Phys. Med. Biol. 52 (2007) 59–73 doi:10.1088/0031-9155/52/1/005

Empirical angle-dependent Biot and MBA models for acoustic anisotropy in cancellous bone

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Received 24 May 2006, in final form 18 October 2006
Published 6 December 2006
Online at stacks.iop.org/PMB/52/59

Abstract

The Biot and the modified Biot–Attenborough (MBA) models have been found useful to understand ultrasonic wave propagation in cancellous bone. However, neither of the models, as previously applied to cancellous bone, allows for the angular dependence of acoustic properties with direction. The present study aims to account for the acoustic anisotropy in cancellous bone, by introducing empirical angle-dependent input parameters, as defined for a highly oriented structure, into the Biot and the MBA models. The anisotropy of the angle-dependent Biot model is attributed to the variation in the elastic moduli of the skeletal frame with respect to the trabecular alignment. The angle-dependent MBA model employs a simple empirical way of using the parametric fit for the fast and the slow wave speeds. The angle-dependent models were used to predict both the fast and slow wave velocities as a function of propagation angle with respect to the trabecular alignment of cancellous bone. The predictions were compared with those of the Schoenberg model for anisotropy in cancellous bone and *in vitro* experimental measurements from the literature. The angle-dependent models successfully predicted the angular dependence of phase velocity of the fast wave with direction. The root-mean-square errors of the measured versus predicted fast wave velocities were 79.2 m s^{-1} (angle-dependent Biot model) and 36.1 m s^{-1} (angle-dependent MBA model). They also predicted the fact that the slow wave is nearly independent of propagation angle for angles about 50°, but consistently underestimated the slow wave velocity with the root-mean-square errors of 187.2 m s^{-1} (angle-dependent Biot model) and 240.8 m s^{-1} (angle-dependent MBA model). The study indicates that the angle-dependent models reasonably replicate the acoustic anisotropy in cancellous bone.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In recent years, quantitative ultrasound (QUS) technologies have played a growing role in the diagnosis of osteoporosis and are gradually becoming an integrated part of the clinical means of bone fracture risk assessment. It is known that ultrasonic wave propagation is responsive to several properties of bone such as density, material properties and microarchitecture (Chaffai *et al* 2002, Nicholson *et al* 2001, Wear 2003, 2005). Most of the current clinical QUS devices measure the speed of sound (SOS) and the broadband ultrasonic attenuation (BUA) at peripheral skeletal sites which contain cancellous bone, such as the calcaneus and finger phalanges (Njeh *et al* 1999). Cancellous bone is a highly porous, anisotropic medium composed of a cellular network of calcified strands or plates called trabeculae, filled with fatty bone marrow. Accordingly, the ultrasonic parameters are sensitive to the anisotropy of cancellous bone, while the underlying physical mechanisms for this remain only partly understood (Hosokawa and Otani 1998, Hughes *et al* 1999, Wear 2000).

The Biot model (1956a, 1956b, 1962) has been applied to gain an insight into ultrasonic wave propagation in cancellous bone, with varying degrees of success (Fellah *et al* 2004, Haire and Langton 1999, Hosokawa 2005, Hosokawa and Otani 1997, 1998, Hughes *et al* 1999, 2003, Lee and Yoon 2006, McKelvie and Palmer 1991, Mohamed *et al* 2003, Wear *et al* 2005, Williams 1992). Recently, Wear *et al* (2005) have successfully applied the Biot model to predict the dependence of phase velocity on porosity in human calcaneus samples. The Biot model was initially employed to interpret ultrasonic tests in porous rock samples. In 1970, the model was taken by Stoll and Bryan (1970) in a form more suitable for water-saturated sediments. It is now regarded as the theory which is most widely accepted for wave propagation in fluid-saturated porous media. The Biot model predicts the existence of two compressional waves, called fast and slow waves, and one shear wave. The fast wave is a bulk wave where the fluid and the solid move in phase. The slow wave corresponds to a wave where the fluid and the solid move out of phase. Potential shortcomings of the Biot model are that it depends on a large number of input parameters which are not necessarily easily measured, and it predicts absorption due to the viscous losses at internal interfaces only.

The Schoenberg model (Schoenberg 1984) is a much simpler, potential alternative to the Biot model. This was first applied to the study of ultrasonic wave propagation in cancellous bone by Hughes *et al* (1999). The Schoenberg model considers parallel solid–fluid layers periodically alternating as shown in figure 1. In the Schoenberg model, the complex architecture of cancellous bone is modelled as a simple layered structure of alternating parallel bone-marrow plates. The model predicts two compressional waves which are equivalent to the fast and the slow waves of the Biot model. An interesting feature of the Schoenberg model, as opposed to the basic Biot model, is that it is essentially an anisotropic model. Hughes *et al* (1999) have successfully employed the Schoenberg model to predict the angular dependences of phase velocities for the fast and the slow waves in bovine cancellous bone. Padilla and Laugier (2000) have interpreted the experimental observation of the slow wave in cancellous bone as a function of propagation angle using the model. However, the Schoenberg

