξ_{c_l} provides a *generic framework into which any bubbly dynamics model may be inserted* (giving $dV_j(t)/dP_l(t)$ appropriate to bubbles in free field or reverberation [4], *in vivo* [5], in structures or sediments, or in clouds of interacting bubbles, *etc.* as the chosen model dictates).

To illustrate this, consider a monodisperse bubble population pulsating in the linear steady-state (Fig. 1). If the propagation were linear and lossless, the graphs of applied pressure (P) against bubble volume (V) would take the form of straight lines. The location of the bubble wall would be plotted by the translation of the point of interest up and down these lines at the driving frequency (Fig. 1, top row). Since a positive applied pressure compresses a bubble in the stiffness regime, here $dV/dP<0$ (Fig. 1, top row, right). If, in this linear lossless regime, ξ_{c_l} (above) is seen as equivalent to c_c (the sound speed in the bubbly water), then $c_c < c_w$. However since a π phase change occurs across the resonance, the opposite is true in the inertia-controlled regime (Fig. 1, top row, left). The sound speed in a polydisperse population can be found through addition of such gradients as directed by the formula for ξ_{c_l} (above). If conditions are linear and lossy (Fig. 1, second row), each acoustic cycle in the steady-state must map out a finite area which is equal to the energy loss per cycle from the First Law of Thermodynamics [3]. The characteristic spine (dashed line, Fig. 1, second row) of each loop can, through summation as directed by the formula for ξ_{c_l} , give the sound speed in a polydisperse population. This is effectively equivalent to the approach of Commander and Prosperetti [1], although they characterised the problem using a complex wavenumber, rather than through the locus in *P-V* space. It is interesting to note that Commander and Prosperetti found greatest difficulties with their theory when strong bubble resonances occur: The second row of Fig. 1 illustrates how this will coincide to conditions where not only is the area mapped out very large, but the characteristic gradient of *dV/dP* is very difficult to identify (in keeping with known through-resonance behaviour of sound speed in monodisperse populations). If conditions are nonlinear and lossless, in steady-state the *P-V* graphs will depart from straight-lines (for example because the degree of compression cannot scale indefinitely; Fig. 1, third row). The gradient *dV/dP* varies throughout the acoustic cycle in a manner familiar from nonlinear acoustic propagation, and appropriate summation (as in ξ_{c_l}) can appropriately describe this propagation and the associated waveform distortion. If conditions are nonlinear and lossy, finite areas are mapped out, and whilst the characteristic spines may present significant challenges, nonlinear propagation may again be identified (the example of the right of the bottom row in Fig. 1 illustrates a strong third harmonic, where the steady-state volume pulsation undertakes three cycles for each period of the driving field).

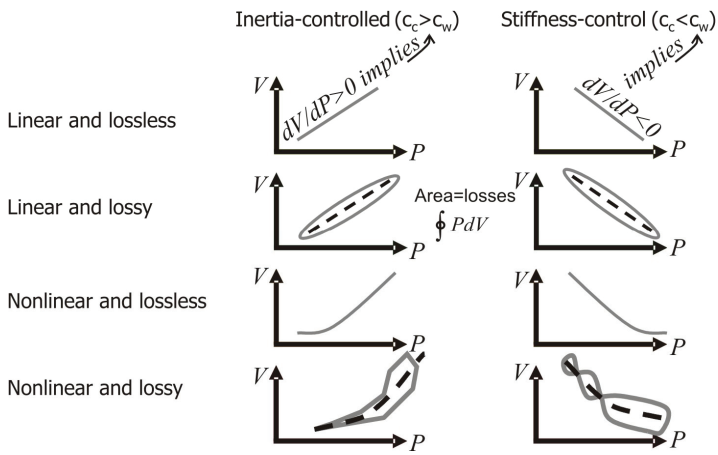

Fig. 1: Schematics of steady-state bubble volume oscillations vs. applied pressure. The left column shows the result for the inertia-controlled regime, and the right column corresponds to the stiffness controlled regime. The four rows correspond to conditions which are (from top downwards): linear and lossless; linear and lossy; nonlinear and lossless; nonlinear and lossy.

