Institute of Sound and Vibration Research

Pavement Analysis Part III: Rayleigh Waves in Layered Media

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UNIVERSITY OF SOUTHAMPTON

INSTITUTE OF SOUND AND VIBRATION RESEARCH

DYNAMICS GROUP

Pavement Analysis
Part III: Rayleigh Waves in Layered Media[1]

by

W.H. Cogill

ISVR Technical Memorandum No. 850

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Authorized for issue by
Dr. M.J. Brennan
Group Chairman

1 *Part I of this series is "Single-layered inverse: first order approximation", ISVR Technical Memorandum No. 833, January 1999.*

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Chapter 1

Experimental Results

This chapter includes some experimental results and their possible interpretation.

1.1 Rayleigh waves on a table top

Accelerometers were placed on the surface of a table. The thickness of the surface of the table was 25mm. Waves of the Rayleigh type were generated in the surface of the table. The accelerometers were placed at a spacing of 32mm, in a radial line from the source of the waves. The difference in the phase of the wave arriving at the two accelerometers was measured. A phase difference of 2π radians corresponds with a wavelength of 32mm and a wavenumber, a reciprocal of the wavelength, of $1/0.032 = 31\ m^{-1}$. The results are shown in Figure 1.1. This figure shows the time series measurement of the wave received at the two accelerometers. Samples were acquired at a rate of 11025 pairs of samples per second. The duration of the time series shown is approximately 3ms.

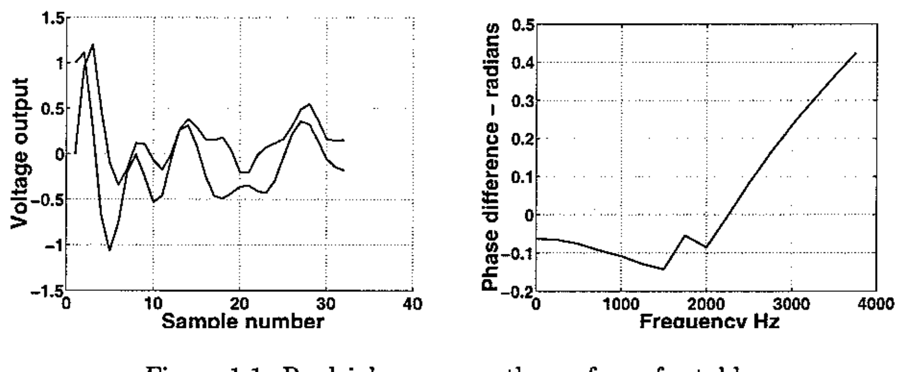

Figure 1.1: Rayleigh waves on the surface of a table.

The axis of the phase difference is 0.7radians $/(2\pi) * 31\ m^{-1} = 3.4\ m^{-1}$ in length. Thus the group velocity between 2000Hz and 4000Hz is $2000/3.4 \simeq 600\ m/s$.

Surface 0.2m	β_1	1320 m/s
Base 0.15m	β_2	1415 m/s
Subgrade	β_3	430 m/s

Table 1.1: LCPC Data: Estimated velocities within the structure.

1.2 Route A6.

The phase velocity of waves of the Rayleigh type was measured on Route A6-Auxerre-Lyon, France [1]. The results of the measurements are shown in Figure 1.2. This figure shows the results of the measurements plotted in two ways. The first shows the phase velocity plotted against the wavelength in metres. The second shows the reciprocal of the wavelength plotted against the frequency in Herz.

The pavement consisted of a surface course 0.2m in thickness, overlying a gravel-cement basecourse 0.15m in thickness. The group velocities $\frac{\delta \omega}{\delta k}$ were read from the plot of the reciprocal of the wavelength against the frequency as shown in Table 1.1.

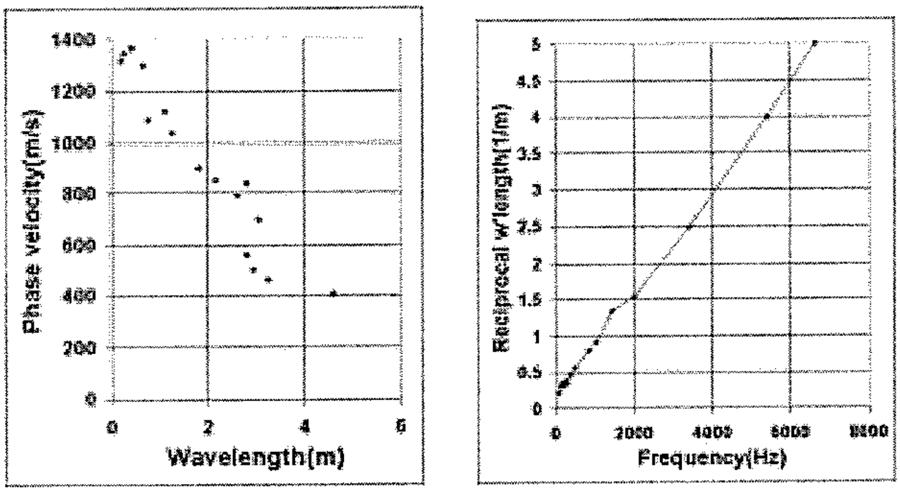

Figure 1.2: Rayleigh waves on route A6-Auxerre-Lyon.

We attempt to match the data in Figure 1.2. We do this by searching for an ideal layered system which yields the same results as those shown in Figure 1.2. The results are shown in Figure 1.3. In Figure 1.3, the first sub-figure shows the phase velocity plotted against the wavelength. The second sub-figure shows the reciprocal of the wavelength plotted against the frequency. All quantities are in relative units. The velocity in the surface layer β_1 is unity, and the thickness of the surface layer is unity.

The theoretical match with the experimental results was obtained with the aid of the programme RAY.FOR (section A.4.2).

The programme RAY.FOR is used in order to calculate the expected response of a pavement surface to vibratory loading. The programme follows the mathematical expressions given by Thrower [12]. The data file for RAY.FOR is

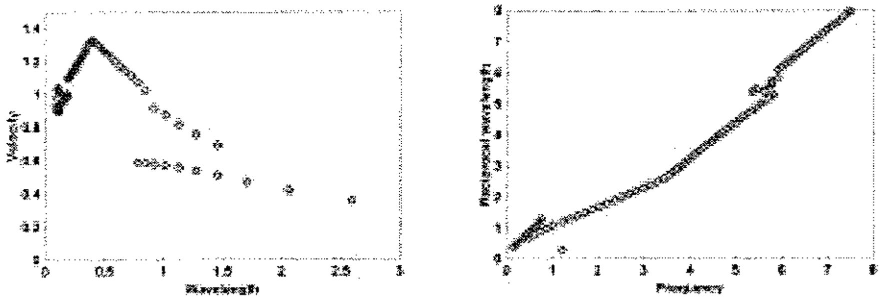

Figure 1.3: Rayleigh waves on route A6-Auxerre-Lyon. Theoretical match.

RAY.DAT which is prepared with the aid of the programme PREP.FOR (section A.4.1).[1]

The data file PREP.DAT used in the programme PREP.FOR was as follows.

```
LCPC DATA E1=11200,E2=34000,E3=1340;H1=0.1,H2=0.2,NU1=NU3=0.25,NU2=0.4
3 11200. .25 34000. .40 1340. .25 0.0
2. 2. 2.
.10 .20
(3.5, .00) (.40, .00) (.10, .01) .02 .10
```

The parameters used in order to obtain the match were estimated as follows. The measured values of the thicknesses of the layers were used as data. The values of Poisson's ratio were guessed as shown in the header line of the data. The densities in the three media were assumed to be equal. The velocities in each of the three media were estimated from the measured values of the reciprocal of the wavelength plotted against the frequency. The match is intended to be read by multiplying the calculated velocities and frequencies by one thousand. The corresponding values of the Young's moduli are as given in the header line of the data file, where the values shown are expressed in kPa.

Successive branches of the response can be computed by selecting suitable starting points for the programme RAY.FOR. This method was used to compute the multiple branches shown in Figure 1.3.

1.3 Hurn Airport

Data acquired from a taxiway at Hurn Airport, Dorset, England, were analysed. The aim was to

- Estimate the thickness of the surface layer
- Confirm the estimate by finding an ideal layered system which yields the observed measurements. This was done with the aid of the forward programme SON.FOR (section A.2).

The figure 1.4 shows the results of the measurements. The figure shows also the results of calculations made with the aid of the programme SON.FOR (section A.2) . The measurements match a structure having a surface layer 0.64 metres in thickness.

[1] The format of the datafile PREP.DAT is given in the appendix A.4.1

1.3.1 Procedure followed

The results shown in Figure 1.4 were obtained as follows. The inverse programme APP_S.FOR(section A.5.1), was used to estimate the thickness of the surface layer. Then the program SON.FOR (section A.2) was used to confirm the thickness of the surface layer, by performing forward calculations. The results of the forward calculations are shown in Figure 1.4. The results for three thicknesses are shown. The thicknesses are 320mm, 640mm and 1280mm.

1.3.2 The inverse programme APP_S.FOR

The inverse programme is based on the solution by Lee for the structure consisting of a surface layer overlying a semi-infinite medium. Lee's solution is represented by a 2 × 2 determinant, in which the transcendental functions are approximated by their first terms only. The limiting velocities are read from the results of the experimental measurements. The velocities are supplied as data to the inverse program. The programme also requires values of the reciprocal wavelength and the frequency, which indicate the curvature of the plot of the reciprocal of the wavelength against the frequency. This final pair of data is subjective, and is inevitably a source of error. The inverse programme APP_S.FOR (section A.5.1)involves a first-order approximation to the frequency equation.

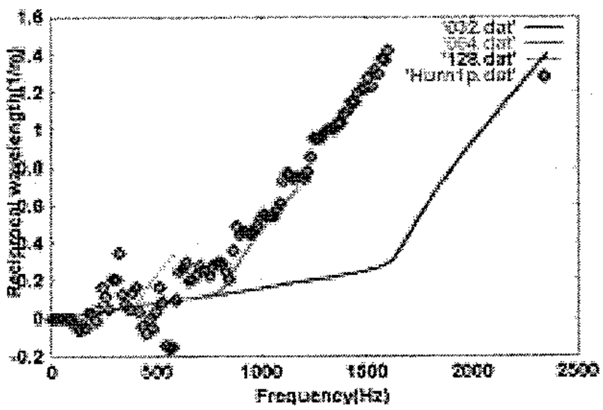

Figure 1.4: Hurn Airport taxiway

The theoretical match in Figure 1.4 was obtained with the aid of the programme APP_S.FOR. The estimate of the thickness of the surface layer was improved with the aid of the programme SON.FOR. The thickness of the surface layer could be measured independently by excavation or by interpreting the measurements of surface deflections in the neighbourhood of a known load.

1.4 Newbury runway

Measurements were performed on a disused airfield runway at Greenham Common, Newbury, Berkshire. The runway had been used by heavy aircraft, and

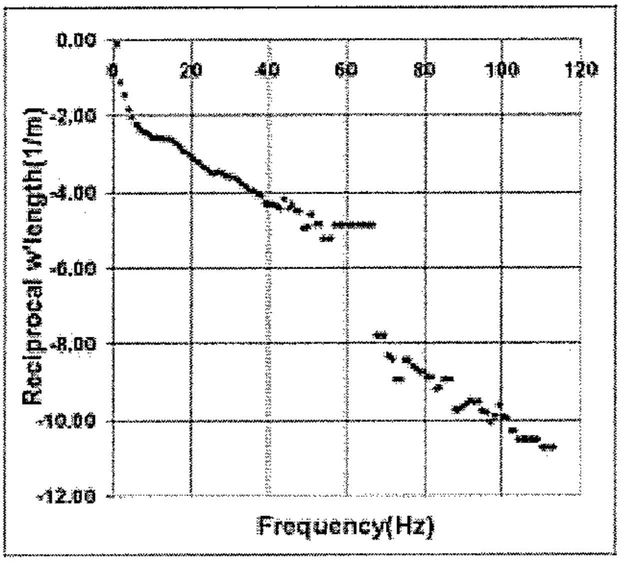

Figure 1.5: Greenham Common, Newbury: Runway showing cracks on the surface

260mm concrete
110mm lean concrete
200mm sand/cement/rock

Table 1.2: Greenham Common Pavement: Structural section.

showed extensive cracking. The results of the measurements are shown in Figure 1.5. This figure shows the reciprocal of the wavelength plotted against the frequency. The spacing between the accelerometers was 0.148m.