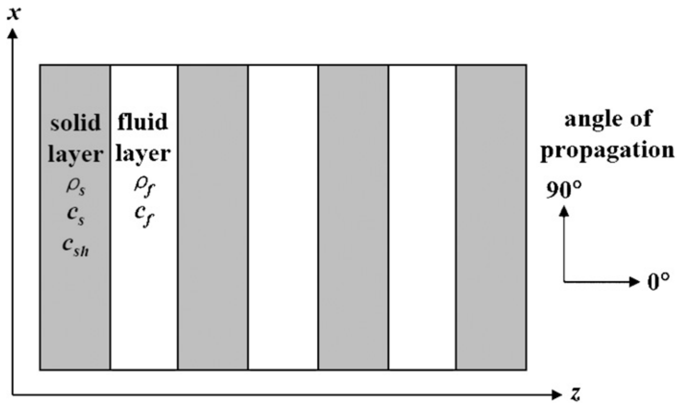

Figure 1. Geometry of the Schoenberg model for acoustic wave propagation in periodically alternating parallel solid–fluid layers.

model takes no account of the effect of viscous absorption of the interstitial fluid on wave propagation.

Roh and Yoon (2004) have proposed a modified Biot–Attenborough (MBA) model for acoustic wave propagation in fluid-saturated porous media with circular cylindrical pores. Lee *et al* (2003) have successfully applied the MBA model to predict the dependences of velocity and attenuation on frequency and porosity in bovine cancellous bone. As previously applied to cancellous bone (Lee *et al* 2003, Lee and Yoon 2006), the MBA model assumes that all of the cylindrical pores in cancellous bone have identical orientation normal to the surface and are parallel to the wave propagation direction. Moreover, consideration is restricted to motion in a single dimension. The MBA model is based on separate treatments of the viscous and the thermal effects of the fluid according to Attenborough (1982, 1983). The Biot model has the merit of including the viscous effect of the interstitial fluid, but it does not take into account the thermal effect. Although Attenborough's theory considers both the viscous and the thermal effects, it does not include the fast wave of the Biot model because it takes the pore frame as a rigid material. In contrast, the MBA model includes the thermal effect specified by an analytic solution and also allows for an elastic solid and fluid medium by means of a parametric fit. Thus, the model is an empirically mixed approach combining the merits of the two approaches of Biot and Attenborough. One drawback of the MBA model is that it relies on the empirical parameters determined from experimental data.

Despite the fact that the Biot and the MBA models have been successfully used in the study of cancellous bone whose trabecular structures are in general anisotropic, they have both been limited by the assumption of isotropy. In contrast, the Schoenberg model is very simple compared with these two models, but it is capable of modelling anisotropic structures. The present study builds on the success of modelling anisotropy with the Schoenberg model, aiming at modifying the Biot and the MBA models in order to account for anisotropic structures of cancellous bone. The modifications were made by introducing empirical angle-dependent input parameters, as defined for a highly oriented structure, into the two models. The angle-dependent models were then used to predict both the fast and the slow wave velocities as a function of propagation angle with respect to the trabecular alignment of anisotropic cancellous bone. The predictions were compared with those of the Schoenberg model and *in vitro* experimental measurements from the literature.

2. Empirical angle-dependent input parameters

2.1. Empirical angle-dependent input parameter of the Biot model

Gibson (1985) has shown that the exponent of the power law for the elastic moduli, n in equation (A.14), varies from 1 to 3, depending on the nature of the skeletal frame and the direction of testing. Testing along the direction of trabecular alignment results in a value of n close to 1. When the structure is more random, i.e., the trabeculae are not aligned in any direction, or when the material is tested in a direction other than that of the major alignment of the trabeculae, n increases to between 2 and 3. The value of n can be determined by curve fitting to the measured phase velocity data as a function of porosity. Williams (1992) has obtained a value of $n = 1.23$ for bovine tibial cancellous bone with an oriented columnar structure (i.e., for propagation parallel to the dominant structural orientation), and a value of $n = 2.35$ for bovine femoral cancellous bone with a random structure (i.e., for propagation perpendicular to the dominant structural orientation). It is conceivable that, making the exponent n be a function of angle (i.e., through the angle-dependent Young's, bulk, and shear moduli of the skeletal frame), the Biot model may have an anisotropic response which models the anisotropic characteristics of ultrasonic wave propagation in cancellous bone. As a first stage in this process, provided that the exponent n is found for a highly oriented structure as a function of propagation angle θ, the Biot model may be modified such that it can model the angle-dependent characteristic of wave propagation through such a structure. If we assume that the exponent n varies with propagation angle θ as $n(\theta) = n_1 \sin^2(\theta) + n_2 \cos^2(\theta)$, then taking $n_1 = 1.23$ and $n_2 = 2.35$ from the work of Williams (1992) gives

$$n(\theta) = 1.23 \sin^2(\theta) + 2.35 \cos^2(\theta), \tag{1}$$

where 0° ($n = 2.35$) and 90° ($n = 1.23$) correspond to the directions of wave propagation perpendicular and parallel to the dominant structural orientation, respectively, as in the Schoenberg model. Equation (1) may enable the Biot model to predict the dependence of phase velocity on the direction of propagation in cancellous bone.