2. APPLICATIONS OF THEORY

This theoretical approach to the non-linear acoustic propagation through bubble clouds has been used to:

- Invert measured acoustic propagation characteristics in the surf zone to determine the bubble size distribution [2,3], and compare the results with inversions undertaken using the linear technique of Commander and McDonald [6], which exploits the linear propagation of Commander and Prosperetti [1];
- Predict the amplitude dependency of attenuation in oceanic bubble clouds [3];
- Predict the pulse-length dependency of attenuation in oceanic bubble clouds [3];

- Compare the errors which might accrue through neglect of the nonlinearity of bubble pulsations in high amplitude fields, with those which occur through neglect of bubble-bubble interaction [3];
- Investigate techniques by which sonar echoes from solid objects (such as mines) could be enhanced in comparison to those from bubbles in the vicinity of the mine, and *vice versa* [3,7].

One obvious scenario where insonification amplitudes sufficiently high to excite nonlinearities might be used is when the void fraction is high, generating severe attenuation. Leighton *et al.* [3] showed that, in such circumstances, the errors which accrue from neglect of the nonlinearity, during propagation which is of sufficiently high amplitude to excite it, can be much greater than the errors which result from neglect of bubble-bubble interactions.

However the ability to incorporate nonlinear bubble dynamics into models of acoustic propagation is not restricted to their use in systems where the void fraction is so high as to make high amplitude insonification an unavoidable necessity. With any bubble cloud, nonlinear pulsations can be generated and the results exploited as an additional diagnostic tool. The scatter from the bubbles might be enhanced or suppressed, relative to those from other structures, by exploiting the nonlinearity [3,7-9].

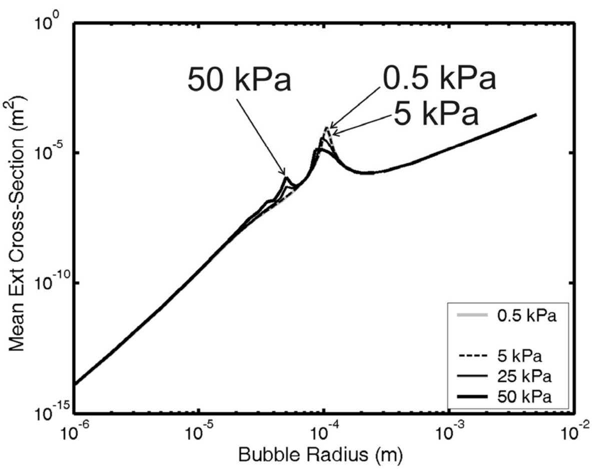

Fig. 2: Acoustic extinction cross-section for a single bubble, as a function of the radius of that bubble, for insonification by a 1 ms duration pulse of 33 kHz centre frequency and 0-peak pulse amplitudes in the range 0.5-50 kPa. The cross-section calculated by the formulation of Leighton et al. [3] varies over time, and the figure plots its mean value. Although the 0.5 kPa and 5 kPa lines differ (particularly close to the fundamental resonance), they are barely distinguishable on this scale. Because the cross-section is not defined during ring-down [3], losses in that period cannot of course be included in this figure. Such ring-down behaviour can however readily be incorporated into calculations of attenuation [3,5]. After Leighton et al. [10].

3. IMPLICATIONS OF THEORY

A not unexpected nonlinear effect in the steady-state would be a decrease in attenuation as the amplitude of the driving pulse increases (equivalent to a decrease in the acoustic absorption cross-section, with commensurate decrease in the acoustic scattering cross-section). This might be expected if the attenuation (and scatter) scale with the amplitude of pulsation of the bubble. That is to say, we are assuming for the moment that, in the bubble population in question, it is the fundamental of the pulsation resonance (rather than, say, a geometrical affect) which is causing attenuation and scatter. As the driving amplitude increases, the amplitude of the bubble pulsation cannot increase proportionately: in the simplest illustration, the displacement on compression cannot of course be greater than the bubble radius. One source of this nonlinearity is the bubble stiffness [7]. Hence if the driving amplitude increases, the bubble response cannot increase proportionately, and we see a decrease in the ratio of the powers scattered and absorbed by the bubble, to the intensity of the incident driving field (the acoustic scatter and absorption cross-sections, respectively). This can be illustrated in Fig. 2, where the peak corresponding to the bubble pulsation fundamental resonance decreases with increasing driving amplitude.