At frequencies below 10Hz, the phase velocity is approximately 2.5 metres per second. At frequencies from 20Hz to 60Hz, the group velocity is approximately 20 metres per second. These velocities are consistent with motion being transmitted between the loose particles of which the pavement is composed. The shape of the plot of the results suggests that the material in the surface layer is stiffer than that in the underlying medium. At frequencies between 80Hz and 115Hz, the group velocity is $\frac{(115-80)}{(11-9)}$ [2] or approximately 18 metres per second.

The programme APP_H.FOR (section A.5.2) yields a value of the thickness of 0.002m. The input data were as follows: $\beta_1 = 20, \beta_2 = 2.5$ Reciprocal wavelength 3.5, frequency 18. The estimate of the thickness of the surface layer is clearly incorrect.

The structure of the pavement was as shown in Table 1.2.

The velocity of waves of the Rayleigh type in concrete is normally 1000m/s or higher. The observed velocity is too low to correspond with a velocity in concrete. The low velocity is due to the cracks in the surface.

[2] The dimensions of this fraction are Hz/m^{-1}

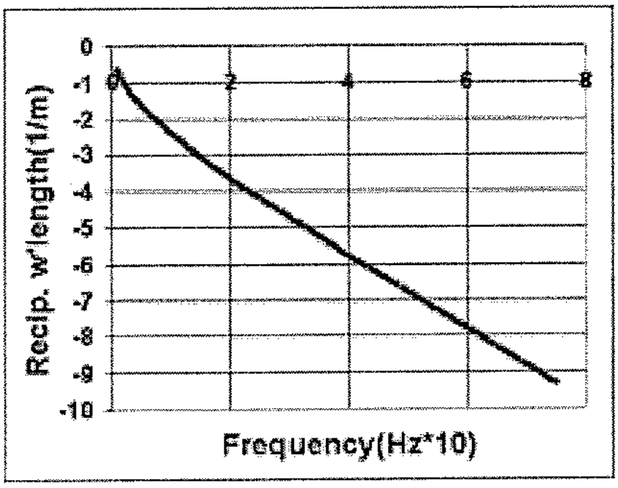

Figure 1.6: Greenham Common, Newbury: Response of idealized system.

NEWBURY 148mm spacing. Data for PREP.DAT
2 30000. .25 30. .25 0.0
2. 2.
0.6
(32., .00) (.1, .00) (.01, .01) .01 .01

Table 1.3: Newbury 148mm spacing. Data for PREP.DAT.

An attempt was made to determine the idealised system yielding the response observed in Figure 1.5. Figure 1.6 was obtained with the aid of the programs PREP.FOR and RAY.FOR (section A.4.2). The data for the programme PREP.FOR were as shown in Table 1.3

The results of the match suggest that the pavement was composed of a surface layer having a thickness of 0.6m. The Young's modulus of the material in the surface layer was one thousand times that of the material in the underlying medium.

1.5 Airport pavement

Measurements of the phase velocity of waves of the Rayleigh type were performed on an airport runway pavement. The pavement was at HMS Daedalus, Solent, Hampshire, England. The results are shown in Figure 1.7. This figure shows the calculated value of the reciprocal of the wavelength plotted against the frequency.

The group velocities were read from Figure 1.7 as shown in Table 1.4:

The group velocities increase as the frequency increases. The lowest group velocity corresponds with the lowest frequency and with the lowest value of the reciprocal wavelength. This velocity represents the properties of the underlying medium. The highest group velocity corresponds with the properties of the

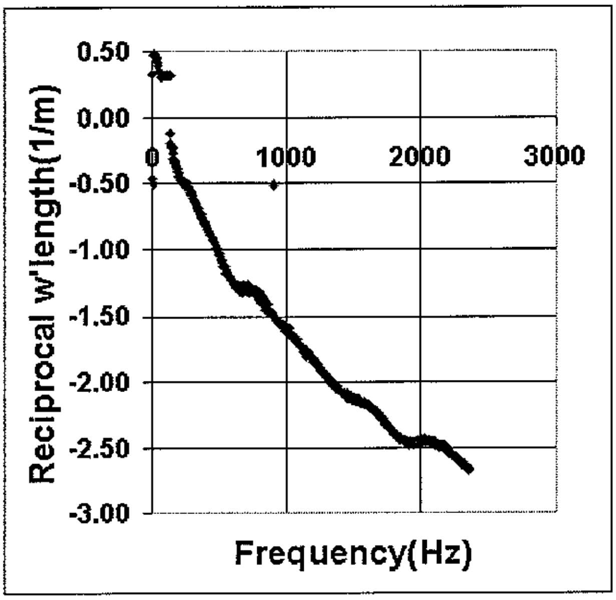

Figure 1.7: HMS Daedalus, Solent: Runway in good condition.

Underlying medium	220m/s
Lower layer	600m/s
Surface layer	1000m/s

Table 1.4: HMS Daedalus: Estimated group velocities of waves of the Rayleigh type within the pavement structure.

β_1	β_2	Recip w'lngth(m^{-1})	Frequency(Hz)	Thickness (m)
1000	150	1.0	300	0.064
1000	300	1.2	800	0.029

Table 1.5: HMS Daedalus: Estimated thickness of surface layer.

surface layer.

The programme APP_H.FOR (section A.5.2)yields the results shown in Table 1.5.

The estimated thicknesses are too small by a factor of ten.

1.6 Parking area

Measurements were performed at a parking area at 50, University Road, Southampton, England. The results are shown in Figure 1.8. This figure shows the reciprocal wavelength plotted against the frequency.

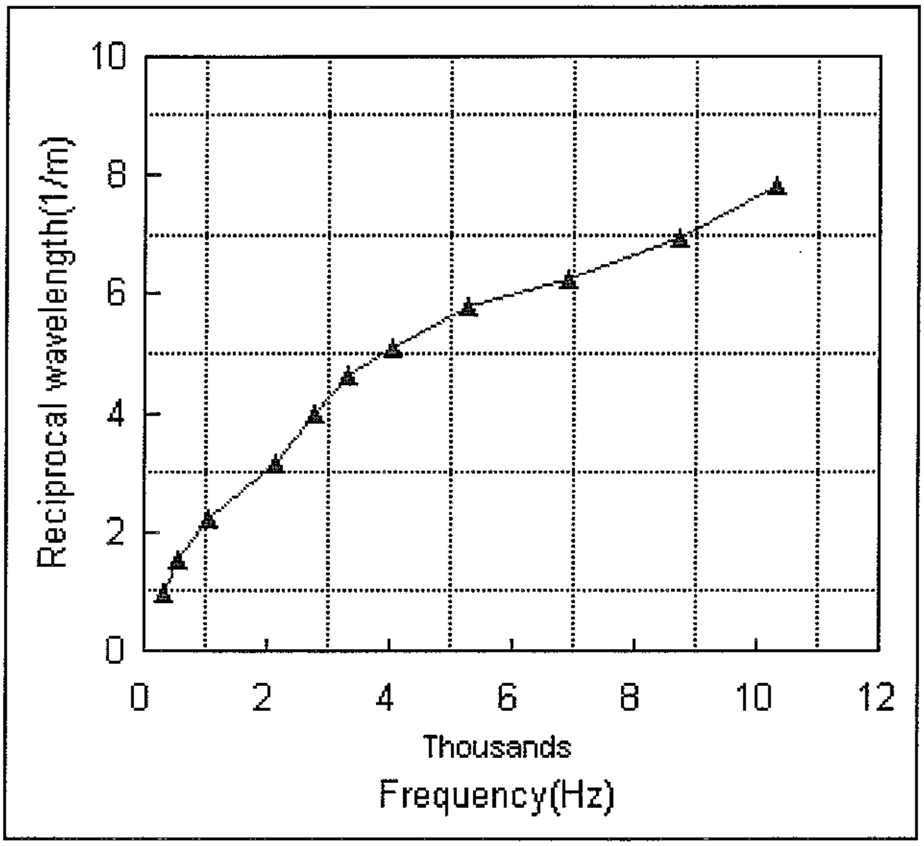

Figure 1.8: Parking area. 1998

The structure of the pavement was as shown in Table 1.6.

25mm bitumen macadam
60mm dense limestone bitumen macadam
150mm limestone quarry scalpings
Prepared subgrade

Table 1.6: Parking area: Structural section of pavement.

Freq(Hz)	Phase (degrees)	λ (m)	$\frac{1}{\lambda}$ '(1/m)	Velocity (m/s)
288	-110	1.05	0.95	302
544	-176	0.65	1.53	356
1024	-255	0.45	2.21	463
2112	-360	0.32	3.13	676
2784	-460	0.25	3.99	697
3328	-535	0.22	4.64	717
4032	-587	0.20	5.10	791
5248	-664	0.17	5.76	910
6880	-720	0.16	6.25	1101
8736	-797	0.14	6.92	1263
10304	-900	0.13	7.81	1319

Table 1.7: Parking area: Numerical results of field measurements.

The group velocities were read from Figure 1.8 as shown in Table 1.8

Table 1.9 shows the estimated thickness of the surface layer obtained with the aid of the programme APP_H.FOR(section A.5.2).

1.7 Conclusions

It appears that the approximate inverse programs APP_H.FOR and APP_S.FOR yield useful results only when the contrast between the surface and underlying media is great. Examples are of an airport runway or freeway pavement. See Figure 1.4 and Figure B.1. This may indicate that an approximation of a higher order than the first is required.

The experimental results (Figures B.1 to B.8) indicate that the model developed corresponds with only very simple structures. It appears that additional parameters are required to represent more closely the results obtained from common structures.

Underlying medium	250m/s
Lower layer	1000m/s
Surface layer	2500m/s

Table 1.8: Parking area: Estimated group velocities of waves of the Rayleigh type within the pavement structure.

$\beta_1(m/s)$	$\beta_2(m/s)$	$\frac{1}{\lambda}$ (m^{-1})	Frequency(Hz)	Thickness(m)	Comment
2500	250	1.00	300	0.064	Surface layer only
1000	300	1.2	800	0.029	Combining surface and lower layers

Table 1.9: Parking area: Estimated thickness of surface layer using APP_H.FOR

The program SIN13.FOR (Section A.3) yields output corresponding to a solution of the equation described previously [4]. However the output does not correspond with a true zero of this equation. Instead the output corresponds with a value of the determinant which is small compared with the initial trial value. However the output obtained agrees with that obtained by means of the program RAY.FOR (Section A.4.2), for which the solution criterion is more rigorous.

Chapter 2

Computations

The programmes SON.FOR (section A.2), SIN13.FOR (section A.3) and RAY.FOR (section A.4.2) have been written in order to calculate the expected response of a layered system to the passage of waves of the Rayleigh type. SON.FOR and SIN13.FOR are intended for a system composed of a single layer overlying a semi-infinite medium; RAY.FOR is intended for a similar system but having more than one layer.