2.2. Empirical angle-dependent input parameter of the MBA model

The phase velocity parameter s_2 in equations (B.7) and (B.9) depends on the direction of loading and varies from 0 to 2 (Lee *et al* 2003, Lee and Yoon 2006). The parameter s_2 has a value close to 0 when the material is loaded along the direction of trabecular alignment and has a value between 1 and 2 in the transverse direction. Thus, in predicting the phase velocity in cancellous bone, it may be regarded that s_2 plays a role equivalent to that of the exponent n of the Biot model. In the same way for determining the exponent n in the Biot model, s_2 can be optimized by curve fitting to the measured phase velocity data as a function of porosity (Lee *et al* 2003). Recently, Lee and Yoon (2006) have obtained a value of $s_2 = 0.8$ for propagation in the direction parallel to the trabeculae of bovine cancellous bone, and a value of $s_2 = 1.6$ in the perpendicular direction. A similar approach that resulted in equation (1) can be taken to obtain an angle-dependent phase velocity parameter $s_2(\theta)$ in the form of

$$s_2(\theta) = 0.5 \sin^2(\theta) + 1.7 \cos^2(\theta), \tag{2}$$

where 0° ($s_2 = 1.7$) and 90° ($s_2 = 0.5$) correspond to the directions of wave propagation perpendicular and parallel to the dominant structural orientation, respectively. The upper and the lower limits of $s_2(\theta)$ at 0° and 90° were determined by curve fitting to the experimental data of phase velocity as a function of propagation angle measured by Hughes *et al* (1999). Making s_2 be a function of propagation angle θ (as given by equation (2)) enables the MBA model to be used for predicting the anisotropic phase velocity in cancellous bone.

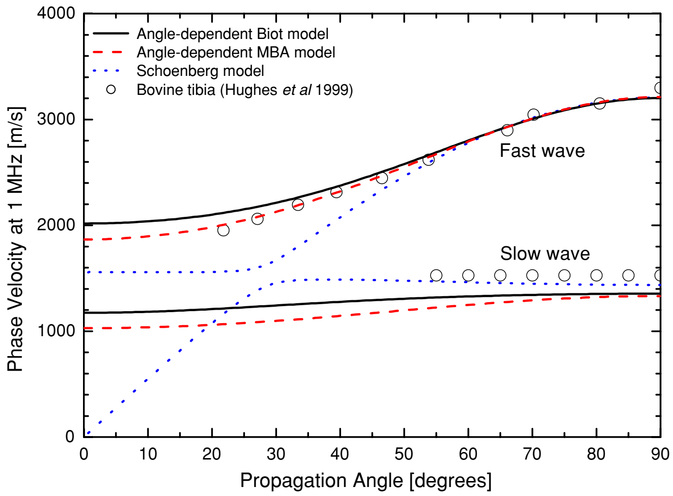

Figure 2. Phase velocities at 1 MHz of the fast and the slow waves as a function of propagation angle predicted by the angle-dependent models and the Schoenberg model for a porosity of 0.65. Experimental measurements at 920 kHz made on one sample of bovine cancellous bone by Hughes *et al* (1999) are also plotted.

3. Results

Figure 2 shows the phase velocities at 1 MHz of the fast and the slow waves as a function of propagation angle predicted by the angle-dependent Biot and MBA models for a porosity of 0.65. The predictions are compared with those of the Schoenberg model for the consistent input parameters. The input parameters of the Biot, the MBA and the Schoenberg models for cancellous bone are summarized in table 1. In the figure, the propagation angle 0° corresponds to the direction perpendicular to the dominant structural orientation, and 90° represents the parallel direction. The solid and the dashed curves stand for the predictions made by the angle-dependent Biot and MBA models, respectively, while the dotted curves represent the Schoenberg model. As seen in the figure, the phase velocity of the slow wave remains almost unchanged. However, the fast wave velocity varies relatively significantly with the angle and has its maximum value at 90°, i.e., for a propagation direction parallel to the dominant structural orientation. For both the fast and the slow waves, the angle-dependent models are monotonically increasing and converge as the angle increases. They are almost in agreement for the fast wave when the angle is higher than 60°. In contrast, the Schoenberg model has a local maximum at about 30° (critical angle) and shows a substantially different behaviour, with the slow wave velocity decreasing to 0 m s^{-1} as the angle is reduced to 0°. It may be worth noting that Padilla and Laugier (2000) have shown that the slow wave would disappear at high angles of incidence to the trabeculae. For the angles larger than the critical angle, the Schoenberg model converges with the angle-dependent models as the angle increases in different ways for the fast and the slow waves; the slow wave velocity data remain higher than the angle-dependent models, while the fast wave velocity data lie under the angle-dependent models.

As figure 2 shows, the three models are also compared with the experimental measurements at 920 kHz made on one sample of bovine cancellous bone by Hughes *et al* (1999), this being the same data as that used to fit the angle-dependent models. It should be

Table 1. Input parameters of the Biot, the MBA and the Schoenberg models for cancellous bone.