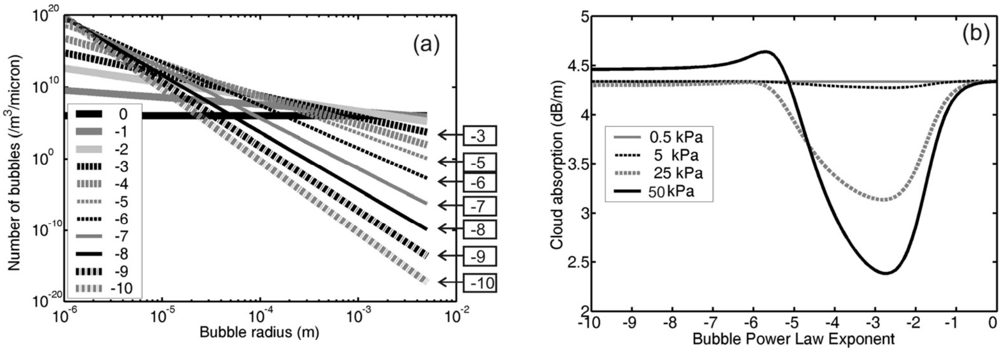

Fig. 3: (a) Various bubble populations, expressed in $n(R_0)$ (bubbles/m^3 per micron bin width in radius) scaled such that the attenuation at low power levels in (b) will be the same for all bubble distributions (b) Cloud absorption for various power law bubble distributions $n(R_0) \propto (R_0)^x$ (for x=0,-1,-2…-10) where the number of bubbles is scaled as described in (a), for insonification as in caption for Fig. 2. Note that the constant offset at high power for -7 to -10 power laws might be due to rounding errors for the bubble response of 1 micron bubble which is amplified by 10^{14} number of bubbles. The difference is not noticeable on the extinction cross-section plot. After Leighton et al. [10].

However the picture is more complicated than the simple correspondence between fundamental resonance pulsation and attenuation/scatter assumed above. It is true that if the bubble population were to be dominated by resonant bubbles, the attenuation would decrease with driving amplitude. However with the decrease of the fundamental resonance peak in the acoustic extinction cross-section, there are corresponding increases in the cross-section corresponding to bubble of particular radii. These are, specifically, about 50, 35 and 25 microns, equivalent to bubbles whose pulsation resonances would be multiples of the insonifying frequency (66, 99, 132 kHz respectively). If the bubble population were to be biased such that there were sufficient numbers of bubbles responding at the second harmonic, the growth in the peak would mean that attenuation could in fact increase with driving amplitude. Fig. 3a illustrates a range of bubble population distributions characterised by a power law, and Fig. 3b plots the predicted attenuation as the amplitude of the driving field increases. Whilst the general trend is that attenuation decreases with driving amplitude (as described above), it is clear that for bubble populations with power law exponents of ~-5.5, increasing the driving amplitude can first decrease and then (at even higher drive amplitudes) increase the attenuation. Therefore by measuring the attenuation at various driving powers, it would for example be possible for a single-frequency source to gain information on the bubble size distribution over an octave or more. At-sea data from a range of sources indicates power laws ranging from -2.0 to -6.4 have been measured (depending on the environmental conditions and bubble size range considered) [9].

4. CONCLUSIONS

Theory for the nonlinear propagation of acoustic waves in bubbly liquids has proved to have numerous uses, from inversion of surf zone data to estimate bubble populations, to predictions of ways of enhancing sonar in bubbly water, to environmental monitoring of bubble populations through exploitation of the nonlinearity as a diagnostic tool. The implications are further explored in another paper in this volume [9].

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