SON.FOR and SIN13.FOR were written in such a manner that all data and output are non-dimensional; RAY.FOR is written to permit some dimensionality to be incorporated.

SON.FOR is written for a system in which the propagation velocities increase with the depth below the surface. SIN13.FOR is intended for the system in which the velocities decrease with the depth below the surface. In RAY.FOR, the component media may be interspersed in any order of stiffness.

The solutions provided by the programmes SON.FOR, SIN13.FOR and RAY.FOR are non-unique. This is true particularly of SON.FOR and of RAY.FOR, which utilize periodic functions in developing the frequency equation. The solution corresponding with the lowest frequency is selected. This solution represents the most probable response of a real structure.

2.1 Scaling

The programmes SON.FOR and SIN13.FOR utilize non-dimensional quantities for both input and output. The results require scaling in order to match the computations with the measurements made on real structures.

The unit of length in SON.FOR and SIN13.FOR is h, the thickness of the layer.

2.1.1 Data Input

The data input to both of the programmes is as follows:

(a) V, the dimensionless trial wavelength. It is measured in units of h/π.

(b) $CB1$, the dimensionless velocity. It is measured in units of β_1.

$$V = \frac{\lambda}{\pi h},\ CB1 = \frac{c}{\beta_1}$$

The calculated dimensionless frequency is therefore

$$\frac{CB1}{V} = \frac{c}{\beta_1}.\frac{\pi h}{\lambda}$$

and the dimensionless reciprocal wavelength is $\frac{1}{V} = \frac{\pi h}{\lambda}$

2.1.1.1 Ratio of densities

Densities do not vary significantly within a normal earthen structure, in comparison with the variation of the velocities of propagation of sound waves.

The ratio of densities $\frac{\rho_2}{\rho_1} = \frac{\beta_1^2}{\beta_2^2}.\frac{\mu_2}{\mu_1}$

i.e. the density is directly proportional to the Lamé shear constant μ, and inversely proportional to the square of the velocity of shear waves β in the material.

2.1.1.2 Example, from 240a2.wk4

Measurements were performed on the surface of a parking area.[1] The results of the measurements are shown in Figure 2.1. This figure shows the reciprocal of the wavelength plotted against the frequency.

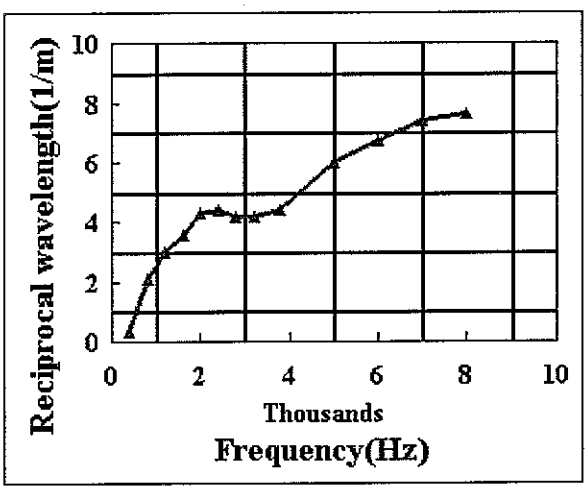

Figure 2.1: Results for analysis obtained from 240a2

$$\beta_1 = 2000m/s,\ \beta_2 = 300m/s,\ h = 0.255m.$$

Unit frequency is $\frac{c}{\lambda} = \frac{\beta_1}{\pi h} = \frac{2000}{0.255\pi} = 2,496Hz.$

Unit reciprocal wavelength is $\frac{1}{\lambda} = \frac{1}{\pi h} = \frac{1}{0.255\pi} = 1.248m^{-1}$

[1] At 50, University Road, The University, Highfield, Southampton, England.

2.1.2 Matching results

We thus obtain a procedure for matching experimental results.

2.1.2.1 Plot the experimental points as wavelength versus velocity;

2.1.2.2 Estimate the ratio of the greatest to the smallest velocities;

2.1.2.3 Write the data for SIN13.FOR(see section A.3) using the ratio of the greatest to the smallest velocities; estimate $\frac{\mu_2}{\mu_1}$ as the reciprocal square of the velocity ratio;

2.1.2.4 Run and plot SIN13.FOR;

The programme SIN13.FOR is an approximation only.

2.1.2.5 Prepare data for RAY.FOR, using PREP.FOR(see section A.4.1). Supply moduli in relative values only, based on the square of the ratio of the maximum to minimum velocities;

2.1.2.6 Run and plot PREP.FOR and RAY.FOR;

2.1.2.7 Go to section 2.1.2.3.

2.1.3 Example of matching experimental results

Figure 2.2 shows the results from Figure 2.1 plotted in an alternative manner. The phase velocity is plotted against the wavelength. The experimental points are denoted by Series1. The purpose of this is to facilitate matching this set of measurements with those expected from an ideal structure. The calculated results obtained from SIN13.FOR and from RAY.FOR are shown as Series2 and Series3 respectively. The behaviour of the structure does not correspond with that expected from an ideal structure. The match gives an indication of the properties of the associated ideal structure.

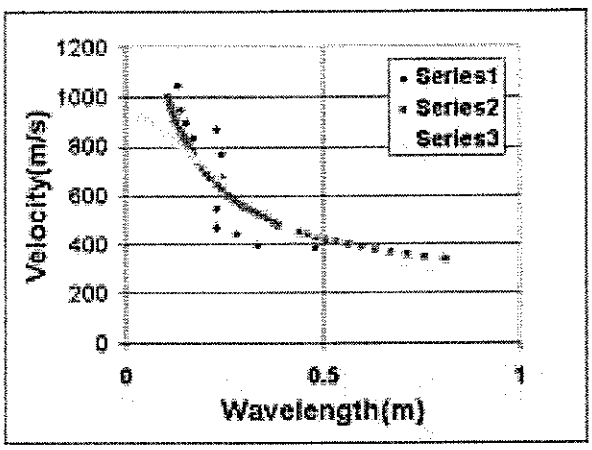

Figure 2.2: Results from 240a2

The following data file was used as input to SIN13.FOR to obtain the indicated output.

```
0.4 8. 0.4 0.01
0.999 0.98 0.97 0.96 0.95 0.94 0.93 999.
0.92 0.91 0.90 0.89 0.87 0.86 0.85 999.
0.84 0.83 0.82 0.81 0.80 0.79 0.78 999.
0.77 0.76 0.73 0.71 0.70 0.69 0.68 999.
0.67 0.64 0.62 0.60 0.59 0.58 0.57 999.
0.56 0.55 0.54 0.53 0.52 0.51 0.50 999.
0.49 0.48 0.45 0.44 0.43 0.42 0.41 999.
0.40 0.39 0.38 0.37 0.36 0.35 0.34 0.0
0.0 0.0 0.0 0.0
```

The following data file was used as input to PREP.FOR to obtain the indicated output from RAY.FOR.

```
240A2. Data for PREP.DAT
2 10000. .25 100. .25 0.0
2. 2.
0.6
(5., .00) (.3, .00) (.01, .01) .0001 .05
```

2.1.3.1 Scaling the outputs obtained from SIN13.FOR and RAY.FOR

The dimensionless velocity obtained from SIN13.FOR was multiplied by 1000, and the dimensionless wavelength was divided by 600.

The dimensionless velocity obtained from RAY.FOR was multiplied by 1000. The output wavelength was divided by 7.0 indicating that the surface layer is 0.6/7.0= 0.08m in thickness. The structure of the pavement is given in Table 1.6.

2.2 SON.FOR

The programme SON.FOR is written in order to calculate the expected response of a single-layered system to the passage of waves of the Rayleigh type. The system considered is that of a compliant layer overlying a stiff semi-infinite medium [2]. This system is the normal progression which occurs in nature. The programme is written in Fortran. It compiles with the aid of Microsoft Fortran Version 5.1.

2.3 SIN13.FOR

The programme SIN13.FOR is written in order to calculate the expected response of a single-layered system to the passage of waves of the Rayleigh type. The system considered is that of a stiff layer overlying a compliant semi-infinite medium. The programme is written in Fortran. It compiles with the aid of Microsoft Fortran Version 5.1.

(1) Title;
(2) Number of media in the structure, Young's modulus and Poisson's ratio of each, Write Switch(0.0=essential output only: 1.0=all information);
(3) Densities of the materials in each medium;
(4) Thicknesses of layers in the structure;
(5) Initial wavelength, Trial velocity, Increment of trial velocity, Ratio of final to initial wavelength, wavenumber step at which calculations are to be made.

Table 2.1: Form of the data input to PREP.DAT

It is written according to Lee's determinant for a system composed of a stiff layer overlying a semi-infinite medium composed of a material having a lower stiffness than the material in the layer [3].

2.4 RAY.FOR

The programme RAY.FOR was written in order to calculate the expected response of a layered system to the passage of waves of the Rayleigh type. The programme is written in Fortran. It compiles with the aid of Microsoft Fortran Version 5.1. The programme PREP.FOR was written in order to prepare a datafile for input to RAY.FOR. The input to PREP.DAT is in free format. As an example, the following data were used in an attempt to obtain Figure 1.3. The input to PREP.DAT requires the data described in Table 2.1.

The contents of PREP.DAT used in order to match the data in Figure 1.2 were as follows:

```
DATA E1=11200,E2=34000,E3=1340;H1=0.1,H2=0.2,NU1=NU3=0.25,NU2=0.4
3 11200. .25 34000. .40 1340. .25 0.0
2. 2. 2.
.10 .20
(3.5, .00) (.40, .00) (.10, .01) .02 .10
```

All calculations in RAY.FOR are made with reference to the velocity β_1, the velocity of shear waves in the surface layer. The value of β_1 is treated as unity in the calculations.

However the true layer thicknesses, as supplied in the data PREP.DAT, were used. This was done in order to avoid complications caused by the exponential decay of displacements with the depth in each layer. These complications could be overcome: see the subroutine EMATRX. This subroutine would require rewriting if the thickness of the surface layer were fixed as unity.

The output from RAY.FOR therefore requires scaling in order to match any real system. The operation of scaling would be more simple if the thickness of the surface layer, in addition to the velocity of shear waves, were treated as unity.

The calculated response is scaled in order to match the results of the observed field measurements. The scale factor is then used to determine the true properties of the media of which the structure is composed. As an example, the calculated results of the structure need to be multiplied by a factor of one thousand in order to match the results of the field measurements. This translates to multiplying the elastic moduli by 10^6, showing that the moduli used in the data are in megapascals. The resulting values fall within the expected ranges of the moduli.

bibliog

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[3] Cogill, W.H., "Single-layered inverse: First-order approximation". ISVR Technical Memorandum No. 833, January 1999. Page 42, Equation (7).

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Appendix A

Program listings

A.1 Introduction

This appendix contains listings of the programmes used in the computations. Sample data are enclosed for each program, except for the two interactive programmes APP_S.FOR and APP_H.FOR.

A.2 SON.FOR

The program SON.FOR was written in order to match experimental results obtained from a system consisting of a layer of compliant material overlying a semi-infinite medium. The material within the semi-infinite medium is less compliant than that in the overlying layer.