Parameter	Biot	MBA	Schoenberg
Density of solid (ρ_s)	1960 kg m^{-3}	1960 kg m^{-3}	1960 kg m^{-3}
Compressional speed of solid (c_s)		3800 m s^{-1}	3800 m s^{-1}
Shear speed of solid (c_{sh})			1800 m s^{-1}
Young's modulus of solid (E_s)	20 GPa		
Poisson's ratio of solid (ν_s)	0.32		
Poisson's ratio of frame (ν_b)	0.32		
Density of fluid (ρ_f)	1000 kg m^{-3}	1000 kg m^{-3}	1000 kg m^{-3}
Compressional speed of fluid (c_f)		1483 m s^{-1}	1483 m s^{-1}
Bulk modulus of fluid (K_f)	2.2 GPa		
Viscosity of fluid (η)	0.001 Pa s		
Kinematic viscosity of fluid (ν)		1×10^{-6} m^2 s^{-1}	
Specific heat ratio of fluid (γ)		1.004	
Prandtl number of fluid (N_{Pr})		7	
Permeability (k)	5×10^{-9} m^2		
Variable (r)	0.25		
Porosity (β)	0.65	0.65	0.65
Pore radius (a)	Equation (A.5)	0.5 mm	
Tortuosity (α)	Equation (A.12)	1	
Exponent (n)	Equation (1)		
Boundary condition parameter (s_1)		1.5	
Phase velocity parameter (s_2)		Equation (2)	

noted that the comparison of the phase velocities at the two different frequencies would not be problematic owing to the relatively nondispersive nature in bovine cancellous bone from 0.5 to 1 MHz (Hosokawa and Otani 1997, Lee *et al* 2003). As may be expected, the fast wave velocities of the angle-dependent models are in a good agreement with the experimental measurements, even if the measured data are only available for a limited range of angles. The root-mean-square errors of the measured versus predicted fast wave velocities were 79.2 m s^{-1} (angle-dependent Biot model), 36.1 m s^{-1} (angle-dependent MBA model) and 247.2 (Schoenberg model). In contrast, the predicted velocities for the slow wave were found to be underestimated compared to the measurement with the root-mean-square errors of 187.2 m s^{-1} (angle-dependent Biot model), 240.8 m s^{-1} (angle-dependent MBA model) and 79.6 (Schoenberg model).

Plotted in figure 3 are the phase velocities at 1 MHz of the fast and the slow waves as a function of porosity predicted by the angle-dependent models and the Schoenberg model for propagation parallel to the dominant structural orientation (propagation angle of 90°). The general trends predicted by all the models are that the phase velocity decreases with porosity for the fast wave, while it increases with porosity for the slow wave. The angle-dependent models are in a good agreement with each other for both the fast and the slow waves. In the fast wave, the angle-dependent models predict the velocity decreased significantly from 3800 to 1483 m s^{-1} with increasing porosity, while the Schoenberg model predicts an almost constant value over a wide range of porosities. In contrast, the dependence of the slow wave velocity on porosity is quite similar for all the three models over a wide range of porosities. It is remarkable that, unlike the other two models, the angle-dependent Biot model predicts a rapid decrease in velocity to 0 m s^{-1} for the porosities higher than about 0.99.

Experimental verification was made by comparing the predictions of the models with measurements made in the direction parallel to the trabeculae of bovine cancellous bone

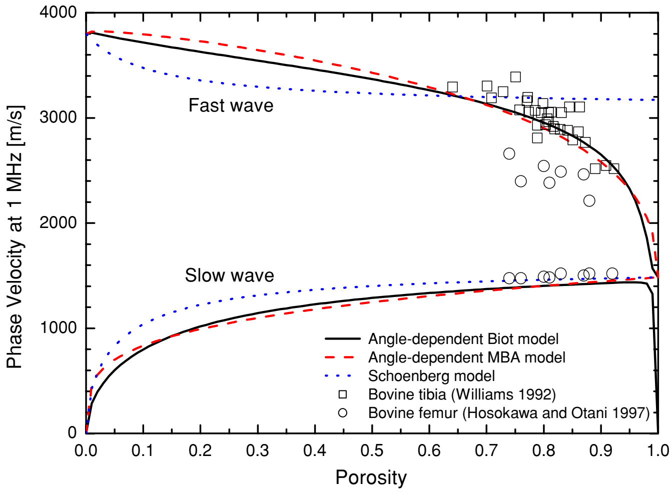

Figure 3. Phase velocities at 1 MHz of the fast and the slow waves as a function of porosity predicted by the angle-dependent models and the Schoenberg model for propagation parallel to the dominant structural orientation (propagation angle of 90°). Experimental measurements made in the direction parallel to the trabeculae of bovine cancellous bone (Hosokawa and Otani 1997, Williams 1992) are also plotted.

(Hosokawa and Otani 1997, Williams 1992). There is a limitation on these data, as measurements are only available for a narrow range of porosities near to physiological porosities. The 32 squares in figure 3 correspond to the bovine tibia samples taken from Williams (1992) and the 8 circles the bovine femur samples from Hosokawa and Otani (1997). The root-mean-square errors of the measured versus predicted fast wave velocities were 234.1 m s^{-1} (angle-dependent Biot model), 231.9 m s^{-1} (angle-dependent MBA model) and 407.6 (Schoenberg model). The fast wave velocities of the angle-dependent models show a reasonable agreement with the experimental data obtained by Williams (1992). However, they are substantially higher than the experimental measurements reported by Hosokawa and Otani (1997). This may be attributed to the fact that the trabecular structure of the bovine femoral samples used by Hosokawa and Otani (1997) was more heterogeneous, with less of an oriented or layered structure as assumed in the models. As mentioned in section 2.1, the value of the exponent n of the Biot model depends on the direction of loading or the direction of propagation, varying from 1 to 3. In fact, Hosokawa and Otani (1997) have obtained $n = 1.46$ in the direction parallel to the trabeculae of bovine femur, while Williams (1992) has found $n = 1.23$ in bovine tibia with an oriented columnar structure. As shown in figure 3, all the three models successfully predicted the porosity dependence of phase velocity of the slow wave with the root-mean-square errors of 90.1 m s^{-1} (angle-dependent Biot model), 85.7 m s^{-1} (angle-dependent MBA model) and 36.5 (Schoenberg model).