A listing of the source program follows, accompanied by a set of sample data.

```
C     003G4      001        025 TTI BASEMENT OET BUILDING WHC33F
C         THERE ARE SEVEN TEN COLUMN DATA FIELDS ON THE VELOCITY CARD.
C         ANY FIELD MAY CONTAIN  999.,  WHICH CAUSES
C         THE PROGRAM TO READ A FURTHER VELOCITY CARD.
C     SON.FOR IS FOR SOFT OVER HARD
C     TEST DATA FOLLOWS: (STARTING POINT V=3.0)
C        0.4 0.101 0.4 100.
C        2.51 2.52 2.53 2.54 2.55 2.56 2.57 999.
C        2.58 2.6 2.75 2.8 2.9 3.1 3.3 999.
C        3.45 3.8 3.9 3.95 4.0 4.05 4.08 999.
C        4.2 4.4 5.1 5.5 6.7 7.5 7.9 999.
C        8.0 8.1 8.2 8.3 8.4 8.5 8.6  999.
C        8.7 8.8 8.9 9.0 9.1 9.15 9.18 999.
C        9.22 9.26  9.28 9.29 9.295 9.297 9.299 0.0
C        0.0 0.0 0.0 0.0
C
C***********************************************************************
C***********************************************************************
C
C  RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM.   JANUARY   1968    WHC33F
C
```

```
C***********************************************************************
C***********************************************************************
      PROGRAM WHC33F
      DIMENSION CB(8)
c     COMPLEX*16 R1K,R2K,S1K,S2K,
c    1R1H,R2H,S1H,S2H,XKR1,XKS1,R2R1,S2S1,XI1,XI2,ETA1,ETA2,XITEM1,
c    2XITEM2
  101 FORMAT (4F10.0)
  102 FORMAT(1H1,33X,'RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM.'
     15X'EWING EQN (4 - 202).'//27X'POISSON''S RATIO IN LAYER = ',
     2F5.2,',      IN HALF SPACE = ', F5.2,',    MU2/MU1 = ',F7.2,////)
  103 FORMAT(8F10.0)
  104 FORMAT(1H0,26X,'VELOCITY     WAVELENGTH     FREQUENCY    WAVE
     1 NUMBER     COUNTER REG.'///)
  105 FORMAT(27X,G10.5,7X,G10.5, 9X,G10.5,6X,G10.5, 9X,I3)
  106 FORMAT(27X,G10.5,7X,G10.5, 9X,G10.5,6X, G10.5,  9X, I3, 3X,
     1  'Y EQUALS ', E18.6)
  107 FORMAT(1H1)
       OPEN(1,FILE='SON.DAT',STATUS='OLD',FORM='FORMATTED')
       OPEN(3,FILE='SON.PRN',STATUS='UNKNOWN',FORM='FORMATTED')
  201 READ(1, * ) B1A1,A1A2,B2A2,XMU2M1
      IF(B1A1.EQ. 0.0) GO TO 999
      SIGMA1 = (1.-2.*B1A1*B1A1)/(2.-2.*B1A1*B1A1)
      SIGMA2 = (1.-2.*B2A2*B2A2)/(2.-2.*B2A2*B2A2)
      WRITE(3,102) SIGMA1,SIGMA2,XMU2M1
      WRITE(3,104)
      WRITE(*,*)  'ENTER A STARTING VALUE OF V'
C "2/V" is 2*pi/lamda = k; so V = lamda / pi
      READ(*,*) V  ! V IS LAMDA / PI
  202 READ(1, * ) (CB(J),J=1,8)
C     REF=10.V
      I=0
C***********************************************************************
C**********STATEMENT 203 IS RE-ENTRY POINT FOR NEW VALUE OF CB1*********
C***********************************************************************
  203 I=I+1
      IF(I.GT.8) GO TO 201
      KOUNT = 0
      CB1=CB(I)
      IF(CB1.EQ.0.) GO TO 201
      IF(CB1.EQ.999.) GO TO 202
C***********************************************************************
      CA1=CB1*B1A1
      CA2=CB1*B1A1*A1A2
      B1B2=B1A1*A1A2/B2A2
      CB2=CB1*B1B2
      write(*,*)ca1,ca2,cb1,cb2
C***********************************************************************
      R1K=SQRT(-1.0+CA1*CA1)
      R2K=SQRT(1.-CA2*CA2)
```

```
      S1K=SQRT(-1.+CB1*CB1)
      S2K=SQRT(1.-CB2*CB2)
      W = 2.*(XMU2M1-1.)
      X = XMU2M1*CB1*CB1*B1B2*B1B2-W
      Y = CB1*CB1+W
      Z = X-CB1*CB1
C*********************************************************************
      XKR1=1./R1K
      XKS1=1./S1K
      R2R1=R2K/R1K
      S2S1=S2K/S1K
C*********************************************************************
C*************START OF ITERATION LOOP**************
C*********************************************************************
  250 R1H = (2./V)*R1K
      KOUNT = KOUNT + 1
      R2H=(2./V)*R2K
      S1H=(2./V)*S1K
      S2H=(2./V)*S2K
C*********************************************************************
      XI1=(2.-CB1*CB1)*(X*COS(R1H)+R2R1*Y*SIN(R1H))
     1        +2.*S1K*(R2K*W*SIN(S1H)-XKS1*Z*COS(S1H))
      XI2=(2.-CB1*CB1)*(S2K*W*COS(R1H)+XKR1*Z*SIN(R1H))
     1        +2.*S1K*(X*SIN(S1H)-S2S1*Y*COS(S1H))
C*********************************************************************
      ETA1=(2.-CB1*CB1)*(R2K*W*COS(S1H)+XKS1*Z*SIN(S1H))
     1      +2.*R1K*(X*SIN(R1H)-R2R1*Y*COS(R1H))
      ETA2=(2.-CB1*CB1)*(X*COS(S1H)+S2S1*Y*SIN(S1H))
     1      +2.*R1K*(S2K*W*SIN(R1H)-XKR1*Z*COS(R1H))
C*********************************************************************
      XITEM1=XI1*ETA2
      XITEM2=XI2*ETA1
C*********************************************************************
      DEL = REAL(XITEM1 - XITEM2)
      IF (KOUNT - 1) 301, 301, 302
  301 XEVEN = V
      V = 1.01 * V
      XODD = V
      YEVEN = DEL
 3011 GO TO 250
C*********************************************************************
  302 IF (KOUNT - 2*(KOUNT/2)) 305, 305, 304
  304 YEVEN = DEL
      XEVEN = V
 3041 GO TO 306
  305 YODD = DEL
      XODD = V
C*********************************************************************
  306 IF(ABS(YODD-YEVEN).LE.. 000001)   GO TO 599
      IF(ABS(XODD - XEVEN) .LE. .001)  GO TO 600
```

```
      V = XEVEN - YEVEN*(XODD - XEVEN)/(YODD- YEVEN)
 3061 GO TO 250
C
C***********************************************************************
C*************END OF ITERATION LOOP*************
C***********************************************************************
  599 FREQ=CB1/V
      VI = 1./V
      WRITE(3,106) CB1, V, FREQ,VI, KOUNT,YODD
      GO TO 203
  600 FREQ=CB1/V
      VI=1./V
      WRITE(3,105) CB1,V,FREQ,VI,KOUNT
      GO TO 203
  999 WRITE(3,107)
      STOP
      END
```

A.2.1 Sample data for program SON.FOR

Sample data follows for the program SON.FOR

```
0.4 0.101 0.4 100.
2.51 2.52 2.53 2.54 2.55 2.56 2.57 999.
2.58 2.6 2.75 2.8 2.9 3.1 3.3 999.
3.45 3.8 3.9 3.95 4.0 4.05 4.08 999.
4.2 4.4 5.1 5.5 6.7 7.5 7.9 999.
8.0 8.1 8.2 8.3 8.4 8.5 8.6 999.
8.7 8.8 8.9 9.0 9.1 9.15 9.18 999.
9.22 9.26 9.28 9.29 9.295 9.297 9.299 0.0
0.0 0.0 0.0 0.0
```

A.2.2 Sample output from program SON.FOR

Abbreviated sample output is shown in Table A.1 for the program SON.FOR

A.3 SIN13.FOR

The program SIN13.FOR was written in order to match experimental results obtained from a system consisting of a layer of stiff material overlying a semi-infinite medium. The material within the semi-infinite medium is less stiff than that in the overlying layer.

A listing of the source program follows, accompanied by a set of sample data. The program is an approximation only. The result is obtained not as true zero of the determinant of the frequency equation. Instead it is obtained when the value of the determinant is a small fraction of its initial trial value.

RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM. EWING EQN (4 - 202). POISSON'S RATIO IN LAYER = .40, IN HALF SPACE = .40, MU2/MU1 = 100.00				
VELOCITY	WAVELENGTH	FREQUENCY	WAVE NUMBER	COUNTER REG.
2.5100	1.3501	1.8592	.74071	7
2.5200	1.3578	1.8560	.73649	4
2.5300	1.3655	1.8529	.73236	4
2.5400	1.3730	1.8499	.72831	4
2.5500	1.3806	1.8470	.72433	4
2.5600	1.3881	1.8443	.72043	4
2.5700	1.3955	1.8417	.71660	4
2.5800	1.4028	1.8391	.71284	4
2.6000	1.4174	1.8344	.70553	3
2.7500	1.5209	1.8081	.65749	6
2.8000	1.5537	1.8021	.64362	4
2.9000	1.6175	1.7929	.61823	5
3.1000	1.7399	1.7817	.57475	6
3.3000	1.8578	1.7762	.53826	5
3.4500	1.9446	1.7742	.51425	5
3.8000	2.1436	1.7727	.46650	6
3.9000	2.2000	1.7727	.45454	5
3.9500	2.2282	1.7728	.44880	4
4.0000	2.2563	1.7728	.44320	4

Table A.1: Sample output from program SON.FOR

```
C     003G4       001         025 TTI BASEMENT OET BUILDING WHC33F
C          THERE ARE SEVEN TEN COLUMN DATA FIELDS ON THE VELOCITY CARD.
C          ANY FIELD MAY CONTAIN  999.,  WHICH CAUSES
C          THE PROGRAM TO READ A FURTHER VELOCITY CARD.
C        Version for hard over soft see Lee,A.R. "The effect of structure
C  upon microseismic disturbance" page 91 eqn (20)
C***********************************************************************
C***********************************************************************
C
C  RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM.   JANUARY  1968    WHC33F
C
C***********************************************************************
C***********************************************************************
c  Trial data follows, works on starting value of 0.4
C        0.4 10.0 0.4 0.01
C        0.58 0.53 0.529 0.528 0.527 0.526 0.525 999.
C        0.524 0.523 0.52 0.51 0.50 0.49 0.48 999.
C        0.47 0.46 0.45 0.42 0.40 0.39 0.38  999.
C        0.36 0.35 0.34 0.33 0.32 0.31 0.30 999.
C        0.290 0.285 0.280 0.275 0.270 0.265 0.250 999.
C        0.283 0.282 0.281 0.280 0.279 0.278 0.277 999.
C        0.276 0.275 0.274 0.273 0.272 0.271 0.270 999.
C        0.269 0.268 0.266 0.264 0.262 0.260 0.258 0.0
C        0.0 0.0 0.0 0.0
```

```
C
      PROGRAM WHC33F
      DIMENSION CB(8)
      REAL*8 R2K,S1K,S2K,
     1R1H,R2H,S1H,S2H,XKR1,XKS1,R2R1,S2S1,XI1,XI2,ETA1,ETA2,XITEM1,
     2XITEM2
  101 FORMAT (4F10.0)
  102 FORMAT(13X,'RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM. SIN13.FOR'
     15X'EWING EQN (4 - 202).'//7X'POISSON''S RATIO IN LAYER = ',
     2F5.2,',     IN HALF SPACE = ', F5.2,',    MU2/MU1 = ',F7.2,////)
  103 FORMAT(8F10.0)
  104 FORMAT(6X,'VELOCITY      WAVELENGTH         FREQUENCY
     1 WAVE NUMBER    COUNTER REG.'///)
  105 FORMAT(5X,G12.4,5X,G12.4,7X,G12.4,4X,G12.4,7X,I3)
  106 FORMAT(5X,G12.4,5X,G12.4,7X,G12.4,4X, G12.4,7X,I3,3X,
     1  'Y EQUALS ', E18.6)
  107 FORMAT(1H1)
        OPEN(1,FILE='SIN13.DAT',STATUS='OLD',FORM='FORMATTED')
        OPEN(3,FILE='SIN13.PRN',STATUS='UNKNOWN',FORM='FORMATTED')
  201 READ(1, * ) B1A1,A1A2,B2A2,XMU2M1
      IF(B1A1.EQ. 0.0) GO TO 999
      SIGMA1 = (1.-2.*B1A1*B1A1)/(2.-2.*B1A1*B1A1)
      SIGMA2 = (1.-2.*B2A2*B2A2)/(2.-2.*B2A2*B2A2)
      WRITE(3,102) SIGMA1,SIGMA2,XMU2M1
      WRITE(3,104)
C "2/V" is 2*pi/lamda = k; so V = lamda / pi
      WRITE(*,*)  'ENTER A STARTING VALUE OF V'
      READ(*,*) V
  202 READ(1, * ) (CB(J),J=1,8)
C     REF=10.V
      I=0
C***********************************************************************
C**********STATEMENT 203 IS RE-ENTRY POINT FOR NEW VALUE OF CB1*********
C***********************************************************************
  203 CONTINUE
C      V = VST  ! RESET V TO THE STARTING VALUE
      I=I+1
      IF(I.GT.8) GO TO 201
      KOUNT = 0
      CB1=CB(I)
      IF(CB1.EQ.0.) GO TO 201
      IF(CB1.EQ.999.) GO TO 202
C***********************************************************************
      CA1=CB1*B1A1
      CA2=CB1*B1A1*A1A2
      B1B2=B1A1*A1A2/B2A2
      CB2=CB1*B1B2
C      WRITE(*,*) CA1,CB1,CA2,CB2
C***********************************************************************
      R1K=SQRT(1.0-CA1*CA1)
```

```
      R2K=SQRT(-1.0+CA2*CA2)
      S1K=SQRT(1.0-CB1*CB1)
      S2K=SQRT(-1.0+CB2*CB2)
      W = 2.*(XMU2M1-1.)
      X = XMU2M1*CB1*CB1*B1B2*B1B2-W
      Y = CB1*CB1+W
      Z = X-CB1*CB1
C*********************************************************************
      XKR1=1./R1K
      XKS1=1./S1K
      R2R1=R2K/R1K
      S2S1=S2K/S1K
C*********************************************************************
C*************START OF ITERATION LOOP**************
C*********************************************************************
  250 CONTINUE
      R1H = (2./V)*R1K
      KOUNT = KOUNT + 1
      R2H=(2./V)*R2K
      S1H=(2./V)*S1K
      S2H=(2./V)*S2K
C*********************************************************************
      XI1=(2.-CB1*CB1)*(X*DCOSH(R1H)+R2R1*Y*DSINH(R1H))
     1     -2.*S1K*(R2K*W*DSINH(S1H)+XKS1*Z*DCOSH(S1H))
      XI2=(2.-CB1*CB1)*(S2K*W*DCOSH(R1H)+XKR1*Z*DSINH(R1H))
     1     -2.*S1K*(X*DSINH(S1H)+S2S1*Y*DCOSH(S1H))
C*********************************************************************
      ETA1=(2.-CB1*CB1)*(R2K*W*DCOSH(S1H)+XKS1*Z*DSINH(S1H))
     1     -2.*R1K*(X*DSINH(R1H)+R2R1*Y*DCOSH(R1H))
      ETA2=(2.-CB1*CB1)*(X*DCOSH(S1H)+S2S1*Y*DSINH(S1H))
     1     -2.*R1K*(S2K*W*DSINH(R1H)+XKR1*Z*DCOSH(R1H))
C******************************************************************** C*********************
      XITEM1=XI1*ETA2
      XITEM2=XI2*ETA1
C*********************************************************************
      DEL = REAL(XITEM1 - XITEM2)
      IF (KOUNT - 1) 301, 301, 302
  301 VEVEN = V
      V = 1.27 * V
      VODD = V
      DELEVEN = DEL
 3011 GO TO 250
C*********************************************************************
  302 IF (KOUNT - 2*(KOUNT/2)) 305, 305, 304
  304 DELEVEN = DEL
      VEVEN = V
 3041 GO TO 306
  305 DELODD = DEL
      VODD = V
C*********************************************************************
```

```
 306 IF(ABS(DELODD-DELEVEN).LE. 0.01)  GO TO 599 ! Failed output
     DIFF =  ABS((DELODD-DELEVEN)/XITEM1)
     IF(DIFF .LE. .000001*DELODD)  GO TO 600   ! Successful output
     V = VEVEN -0.5*DELEVEN*(VODD - VEVEN)/(DELODD- DELEVEN) !Convergen
3061 GO TO 250
C
C**********************************************************************
C*************END OF ITERATION LOOP*************
C**********************************************************************
 599 FREQ=CB1/V
     VI = 1./V
     WRITE(3,106) CB1, V, FREQ,VI, KOUNT,DELODD
     GO TO 203
 600 FREQ=CB1/V
     VI=1./V
     WRITE(3,105) CB1,V,FREQ,VI,KOUNT
     GO TO 203
 999 WRITE(3,107)
     STOP
     END
```

A.3.1 Sample data for program SIN13.FOR

Sample data follows for the program SIN13.FOR

```
0.4 8. 0.4 0.01
0.999 0.98 0.97 0.96 0.95 0.94 0.93 999.
0.92 0.91 0.90 0.89 0.87 0.86 0.85 999.
0.84 0.83 0.82 0.81 0.80 0.79 0.78 999.
0.77 0.76 0.73 0.71 0.70 0.69 0.68 999.
0.67 0.64 0.62 0.60 0.59 0.58 0.57 999.
0.56 0.55 0.54 0.53 0.52 0.51 0.50 999.
0.49 0.48 0.45 0.44 0.43 0.42 0.41 999.
0.40 0.39 0.38 0.37 0.36 0.35 0.34 0.0
0.0 0.0 0.0 0.0
```

A.3.2 Sample output for program SIN13.FOR

Selected sample output is shown in Table A.2 for the program SIN13.FOR

A.4 PREP.FOR and RAY.FOR

The program RAY.FOR was written in order to match experimental results obtained from a multi-layered system. The system can be composed of up

RAYLEIGH WAVES IN A SINGLE-LAYERED SYSTEM. EWING EQN (4 - 202). POISSON'S RATIO IN LAYER = .40, IN HALF SPACE = .40, MU2/MU1 = .01				
VELOCITY	WAVELENGTH	FREQUENCY	WAVE NUMBER	COUNTER
.5800	1.155	.5020	.8654	31
.5300	1.306	.4060	.7660	7
.5290	1.322	.4003	.7567	4
.5280	1.325	.3985	.7548	4
.5270	1.328	.3968	.7530	4
.5260	1.331	.3951	.7512	4
.5250	1.334	.3935	.7494	4
.5240	1.337	.3918	.7477	4
.5230	1.341	.3901	.7459	4
.5200	1.350	.3852	.7407	4
.5100	1.382	.3691	.7236	4

Table A.2: Sample output from program SIN13.FOR

to five media, consisting of four layers overlying a semi-infinite medium. The stiffnesses of the materials within the system may be in any order.

The data is prepared by means of a program PREP.FOR.

A.4.1 PREP.FOR

The program PREP.FOR is written in order to prepare the data file RAY.DAT required for the program RAY.FOR A listing of the program PREP.FOR follows.

```
      PROGRAM PREP
C       THIS PROGRAM PREPARES DATA FOR RAY. THE INPUT IS IN FILE PREP.DAT
C     THE DATA LINES IN PREP.DAT ARE TITLE,
C       NS,(EM(I),V(I), I=1,NS),SWTCH
C       (RHO(I),I=1,NS)
C       (HH(I),I=1,N)
C       WLNGTH,CD1TRA,CINCR ALLCOMPLEX; WLFAC,DELFAC BOTH REAL
C
      DIMENSION EM(5),V(5),RHO(5),HH(5),TITLE(20)
      COMPLEX WLNGTH,CB1TRL,CINCR
C       OPEN FILE PREP.DAT WHICH CONTAINS DATA IN FREE FORMAT.
      OPEN(1,FILE='PREP.DAT',STATUS='OLD')
C       OPEN FILE RAY.DAT TO ACCEPT DATA IN FIXED FORMAT.
      OPEN(3,FILE='RAY.DAT',STATUS='UNKNOWN')
      REWIND 3
 101  FORMAT(20A4)
 300        WRITE (*,*) 'ENTER TITLE'
      READ(1,101,END=999) (TITLE(I), I = 1,19)
            WRITE(*,*)'NS,(EM(I),V(I),I=1,NS),SWTCH'
      READ(1,*)NS,(EM(I),V(I),I=1,NS),SWTCH
            WRITE(*,*)'(RHO(I),I=1,NS)'
      READ(1,*)(RHO(I),I=1,NS)
            WRITE(*,*) 'HH(I),I=1,N)'
      N = NS-1
```

```
      READ(1,*)(HH(I),I=1,N)
           WRITE(*,*)'WLNGTH,CB1TRL,CINCR,COMPLEX,WLFAC,DELFAC,REAL'
c          WRITE(*,*)'ENTER WLNGTH,CB1TRL,CINCR ALL COMPLEX'/
c     1 'WLFAC,DELFAC BOTH REAL'
      READ (1,*) WLNGTH,CB1TRL,CINCR,WLFAC,DELFAC
 2001 FORMAT(19A4,F4.0)
 201  FORMAT(I3,3X,6(F6.0,F6.2))
 202  FORMAT (6X,12F6.0)
 203  FORMAT (10F6.2)
 204  FORMAT (8F6.2)
      WRITE (3,2001)(TITLE(I),I=1,19),SWTCH
      WRITE (3,201) NS,(EM(I),V(I),I=1,NS)
      WRITE (3,202) (RHO(I),I=1,NS)
      N = NS - 1
      WRITE (3,203) (HH(I),I=1,N)
      WRITE (3,204) WLNGTH,CB1TRL,CINCR,WLFAC,DELFAC
c      GO TO 300
 999  CONTINUE
      CLOSE(UNIT=3)
           STOP
      END
```

The contents of PREP.DAT are as follows:

```
(1) Title;
(2) Number of media in the structure, Young's modulus and Poisson's ratio of each,
Write Switch(0.0=essential output only: 1.0=all information);
(3) Densities of the materials in each medium;
(4) Thicknesses of layers in the structure;
(5) Initial wavelength, Trial velocity, Increment of trial velocity,
Ratio of final to initial wavelength,
wavenumber step at which calculations are to be made.
```

The input to the program PREP.FOR is as follows.

```
LCPC DATA E1=11200,E2=34000,E3=1340;H1=0.1,H2=0.2,NU1=NU3=0.25,NU2=0.4
3 11200. .25 34000. .40 1340. .25 0.0
2. 2. 2.
.10 .20
(3.5, .00) (.40, .00) (.10, .01) .02 .10
```

A.4.2 RAY.FOR

The program PREP.FOR outputs a data file labelled RAY.DAT. This file is read as input to the program RAY.FOR