Figure 4 displays the phase velocities at 1 MHz of the fast and the slow waves as a function of porosity predicted by the angle-dependent models and the Schoenberg model for propagation perpendicular to the dominant structural orientation (propagation angle of 0°). As seen in the case for the propagation angle of 90° (figure 3), although there are large differences between the models, the general trends on the velocity against porosity are preserved, i.e., the phase velocity decreases with porosity for the fast wave, while it increases with porosity for the slow wave. As seen in figure 4, for the range of physiological porosities, the predictions of the three models for the fast wave velocity converge as the porosity approaches 1. It is noted

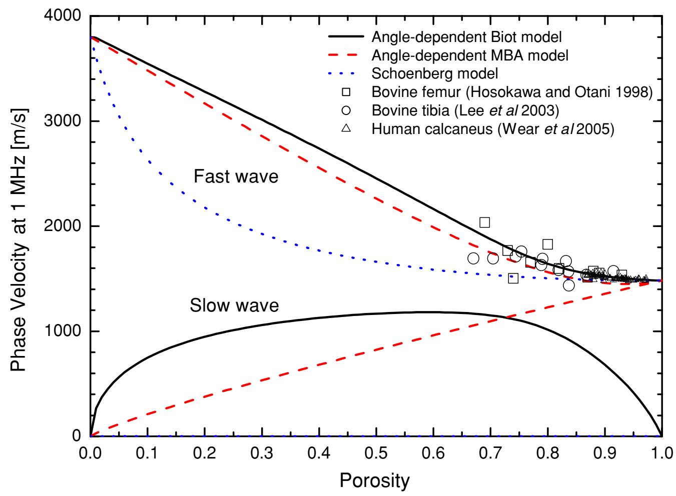

Figure 4. Phase velocities at 1 MHz of the fast and the slow waves as a function of porosity predicted by the angle-dependent models and the Schoenberg model for propagation perpendicular to the dominant structural orientation (propagation angle of 0°). Experimental measurements made in the direction perpendicular to the trabeculae of bovine (Hosokawa and Otani 1998, Lee *et al* 2003) and human (Wear *et al* 2005) cancellous bones are also plotted.

that the predicted values of the velocity are much lower than those for propagation parallel to the dominant structural orientation (see figure 3). Regardless of the propagation angle, the angle-dependent models are capable of predicting velocities of both compressional wave types. However, the Schoenberg model predicts only the fast wave when the propagation direction is perpendicular to the dominant structural orientation. More precisely, as indicated in figure 2, the Schoenberg model yields the phase velocity of the slow wave to be 0 m s^{-1} at 0°. Such directional dependence may be due to inertial coupling between the solid and the fluid in Schoenberg's layered structure, as previously described by Hughes *et al* (1999). For propagation perpendicular to the dominant structural orientation, the inertial coupling is large, and the motions of the solid and the fluid are fully locked together. Only the fast wave propagates because the relative motion associated with the slow wave is impeded.

The predicted velocities of the fast wave are compared with several experimental measurements made in the direction perpendicular to the trabeculae of bovine (Hosokawa and Otani 1998, Lee *et al* 2003) and human (Wear *et al* 2005) cancellous bones. The 10 squares in figure 4 correspond to the bovine femur samples (with porosities from 0.69 to 0.93) taken from Hosokawa and Otani (1998), the 12 circles the bovine tibia samples (from 0.67 to 0.92) from Lee *et al* (2003) and the 53 triangles the human calcaneus samples (from 0.86 to 0.98) from Wear *et al* (2005). It is shown that the angle-dependent models agree reasonably well with *in vitro* measurements for the physiological range of porosities. The root-mean-square errors of the measured versus predicted fast wave velocities were 61.8 m s^{-1} (angle-dependent Biot model), 78.1 m s^{-1} (angle-dependent MBA model) and 99.9 (Schoenberg model).

4. Discussion

In the present study, the parameter values of the Biot model for the solid bone were taken from the work of Williams (1992) because $n(\theta)$ of equation (1) were determined from the values of the exponent n optimized for the direction of propagation by Williams (1992). All parameter values for the pore fluid are equal to those for water because the predictions are compared with the experimental measurements for defatted, water-saturated bone samples from the literature. The three models have consistent input parameters for a fair comparison between them, as shown in table 1. One feature of this range of models has been the numerous parameters used as input or as fitting parameters. For example, the Biot model requires 14 parameters including frequency, of which around the four are not easy to estimate. The sensitivity of each physical parameter of cancellous bone used in the Biot model has been reported by Fellah *et al* (2004). Numerical simulations of transmitted waves in the time domain were run by varying the parameters of cancellous bone and the variation applied to the governing parameters was 20%. They showed the importance of the values of these parameters in fast and slow wave arrival times (speeds) and attenuation, respectively. In contrast, the MBA model uses commonly known input parameters, at the cost of introducing simplifications such as an enforced tortuosity of unity (Lee *et al* 2003). Our future work is to gain an insight into the sensitivity of the predictions of the fast and the slow wave velocities to the values of the physical parameters that are input into the angle-dependent models.

The Biot and the MBA models for acoustic wave propagation in porous media have been found useful to predict wave properties in cancellous bone (Fellah *et al* 2004, Haire and Langton 1999, Hosokawa 2005, Hosokawa and Otani 1997, 1998, Hughes *et al* 1999, 2003, Lee *et al* 2003, Lee and Yoon 2006, McKelvie and Palmer 1991, Mohamed *et al* 2003, Wear *et al* 2005, Williams 1992). However, neither of the models, as previously applied to cancellous bone, allows for the angular dependence of acoustic properties with direction. In the present study, these two models have been modified in an attempt to predict the anisotropic phase velocity as a function of propagation angle with respect to the trabecular alignment of cancellous bone. The anisotropy of the angle-dependent Biot model is attributed to the variation in the elastic moduli of the skeletal frame with respect to the trabecular alignment. The angle-dependent MBA model employs a simple empirical way of using the parametric fit for the fast and the slow wave speeds.

Significant differences can be observed between the angle-dependent models and the Schoenberg model for the fast wave velocity as a function of porosity for propagation parallel (figure 3) and perpendicular (figure 4) to the dominant structural orientation. This discrepancy may be attributable to the fact that they were derived from fundamentally different perspectives. Although the Schoenberg model has been relatively successful in predicting the anisotropic nature of the fast and the slow wave velocities, it clearly gives an oversimplification of the structure of cancellous bone, which lends itself to the modifications of the Biot and the MBA models. Bone consisting of solid, parallel plates exists in very few skeletal sites over areas of less than a few centimetres, and is more usually found having arching plates filled with perforations. Furthermore, the omission of fluid viscosity by Schoenberg's approach prevents it from accounting for viscous effects; similarly thermal effects are neglected. By contrast, the Biot-based models discussed here provide a comprehensive and more realistic description of the porous structure and fluid dynamics, but which is perhaps too detailed to be of practical use. Those different features described may result in substantial discrepancy between the angle-dependent models and the Schoenberg model. However, the models help to provide a greater insight into the nature of propagation in cancellous bone, which can be built on in future research.

Analysis of the angle-dependent Biot model reveals that the angular dependence of wave properties may be attributed to the mechanical anisotropy of the skeletal frame which is based on Gibson's cellular solid model (Gibson and Ashby 1997). Indeed, the introduction of the angle-dependent mechanical properties of the skeletal frame, equations (A.14)–(A.16), leads to the prediction of an anisotropic response of cancellous bone. This empirical approach essentially assumes the elastic isotropy of cancellous bone, whereas cancellous bone is highly anisotropic and inhomogeneous due to the varying composition of its constituents. Recently, an interesting method of data analysis for a set of elastic constant measurements has been reported, dealing with variable composition anisotropic elastic materials, such as cancellous bone, hardwood and softwood, whose elastic coefficients depend upon the particular composition of the material (Cowin and Yang 1997, Yang *et al* 1999). This model permits the identification of the type of elastic symmetry to be accomplished independent of the examination of the variable composition of the material. This method of analysis is a valid approach to the construction of anisotropic stress–strain relations for other anisotropic and compositionally variable materials. In this regard, a truer model of ultrasonic wave propagation in cancellous bone should consider the compositional dependence of the elastic constants.

The following discussion concerns the usefulness of the models and their relevance to clinical analysis *in vivo*. A useful theoretical model may be defined as one having parameters that may be adequately determined, and from which information about bone status may be extracted via ultrasonic measurements. The anisotropy of a porous matrix originates from two elements: the compressibility of the skeletal frame and the motion of the pore fluid. Both elements would be affected by the erosion of the structure. In healthy bone, the two-dimensional structure of the trabecular network is highly anisotropic, forming oriented plates. These gradually erode to a rod-like morphology as the trabeculae disintegrate and the pore spacing increases. Hence, Young's moduli in the orthogonal direction will have significantly different values in healthy bone. The erosion model suggests that their values will become closer as the cancellous structure approaches isotropy in osteoporotic bone. This geometric transition may be incorporated into the structural definition of propagation models to predict wave property variations with pathological changes. This mechanism may assist understanding of the ways in which changes in the cancellous structure of bone with the onset of osteoporosis may affect the wave properties.

The Biot model, without doubt, is known to be a useful tool to enable us to gain an insight into the characteristic features of wave propagation in cancellous bone. However, it requires a great number of input parameters that are not known with high certainty. In addition, the Biot model predicts absorption due to the viscous losses at internal interfaces only, so it consistently underestimates attenuation by several orders of magnitude (Hosokawa and Otani 1997), as it neglects signal losses such as reflection from the flat surfaces of samples, scattering, absorption of fluid and matrix. Furthermore, the attenuation of the fast wave at high porosities is much higher than that of the slow wave in spite of the opposite prediction obtained by the Biot model (Hosokawa and Otani 1997). These differences are indicative of non-Biot dissipative mechanisms. The pore fluid of the Schoenberg model is assumed to be inviscid, and so viscous absorption is not predicted. This makes it difficult to do a direct comparison on attenuation between the angle-dependent models and the Schoenberg model. Therefore, further studies on the influence of structural anisotropy on attenuation in cancellous bone using the angle-dependent models may prove to be enlightening. Furthermore, a future work will also investigate anisotropy in structural parameters, such as the tortuosity in the Biot model, in more depth.