A listing of the program RAY.FOR follows. The data is supplied by the output from PREP.FOR The output from RAY.FOR is the file RAY.PRN The file RAY.PRN shows the phase velocities, the wavelengths, the wavenumbers and the frequencies for each point which is calculated.

```
c     ** N-LAYERED RAYLEIGH WAVE PROGRAM.   MOD LEVEL 3. MAR 1970
      PROGRAM RAYL
C
C     X003G4      *10      025 COGILL     TTI  TEXAS TRANSPORTATION INSTI
C     X003G4      *30      025 COGILL     TTI  TEXAS TRANSPORTATION INSTI
      REAL PFREQ(100),PVEL(100),PWNGTH(100),PXN(100)
      REAL EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),HH(6),
     1 FREQ1,FREINC
      REAL ABS,SQRT,XLMDA(6)
      COMPLEX     P1,XK,A(6),R(6),S(6),T(6),E(6,4,4),
     1 PM(6,4,4),CINV(6,4,4),XJ(4,4),VALUE,DELXJ(2,2),
     2 T6,T7,G(4,4)
      COMPLEX  CB1RES,CINRES,WLNTHM,DELXN
      COMPLEX  CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CMPLX,XN,
     1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CVN,CINCR1
      COMPLEX  CSQRT,CCOS,CSIN,CRH,CSH,SRH,SSH
      COMMON ALPHA,BETA,XK,XMU,XLMDA
      COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
     1 CVN,CVP,WLNGTH,WSWTCH,NTRLS,ICYCLE,ISWTCH
      REAL TITLE(18)
      CHARACTER*4 ASTER
 101  FORMAT(I3,3X,12F6.0)
 102  FORMAT(6X,12F6.0)
 103  FORMAT(10F6.0)
 104  FORMAT(8F6.0)
 105  FORMAT(18A4,F8.0)
C     CALL ERRSET (208,0,-1,1,1,207)
       open(1,file='ray.dat',status='old',form='formatted')
       open(3,file='ray.prn',status='unknown',form='formatted')
       rewind 1
       rewind 3
 1001 READ(1,105,end = 999)(TITLE(I),I=1,18), WSWTCH
C
C        **WSWTCH** IS AN OUTPUT SWITCH WHICH CONTROLS THE
C     PRINTING OF THE CONVERGENCE PARAMETER 'VALUE'.
C        IF WSWTCH = 0 (OR LEFT BLANK) 'VALUE' IS NOT PRINTED
C           WSWTCH = 1.0 'VALUE' IS PRINTED WITH THE FINAL OUTPUT
C           WSWTCH = 2.0 'VALUE' AND ITS CORRESPONDING TRIAL VELOCITY
C           'CB1' ARE PRINTED AFTER EACH CYCLE OF CALCULATION.
C
      NPAGE = 1
      ASTER = '****'
      WRITE (3,204)  (TITLE(I),I=1,18),
     2 NPAGE
      READ(1,101) NS,(EM(I),V(I), I=1,6)
      IF(NS.EQ.0) GO TO 9992
      READ(1,102) (RHO(I), I = 1,6)
C201  FORMAT(' TRIAL VELOCITY OUTSIDE ALLOWABLE RANGE')
```

```
C202  FORMAT (1H1)
 204  FORMAT(1H1//,18A4,1X7H    PAGE,(I3))
 205  FORMAT(4X,'THE PROBLEM PARAMETERS ARE'//
    2       ('LAYER',I3,'  HAS MODULUS ',F10.0,
    3       '  POISSON''S RATIO ',F5.3,'  DENSITY  ',F5.0,
    4       '  AND THICKNESS ',F6.2,' UNITS'))
 206  FORMAT('LAYER',I3,'  HAS MODULUS ',F10.0,
    2       '  POISSON''S RATIO ',F5.3,'  DENSITY  ',F5.0,
    3       '  AND IS SEMI-INFINITE.  '/)
 207  FORMAT(4X,'VELOCITY      WAVELENGTH',7X,'FREQUENCY   WAVE NUMBER
    1    COUNTER REG.'///)

 209  FORMAT(1X,2F7.3,2F7.3,4X,2F7.3, 2F7.3,3X,I4,E8.2)
C     NLINE = 17 + NS
      IF(NS-6) 10,10,1
 1    READ(1,102) (EM(I),V(I),I=7,NS)
      READ(1,102) (RHO(I), I = 7,NS)
 10   N=NS-1
      DO 1011 J = 1,6
 1011 HH(J) = 0.0
      READ(1,103) (HH(I),I=1,N)
      WRITE(3,205) (I,EM(I),V(I),RHO(I),HH(I), I = 1,N)
      WRITE(3,206) NS,EM(NS),V(NS),RHO(NS)
      WRITE(3,207)
      DO 120 I =1,100
      PFREQ(I) =0.0
      PVEL(I) =0.0
      PWNGTH(I) =0.0
      PXN(I) =0.0
 120  CONTINUE
C
C        THE FIVE CARDS STARTING WITH STATEMENT 126 ARE INSERTED
C     IN ORDER TO PERMIT THE OPERATION OF THE FREE PLATE OPTION.
C              ISWTCH = 1        FOR SEMI-INFINITE SYSTEM
C              ISWTCH = 2        FOR COMPOUND PLATE
C
 126  IF(EM(NS)) 127,127,128
 127  NS = NS - 1
      ISWTCH = 2
      GO TO 1111
 128  ISWTCH = 1
 1111 DO 11 I=1,NS
      XLMDA(I)=V(I)*EM(I)/((1.+V(I))*(1.-2.*V(I)))
      XMU(I)=EM(I)/(2.*(1.+V(I)))
 11   CONTINUE
C
C     WE SHALL HOLD THE WAVELENGTH CONSTANT DURING EACH ITERATION, A
C     INTERPOLATE TO FIND THE PHASE VELOCITY 'CB1'.
C
      DO 111 I=1,NS
```

```
      BETA(I)=SQRT(XMU(I)/XMU(1))*SQRT(RHO(1)/RHO(I))
 111  ALPHA(I)=SQRT((XLMDA(I)+2.*XMU(I))/(XLMDA(1)+
     12.*XMU(1)))*SQRT((XLMDA(1)+2.*XMU(1))/XMU(1))*SQRT(RHO(1)/RHO(I)
     1)
C     THIS (DO 111) ROUTINE DETERMINES BETA(I), ALPHA(I) AS MULTIPLES
C     OF BETA(1)
      DELTAP = CMPLX(0.0,0.0)
      DELTAN = CMPLX(0.0,0.0)
      CVP    = CMPLX(0.0,0.0)
      CVN    = CMPLX(0.0,0.0)
C1212 READ(1,104) WLNGTH,FREQ1,FREINC
C     CB1TRL = WLNGTH * FREQ1
C     CINCR = WLNGTH * FREINC
      READ (1,104) WLNGTH,CB1TRL,CINCR,WLFAC,DELFAC
C
C        'WLFAC' IS THE RATIO OF THE MINIMUM TO THE MAXIMUM WAVELENGTH.
C     'DELFAC' IS THE WAVENUMBER INTERVAL AT WHICH POINTS ARE TO BE
C     CALCULATED.  THE QUOTIENT 1/DELFAC  GIVES THE NUMBER OF
C     POINTS TO BE CALCULATED.
C
      IPLOT = 0
      CB1RES = CB1TRL
      CINRES = CINCR
      WLNTHM = WLFAC * WLNGTH
      DELXN = DELFAC
      GO TO 1213
 1212 WLNGTH = WLNGTH/(1 + DELXN*WLNGTH)
      IF(REAL(WLNGTH).LE. REAL(WLNTHM)) GO TO 9991
      CB1TRL = CB1RES
      CINCR = CINRES
 1213 CONTINUE
      IF(REAL(WLNGTH) .EQ.0.0 .AND. AIMAG(WLNGTH) .EQ. 0.0) GO TO 1001
      CALL TRAVEL(N,NS,HH,A,R,S,T,E,G,DELXJ,ITRACK)
C900  CB1 = 0.5*(CV1+CV2)
      XN = 1./WLNGTH
      FREQ = CB1 * XN
C     THE FOLLOWING 'IF' HAS BEEN DE-ACTIVATED TO SHORT CIRCUIT A
C     LOOP THROUGH THE SPIRAL SEARCH, WHICH CAN OCCUR IF THE
C     IMAGINARY PART OF 'CB1' IS ZERO.     WHC 71.024
C     IF(ABS(AIMAG(CB1)).LT. 0.0001 .AND. NTRLS .LT. 10) GO TO 1213
      IPLOT = IPLOT+ 1
      PFREQ(IPLOT) = FREQ
      PVEL(IPLOT) = CB1
      PWNGTH(IPLOT) = WLNGTH
      PXN(IPLOT) = XN
C     OUTPUT FORMAT BASED ON WHC33B
      IF (WSWTCH .EQ. 1.0) GO TO 901
      GO TO 902
 901  DIFF=AMIN1(CABS(DELTA1),CABS(DELTA2),CABS(DELTAN),CABS(DELTAP))
      WRITE (3,209) CB1,WLNGTH,FREQ,XN,NTRLS,DIFF
```

```
      GO TO 1212
 902  WRITE (3,209) CB1,WLNGTH,FREQ,XN,NTRLS
      GO TO 1212
C999  WRITE (3,201)
 9991 CONTINUE
 9992 CONTINUE
C9991 CALL FPLOT (PVEL,PWNGTH,PFREQ,PXN,TITLE,IPLOT)
C9992 CALL CLOSE
      GO TO 1001
  999 stop
      END
      SUBROUTINE TRAVEL(N,NS,HH,A,R,S,T,E,G,DELXJ,ITRACK)   TRAVEL
      REAL EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),HH(6),
     1 FREQ1,FREINC
      REAL ABS,SQRT,XLMDA(6)
      COMPLEX        XK,A(6),R(6),S(6),T(6),E(6,4,4),
     1 PM(6,4,4),CINV(6,4,4),XJ(4,4),VALUE,DELXJ(2,2),
     2 T6,T7,G(4,4)
      COMPLEX  CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CMPLX,XN,
     1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CVN
      COMPLEX CELTA,CELTA1,CELTA2
      COMPLEX CB1REC
      COMPLEX  CSQRT,CCOS,CSIN,CRH,CSH,SRH,SSH
      COMMON ALPHA,BETA,XK,XMU,XLMDA
      COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
     1 CVN,CVP,WLNGTH,WSWTCH,NTRLS,ICYCLE,ISWTCH
 203  FORMAT('   CB1 = ', 2E14.4)
 208  FORMAT(20X,'DELTA = ',2D14.4,I4,'   CB1REC = ',2E14.4)
 210  FORMAT('    RESIGNED AFTER  ',I4, '  TRIALS'/)
 211  FORMAT('         RESIGNED   NCIRC = ',I4,  '   ATTEMPTS')
      NINTP = 0
      NTRLS=0
      ITRACK = 1
        NREV = 5
C
C        IF THE PROGRAM OSCILLATES, MAKE THE PREVIOUS CARD 'NREV = 7'.
C      IF 'NREV = 1',  THE PROGRAM IS SENSITIVE AND OSCILLATES READILY.
C
 121  CONTINUE
      IF(MOD(ITRACK,4).EQ. 1) ICYCLE = 1
      IF(MOD(ITRACK,4).EQ. 2) ICYCLE = 2