5. Conclusions

In the present study, modifications were made on the Biot and the MBA models in order to account for acoustic anisotropy in cancellous bone, by introducing empirical angle-dependent input parameters, as defined for a highly oriented structure, into the two models. The angle-dependent models were used to predict both the fast and slow wave velocities as a function of propagation angle with respect to the trabecular alignment of cancellous bone. The predictions were compared with those of the Schoenberg model for anisotropy in cancellous bone and *in vitro* experimental measurements from the literature. The angle-dependent models successfully predicted the angular dependence of phase velocity of the fast wave with direction. They also predicted the fact that the slow wave is nearly independent of propagation angle for angles about 50°, but consistently underestimated the slow wave velocity. The study indicates that the angle-dependent models reasonably replicate the acoustic anisotropy in cancellous bone.

Acknowledgments

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) (KRF-2005-214-C00060). The authors thank the referees for their cogent and insightful comments.

Appendix A. Biot model

The characteristic frequency equation of the Biot model is given by (Stoll and Bryan 1970)

$$\begin{vmatrix} Hl^2 - \rho\omega^2 & \rho_f\omega^2 - Cl^2 \\ Cl^2 - \rho_f\omega^2 & (m - \mathrm{i}F\eta/k\omega)\omega^2 - Ml^2 \end{vmatrix} = 0 \tag{A.1}$$

with $l = l_r + \mathrm{i}l_i$, where ω is the angular frequency of the wave, ρ is the total (effective) density of the medium, ρ_f is the density of the pore fluid, m is a parameter equal to $\alpha\rho_f/\beta$ with the tortuosity α and the porosity β which is the ratio of the volume of the pores to the total volume of the medium, η is the viscosity of the fluid, and k is the permeability coefficient. The factor $F(\kappa)$ is

$$F(\kappa) = F_r(\kappa) + \mathrm{i}F_i(\kappa) = \frac{1}{4}\left[\frac{\kappa T(\kappa)}{1 - 2T(\kappa)/\mathrm{i}\kappa}\right], \tag{A.2}$$

where

$$T(\kappa) = \frac{(-\sqrt{-\mathrm{i}})J_1(\kappa\sqrt{-\mathrm{i}})}{J_0(\kappa\sqrt{-\mathrm{i}})}, \tag{A.3}$$

$$\kappa = a(\omega\rho_f/\eta)^{1/2} \tag{A.4}$$

and

$$a = (8\alpha k/\beta)^{1/2}. \tag{A.5}$$

The functions J_0 and J_1 are cylindrical Bessel functions of the zeroth and first order, and a is a parameter depending on the sizes and the shapes of the pores and having the dimension of length. In equation (A.1), the coefficients H, C and M are given by

$$H = \frac{(K_s - K_b)^2}{D - K_b} + K_b + \frac{4}{3}\mu_b, \tag{A.6}$$

$$C = \frac{K_s(K_s - K_b)}{D - K_b} \tag{A.7}$$

and

$$M = \frac{K_s^2}{D - K_b}, \tag{A.8}$$

where

$$D = K_s\left[1 + \beta\left(\frac{K_s}{K_f} - 1\right)\right]. \tag{A.9}$$

In equations (A.6)–(A.9), K_f is the bulk modulus of the fluid, K_s is the bulk modulus of the solid material comprising the skeletal frame, K_b is the complex bulk modulus of the skeletal frame and μ_b is the complex shear modulus of the skeletal frame. The dispersion relation of equation (A.1) yields the wavenumbers, l_{fast} and l_{slow}, for the fast and the slow compressional waves in a porous medium which are expressed as

$$\frac{l_{\text{fast}}}{\omega} = \left\{\frac{(Hm' + M\rho - 2C\rho_f) - \left[(Hm' + M\rho - 2C\rho_f)^2 - 4(HM - C^2)\left(\rho m' - \rho_f^2\right)\right]^{1/2}}{2(HM - C^2)}\right\}^{1/2} \tag{A.10}$$

and

$$\frac{l_{\text{slow}}}{\omega} = \left\{\frac{(Hm' + M\rho - 2C\rho_f) + \left[(Hm' + M\rho - 2C\rho_f)^2 - 4(HM - C^2)\left(\rho m' - \rho_f^2\right)\right]^{1/2}}{2(HM - C^2)}\right\}^{1/2}, \tag{A.11}$$

where m' is equal to $m - \mathrm{i}F\eta/k\omega$. From equations (A.10) and (A.11), the phase velocities of the fast and the slow waves can be found from ω/l_r as a function of frequency. The tortuosity α is defined as the ratio of the length of the true path of flow for the pore fluid to the shortest distance between the inflow and the outflow and is determined by (Berryman 1980)

$$\alpha = 1 - r(1 - 1/\beta), \tag{A.12}$$

where r is a variable calculated from a microscopic model of a frame moving in the fluid. The intrinsic bulk modulus K_s of the solid bone material comprising the skeletal frame is calculated from Young's modulus E_s of the solid bone by assuming the material to be isotropic (Katz and Meunier 1987, Lang 1969):