      IF(MOD(ITRACK,4).EQ. 3) ICYCLE = 3
      IF(MOD(ITRACK,4).EQ. 0) ICYCLE = 4
C     IF(REAL(CB1) .EQ. 1.0) GO TO 1201
      GO TO (1203,1204,1203,1204),ICYCLE
 1203 CB1 = CMPLX(REAL(CB1TRL)+REAL(CINCR),AIMAG(CB1TRL))
      GO TO 1205
 1204 CB1 = CMPLX(REAL(CB1TRL),AIMAG(CB1TRL)+AIMAG(CINCR))
 1205 CONTINUE
C     GO TO 122
```

```
C        STATEMENT 1201 IS A BACKTRACKER IN CASE EITHER A(I), R(I) OR
C     S(I) IS ZERO.  IF R(I) OR S(I) IS ZERO, A ZERO DIVIDE OCCURS IN
C     SUBROUTINE 'GMATRX'.  IF A(I) IS ZERO, THE RESULT IS NOT OF
C     INTEREST FOR THE PRESENT.
C        THE SEARCH FOR A VELOCITY BACKTRACKS TO THE MOST RECENT 'CB1'
C     YIELDING NON-ZERO A(I), R(I), S(I).  IT RE-COMMENCES FORWARD
C     TRACKING USING AN INTERVAL ONE TENTH OF THE PREVIOUS ONE.
C1201 CB1 = CB1TRL + 0.1 * CINCR
C     CINCR = 0.1 * CINCR
 122  CONTINUE
      WLNGTH = CMPLX(REAL(WLNGTH), REAL(WLNGTH)*AIMAG(CB1)/REAL(CB1))
      XK = 6.2831852/WLNGTH
      IF(NTRLS .GT.140) GO TO 8901
      IF(WSWTCH .EQ. 0.0) GO TO 123
      WRITE(3,203) CB1
C     IF(CB1.GT.1.0,0.0 .OR.CB1.LT.BETA(NS),0.0) GO TO 999
 123  CONTINUE
C
      DO 129 I = 1, NS
      A(I) = 0.0
      R(I) = 0.0
      S(I) = 0.0
      T(I) = 0.0
 129  CONTINUE
 131  CONTINUE
      DO 130 I = 1,NS
      A(I)=XK*XK*(1.-CB1*CB1/(2.*((BETA(I)/BETA(1))**2)))
      R(I)=CSQRT( XK*XK*(1.-CB1*CB1/((ALPHA(I)/BETA(1))**2)))
      S(I)=CSQRT( XK*XK*(1.-CB1*CB1/((BETA(I)/BETA(1))**2)))
      T(I)=XMU(I)/XMU(1)
C THE FOLLOWING 'IF' HAS BEEN DE-ACTIVATED TO OBVIATE ERROR MD-3  69004
C     IF(A(I).EQ.CMPLX(0.0,0.0).OR.R(I).EQ.CMPLX(0.0,0.0).OR.
C    1  S(I).EQ.CMPLX(0.0,0.0)) GO TO 1201
 130  CONTINUE
      CALL GMATRX   (A,R,S,T,G,NS)
C
C     SET UP THE 'E' MATRICES, THE PRODUCTS OF THE D'S AND THE INVERSE
C     'C' 'S.
C
      CALL EMATRX   (HH,A,R,S,T,E,N)
 137  CALL PROMAT(N,NS,E,G,DELXJ)
C
C     WE NOW HAVE THROWER'S 'J' MATRIX COMPLETE, FOR THE PARTICULAR
C     WAVELENGTH 'WLNGTH' AND THE TRIAL VELOCITY 'CB1'.
C
      CALL CHECK(DELXJ,VALUE)
 138  CONTINUE
      IF(WSWTCH .EQ. 2.0)  GO TO 1371
      GO TO 139
1371  WRITE (3,208) VALUE,ICYCLE,CB1REC
```

```
 139  IF(NTRLS .NE. 0) GO TO 140
      DELTA1= VALUE
      CV1 = CB1
      GO TO 160
 140  DELTA2= VALUE
      CV2 = CB1
      DIFVEL = CABS(CV2-CV1)
      SUMVEL = CABS(CV1 + CV2)
      GO TO (1412,1412,1411,1411),ICYCLE
 1411 PHI = PHI + PINCR
C
C        ENTER SPIRAL SEARCH FOR IMPROVED ROOT
C
C       EJECT IF CB1 IS PURELY REAL
 204  FORMAT('  RAD = ',E14.4)
      IF(ABS(AIMAG(CB1)).LT. 0.001)  GO TO 9009
      IF(NCIRC) 1416,1415,1416
 1415 RAD = AIMAG(CB1REC)
 1416 CONTINUE
      CELTA = VALUE
      IF(NCIRC.EQ.0) GO TO 1420
      CELTA1 = CELTA2
 1420 CELTA2 = CELTA
      IF(NCIRC .EQ. 0) GO TO 1429
C     SIGN CHANGE TEST FOLLOWS
      IF(SIGN(1.,REAL (CELTA1)) - SIGN(1., REAL(CELTA2))) 1421,1429,1421
 1421 IF(SIGN(1.,AIMAG(CELTA1)) - SIGN(1.,AIMAG(CELTA2))) 1422,1429,1422
 1422 PHI1 = PHI - 1.5*PINCR
      CINCR = CMPLX(XKONST*RAD*COS(PHI1),XKONST*RAD*SIN(PHI1))
      PHI = 2 * PI
C     'CV1', 'CV2' ARE NEEDED AT STATEMENT 900, IN THE MAIN PROGRAM.
      CV1 = CB1REC
      CB1REC = CB1REC + 0.3 * CINCR
      CV2 = CB1REC
      PINCR = -PINCR
 1429 NCIRC = NCIRC + 1
      IF(NCIRC .GT. 40) GO TO 9008
      IF(PHI - 1.9 * PI) 1417,1413,1413
 1417 IF(PHI+1.9*PI) 1413,1413,1414
 1413 XKONST =  0.7 * XKONST
      IF(XKONST*RAD .LT. 0.002)GO TO 9009
      PHI = 0.0
 1414 CINCR=CMPLX(XKONST*RAD*COS(PHI),XKONST*RAD*SIN(PHI))
      CB1 = CB1REC + CINCR
      GO TO 122
 1412 CONTINUE
      IF(DIFVEL .LT.0.0005   * SUMVEL) GO TO 900
      IF(NINTP .NE. 0) GO TO 300
      GO TO (1401,1402,1402,1401), ICYCLE
 1401 IF(SIGN(1.,REAL(DELTA2))-SIGN(1.,REAL(DELTA1)))  300,150,300
```

```
 1402 IF(SIGN(1.,AIMAG(DELTA2))-SIGN(1.,AIMAG(DELTA1))) 300,150,300
C1403 CONTINUE
 150  GO TO (1501,1502,1502,1501), ICYCLE
 1501 IF(ABS(REAL(DELTA1))-ABS(REAL(DELTA2))) 151,152,152
 1502 IF(ABS(AIMAG(DELTA1))-ABS(AIMAG(DELTA2))) 151,152,152
C1503 CONTINUE
C     ROUTINE TO OVERCOME SIGNIFICANCE ERROR
 151  NREV = NREV - 1
      IF(NREV) 1511,152,152
 1511 CINCR = - 2.*CINCR
 152  DELTA1 = DELTA2
      CV1 = CV2
 160  NTRLS = NTRLS + 1
      CB1TRL = CB1
C     ITRACK = ITRACK + 1
      GO TO 121
C
C     INTERPOLATE ROUTINE,  RETAINING THE MOST RECENT DELTAS
C     OF OPPOSITE SIGN
C
C
C     PREPARE FOR INTERPOLATION ROUTINE.  THE PROGRAM PERFORMS THE
C     FOLLOWING OPERATIONS IN A CYCLIC FORM.
C(1)TRACK 'CB1' ALONG ITS REAL AXIS, SEEK ZERO ON REAL AXIS OF 'DELTA'.
C(2)TRACK 'CB1' ALONG ITS IMAG AXIS, SEEK ZERO ON IMAG AXIS OF 'DELTA'.
C(3)TRACK 'CB1' ALONG ITS REAL AXIS, SEEK ZERO ON IMAG AXIS OF 'DELTA'.
C(4)TRACK 'CB1' ALONG ITS IMAG AXIS, SEEK ZERO ON REAL AXIS OF 'DELTA'.
C
C
C     FIRST CONNECT THE REAL AND IMAGINARY PARTS OF THE INTERPOLATED
C     'DELTA1'.  THE SEQUENCE IS CONTROLLED BY 'ICYCLE', AND IS SET
C     BY THE 'MOD' SWITCH FOLLOWING STATEMENT 121.
C
 300  CONTINUE
      GO TO (3001,3002,3002,3001),ICYCLE
 3001 IF(REAL(DELTA1)) 301,302,302
 3002 IF(AIMAG(DELTA1)) 301,302,302
C3003 CONTINUE
 301  CONTINUE
      GO TO (3011,3012,3012,3011),ICYCLE
 3011 DELTAN = CMPLX(REAL(DELTA1),AIMAG(DELTAN))
      GO TO 3013
 3012 DELTAN = CMPLX(REAL(DELTAN),AIMAG(DELTA1))
 3013 CONTINUE
C     NOW CONNECT THE PARTS OF 'CV1' AND 'CVN'.
      GO TO (3014,3015,3014,3015),ICYCLE
 3014 CVN = CMPLX(REAL(CV1),AIMAG(CVN))
      GO TO 3016
 3015 CVN = CMPLX(REAL(CVN),AIMAG(CV1))
 3016 CONTINUE
```

```
      GO TO 303
 302  CONTINUE
      GO TO (3021,3022,3022,3021),ICYCLE
 3021 DELTAP = CMPLX(REAL(DELTA1),AIMAG(DELTAP))
      GO TO 3023
 3022 DELTAP = CMPLX(REAL(DELTAP),AIMAG(DELTA1))
 3023 CONTINUE
C     NOW CONNECT THE PARTS OF 'CV1' AND 'CVP'.
      GO TO (3024,3025,3024,3025),ICYCLE
 3024 CVP = CMPLX(REAL(CV1),AIMAG(CVP))
      GO TO 3026
 3025 CVP = CMPLX(REAL(CVP),AIMAG(CV1))
 3026 CONTINUE
C
C     NOW CONNECT THE REAL AND IMAGINARY PARTS OF THE INTERPOLATED
C     'DELTA2'
C
 303  CONTINUE
      GO TO (3031,3032,3032,3031),ICYCLE
 3031 IF(REAL(DELTA2)) 305,306,306
 3032 IF(AIMAG(DELTA2)) 305,306,306
C3033 CONTINUE
 305  CONTINUE
      GO TO (3051,3052,3052,3051),ICYCLE
 3051 DELTAN = CMPLX(REAL(DELTA2),AIMAG(DELTAN))
      GO TO 3053
 3052 DELTAN = CMPLX(REAL(DELTAN),AIMAG(DELTA2))
 3053 CONTINUE
C     NOW CONNECT THE PARTS OF 'CV2' AND 'CVN'.
      GO TO (3054,3055,3054,3055),ICYCLE
 3054 CVN = CMPLX(REAL(CV2),AIMAG(CVN))
      GO TO 3056
 3055 CVN = CMPLX(REAL(CVN),AIMAG(CV2))
 3056 CONTINUE
      GO TO 307
 306  CONTINUE
      GO TO (3061,3062,3062,3061),ICYCLE
 3061 DELTAP = CMPLX(REAL(DELTA2),AIMAG(DELTAP))
      GO TO 3063
 3062 DELTAP = CMPLX(REAL(DELTAP),AIMAG(DELTA2))
 3063 CONTINUE
C     NOW CONNECT THE PARTS OF 'CV2' AND 'CVP'.
      GO TO (3064,3065,3064,3065),ICYCLE
 3064 CVP = CMPLX(REAL(CV2),AIMAG(CVP))
      GO TO 3066
 3065 CVP = CMPLX(REAL(CVP),AIMAG(CV2))
 3066 CONTINUE
 307  NINTP = 1
      GO TO (3071,3072,3072,3071),ICYCLE
 3071 DP = REAL(DELTAP)
```

```
      DN = REAL(DELTAN)
      GO TO 3073
 3072 DP = AIMAG(DELTAP)
      DN = AIMAG(DELTAN)
 3073 CONTINUE
      GO TO (3074,3075,3074,3075),ICYCLE
 3074 CP = REAL(CVP)
      CN = REAL(CVN)