$$K_s = \frac{E_s}{3(1 - 2\nu_s)}, \tag{A.13}$$

where ν_s is Poisson's ratio of the solid bone. Once the intrinsic material properties have been measured or calculated, it is possible to calculate the parameter values for the skeletal frame by using a power law relating Young's modulus E_b (of the skeletal frame) to E_s (of the solid bone) through the volume fraction $(1 - \beta)$ of bone (Ashman and Rho 1988, Gibson 1985, Gibson and Ashby 1997):

$$E_b = E_s(1 - \beta)^n, \tag{A.14}$$

$$K_b = \frac{E_b}{3(1 - 2\nu_b)} \tag{A.15}$$

and

$$\mu_b = \frac{E_b}{2(1 + \nu_b)}, \tag{A.16}$$

where K_b and μ_b are the bulk and the shear moduli of the skeletal frame, respectively. The power index n is a variable depending on the alignment of the structure (Gibson 1985), and ν_b is Poisson's ratio of the skeletal frame.

Appendix B. MBA model

In the MBA model, for simplicity, acoustic wave propagation through a circular cylindrical pore is assumed to be one-dimensional along the axis of the pore (Lee *et al* 2003, Roh and Yoon 2004). Care is taken to treat the boundary condition at a nonrigid pore frame, which should be different from that at a rigid frame. The continuity equation for a wave propagating through a pore filled with fluid is given by

$$-\rho_f \frac{\partial \langle v \rangle}{\partial x} = \frac{\partial \rho}{\partial t}, \tag{B.1}$$

where ρ_f is the density of the fluid and $\langle v \rangle$ is the average particle velocity over the cross section of the pore for propagation in the x direction of the central axis of the pore. The equation of motion in terms of the complex (or frequency-dependent) density $\rho_c(\omega)$ of the pore fluid is then expressed as

$$\frac{\partial p}{\partial x} = \rho_c(\omega) \frac{\partial \langle v \rangle}{\partial t}, \tag{B.2}$$

where p is the acoustic pressure and ω is the angular frequency of the wave. The complex density $\rho_c(\omega)$ is written as

$$\rho_c(\omega) = \rho_f [1 - 2(\lambda\sqrt{\mathrm{i}})^{-1} T'(\lambda\sqrt{\mathrm{i}})]^{-1}, \tag{B.3}$$

where

$$T'(\lambda\sqrt{\mathrm{i}}) = \frac{J_1(\lambda\sqrt{\mathrm{i}})}{J_0(\lambda\sqrt{\mathrm{i}})}. \tag{B.4}$$

The dimensionless parameter $\lambda(\omega)$ related to the thickness of the viscous boundary layer at the pore wall is given by

$$\lambda(\omega) = a s_1 (\omega/\nu)^{1/2}, \tag{B.5}$$

where a is the radius of the circular cylindrical pore, ν is the kinematic viscosity of the pore fluid and s_1 is the boundary condition parameter representing the rigidity of the pore frame. Assuming that the pore fluid is a nonviscous conducting fluid, the heat flow may only occur in the transverse direction, and the acoustic pressure p will be uniform over the cross section of the pore. The thermal equations in this case may be arranged so as to give the complex compressibility $C_c(\omega)$ of the pore fluid:

$$C_c(\omega) = \frac{1}{\rho_f} \frac{\mathrm{d}\rho}{\mathrm{d}p} = \left(\gamma \rho_f c_f^2\right)^{-1} \left[1 + 2(\gamma - 1)\left(N_{\mathrm{Pr}}^{1/2} \lambda\sqrt{\mathrm{i}}\right)^{-1} T'\left(N_{\mathrm{Pr}}^{1/2} \lambda\sqrt{\mathrm{i}}\right)\right], \tag{B.6}$$

where c_f, γ and N_{Pr} are the equilibrium compressional speed, the specific heat ratio and the Prandtl number of the pore fluid, respectively. The wavenumber l_{fast} for the fast compressional wave in a nonrigid porous medium with bulk cylindrical pores can then be expressed by using the empirical formula

$$l_{\mathrm{fast}} = \alpha \left[\frac{l_c^2 l_s^2}{(1-\beta)^{s_2} l_c^2 + \beta^{s_2} l_s^2} \right]^{1/2}, \tag{B.7}$$

where α is the tortuosity, β is the porosity, $l_s = \omega / c_s$ is the wavenumber of the pore frame and s_2 is the phase velocity parameter representing the form of the phase velocity curve as a function of porosity. The complex wavenumber $l_c(\omega)$ of the pore fluid is given by

$$l_c(\omega) = \omega [C_c(\omega) \rho_c(\omega)]^{1/2}. \tag{B.8}$$

In a similar way, an empirical expression for the slow wave can be obtained using the value of l_c from the MBA model. For a low porosity material, the Biot model shows that the slow wave velocity tends to zero. Hence, the variation of the slow wave velocity can be fitted by an equation of the form:

$$l_{\text{slow}} = \alpha \left[\frac{l_c^2 l_g^2}{(1-\beta)^{s_2} l_c^2 + \beta^{s_2} l_g^2} \right]^{1/2}, \tag{B.9}$$

where l_{slow} is the wavenumber for the slow compressional wave and $l_g = \omega/c_g$ is the wavenumber of a hypothetical fluid with a very low wave speed. The phase velocities of the fast and the slow waves can then be obtained from the real components of l_{fast} and l_{slow} as a function of frequency.

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