      GO TO 3076
 3075 CP = AIMAG(CVP)
      CN = AIMAG(CVN)
 3076 CONTINUE
      IF(CP.EQ.0.0.AND.CN.EQ.0.0.OR.DP.EQ.0.0.AND.DN.EQ.0.0)GO TO 900
      FN = XTERPL(DP,DN,CP,CN)
      GO TO (3077,3078,3077,3078),ICYCLE
 3077 CB1 = CMPLX(FN,AIMAG(CB1))
      GO TO 3079
 3078 CB1 = CMPLX(REAL(CB1),FN)
 3079 CONTINUE
      NREV = 3
      NTRLS = NTRLS + 1
      DELTA1 = DELTA2
      CV1 = CV2
      GO TO 122
C
 900  CONTINUE
      NINTP = 0
      GO TO (3082,3081,3082,3082) , ICYCLE
 3081 CB1REC = CB1
      CB1TRL = CB1
      NCIRC = 0
      XKONST = 1.0
      PHI = 0
      PI = 3.14159265
      PINCR = PI/4
 3082 CONTINUE
      GO TO (9081,9084,9081,9082),ICYCLE
 9081 CINCR = CMPLX(-.5*REAL(CINCR),AIMAG(CINCR))
      GO TO 9083
 9082 CINCR = CMPLX(REAL(CINCR),-.5*AIMAG(CINCR))
      GO TO 9083
 9084 CINCR = CMPLX(AIMAG(CB1REC),AIMAG(CINCR))
 9083 CONTINUE
      GO TO (9091,9092,9091,9092),ICYCLE
 9091 CB1TRL = CMPLX(REAL(CB1)             ,AIMAG(CB1TRL))
      GO TO 9093
 9092 CB1TRL = CMPLX(REAL(CB1TRL),AIMAG(CB1))
 9093 CONTINUE
      GO TO (9001,9001,9001,9009), ICYCLE
 9001 ITRACK = ITRACK + 1
```

```
     GO TO 121
9008 WRITE(3,211) NCIRC
     RETURN
9009 CONTINUE
       RETURN
8901 WRITE(3,210) NTRLS
     RETURN
C     DEBUG TRACE,SUBCHK,SUBTRACE,INIT(CV1,CV2,CB1REC,CB1TRL,VALUE,NCIR
C    *,CELTA,CELTA1,CELTA2,PHI,CINCR,PHI1,DIFVEL,SUMVEL)
C     AT 121
C     TRACE ON
C     AT 123
C     TRACE OFF
C     AT 140
C     TRACE ON
C     AT 1412
C     TRACE OFF
C     AT 900
C     TRACE ON
C     AT 9009
C     TRACE OFF
     END
     SUBROUTINE GMATRX    (A,R,S,T,G,NS)                           GMATRX
     REAL EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),HH(6),
    1 FREQ1,FREINC
     COMPLEX          XK,A(6),R(6),S(6),T(6),E(6,4,4),
    1 PM(6,4,4),CINV(6,4,4),XJ(4,4),VALUE,DELXJ(2,2),
    2 T6,T7,G(4,4)
     REAL ABS,SQRT,XLMDA(6)
     COMPLEX  CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CMPLX,XN,
    1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CVN
     COMPLEX  CSQRT,CCOS,CSIN,CRH,CSH,SRH,SSH
     COMMON ALPHA,BETA,XK,XMU,XLMDA
     COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
    1 CVN,CVP,WLNGTH,WSWTCH,NTRLS,ICYCLE,ISWTCH
C
C     DEFINITION OF MATRIX 'G', AS GIVEN IN THROWER'S PAPER
C
     DO 1 J = 1,4
     DO 1 I = 1,4
 1   G(I,J) = 0.0
C11   DO 10 I = NS,NS
     I=NS
C
C
     G(1,1) =        1./(2.*T(I)*(A(I)-XK**2))
     G(1,2)   =    XK/(2.*T(I)*R(I)*(A(I)-XK**2))
     G(1,3)   =    A(I)/(R(I)*(A(I)-XK**2))
     G(1,4)   =   XK/(A(I) - XK**2)
     G(2,1)  =     XK/(2.*T(I)*S(I)*(A(I)-XK**2))
```

```
      G(2,2) = G(1,1)
      G(2,3) = G(1,4)
      G(2,4) =       A(I)/(S(I)*(A(I)-XK**2))
 10   CONTINUE
      RETURN
      END
      SUBROUTINE EMATRX   (HH,A,R,S,T,E,N)                    EMATRX
      REAL EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),HH(6),
     1 FREQ1,FREINC
      REAL ABS,SQRT,XLMDA(6)
      COMPLEX         XK,A(6),R(6),S(6),T(6),E(6,4,4),
     1 PM(6,4,4),CINV(6,4,4),XJ(4,4),VALUE,DELXJ(2,2),
     2 T6,T7,G(4,4)
      COMPLEX  CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CMPLX,XN,
     1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CVN
      COMPLEX  CSQRT,CCOS,CSIN,CRH,CSH,SRH,SSH,CEXP
      COMMON ALPHA,BETA,XK,XMU,XLMDA
      COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
     1 CVN,CVP,WLNGTH,WSWTCH,NTRLS,ICYCLE,ISWTCH
C      SET UP THE ELEMENTS OF THROWER'S "E" MATRIX
C
      DO 401 I = 1, 6
      DO 401 J = 1, 4
      DO 401 K = 1, 4
 401  E(I,J,K) = 0.0
 402  DO 499 I = 1, N
      AIXK = A(I) - XK*XK
C      T(I) =T(I)
C      S(I) =S(I)
      R1 = CEXP(R(I)*HH(I))
      R2 = CEXP(-R(I)*HH(I))
      CRH = 0.5*(R1 + R2)
      SRH = 0.5*(R1 - R2)
      S1 = CEXP(S(I)*HH(I))
      S2 = CEXP(-S(I)*HH(I))
      CSH =0.5*(S1+S2)
      SSH =0.5*(S1-S2)
      E(I,1,1) = (A(I)*CRH - XK*XK*CSH)/AIXK
      E(I,1,2)=XK*(A(I)*SRH/R(I)-S(I)*SSH)/AIXK
      E(I,1,3)= 2.*T(I)*(A(I)*A(I)*SRH/R(I)-XK*XK*S(I)*SSH)/AIXK
      E(I,1,4) = 2.*T(I)*XK*A(I)*(CRH - CSH)/AIXK
      E(I,2,1) = -XK*(R(I)*SRH - A(I)*SSH/S(I))/AIXK
      E(I,2,2) = (A(I)*CSH - XK*XK*CRH)/AIXK
      E(I,2,3) = -2.*T(I)*XK*A(I)*(CRH - CSH) /AIXK
      E(I,2,4) = -2.*T(I)*(XK*XK*R(I)*SRH - A(I)*A(I)*SSH/S(I))/AIXK
      E(I,3,1) = (R(I)*SRH - XK*XK*SSH/S(I))/(2.*T(I)*AIXK)
      E(I,3,2) = XK*(CRH-CSH)/(2.*T(I)*AIXK)
      E(I,3,3) = (A(I)*CRH - XK*XK*CSH)/AIXK
      E(I,3,4) = XK*(R(I)*SRH - A(I)*SSH/S(I))/AIXK
      E(I,4,1) = -XK*(CRH - CSH)/(2.*T(I)*AIXK)
```

```
      E(I,4,2) = -(XK*XK*SRH/R(I)-S(I)*SSH)/(2.*T(I)*AIXK)
      E(I,4,3) = - XK*(A(I)*SRH/R(I) -S(I)*SSH)/AIXK
      E(I,4,4) = -(XK*XK*CRH-A(I)*CSH)/AIXK
  499 CONTINUE
      RETURN
      END
      SUBROUTINE PROMAT(N,NS,E,G,DELXJ)                          PROMAT
      REAL EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),HH(6),
     1 FREQ1,FREINC
      REAL ABS,SQRT,XLMDA(6)
      COMPLEX        XK,A(6),R(6),S(6),T(6),E(6,4,4),
     1 PM(6,4,4),CINV(6,4,4),XJ(4,4),VALUE,DELXJ(2,2),
     2 T6,T7,G(4,4)
      COMPLEX  CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CMPLX,XN,
     1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CVN
      COMPLEX  CSQRT,CCOS,CSIN,CRH,CSH,SRH,SSH
      COMMON ALPHA,BETA,XK,XMU,XLMDA
      COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
     1 CVN,CVP,WLNGTH,WSWTCH,NTRLS,ICYCLE,ISWTCH
C201  FORMAT('XJ(',I2,',',I2,') = ', D14.8)
C
C        CONTINUED PRODUCT OF THE 'E' MATRICES.
C
C     START SETTING UP THE PRODUCT MATRICES "PM".  THEY ARE OBTAINED
C     USING THE CONTINUED PRODUCT OF THE 'E' MATRICES.
C     IS PRE-MULTIPLIED BY THE 'G' MATRIX FOR THE SEMI-INFINITE MEDIUM.
C                    NS  =  NO. OF MEDIA
C                    N   =  NO. OF LAYERS
C                        =  NS  -  1  .
      DO 26 K = 1,N
      DO 20 J = 1,4
      DO 20 M = 1,4
   20 PM(K,M,J) = 0.0
      IF(K-1) 21,21,23
C     PM(1,I,J) = E(1,I,J) IS THE 'E' MATRIX FOR THE TOP LAYER
   21 DO 22 J = 1,2
      DO 22 I = 1,4
   22 PM(K,I,J) = E(K,I,J+2)
      GO TO 26
   23 CONTINUE
C
C     PRE-MULTIPLY THE PRODUCT MATRIX THUS FAR ESTABLISHED BY THE 'E'
C     MATRICES FOR THE SUCCESSIVELY LOWER LAYERS
C
C     FOR AN EXPLANATION OF THE STRANGE APPEARANCE OF 'PM' AT THIS ST
C     SEE THROWER'S EQUATION (19).  AS THE INITIAL 'PM' MATRIX
C     (THE CONTRIBUTION FROM THE TOP LAYER) IS ONLY A 4(ROW) X 2(COLUM
C     MATRIX, SUBSEQUENT 'PM' MATRICES ARE ONLY 4(ROW) X 2(COLUMN)
C     MATRICES.  THIS CONSTITUTES THE RATHER ASTOUNDING ECONOMY
C     OF THE METHOD.
```

```
C
      DO 25 J = 1,2
      DO 25 I = 1,4
      T6 = E(K,I,1)*PM(K-1,1,J) + E(K,I,2)*PM(K-1,2,J)
     1   +E(K,I,3)*PM(K-1,3,J) + E(K,I,4)*PM(K-1,4,J)
      PM(K,I,J) = T6
   25 CONTINUE
C
C     NOW PRE-MULTIPLY THE PRODUCT MATRIX 'PM' BY THE 'G' MATRIX,
C     A 2 X 4 RECTANGULAR MATRIX DEFINED IN THROWER'S PAPER.  THIS
C     YIELDS THE 'TEST' DETERMINANT DENOTED BY 'DELXJ'.
C
   26 CONTINUE
      GO TO (27,36), ISWTCH
   27 DO 33 I = 1,2
      DO 33 J = 1,2
   33 DELXJ(I,J) = 0.0
      DO 35 J = 1,2
      DO 35 I = 1,2
      T7 = G(I,1)*PM(N,1,J) + G(I,2)*PM(N,2,J)
     1   +G(I,3)*PM(N,3,J) + G(I,4)*PM(N,4,J)
      DELXJ(I,J) = T7
   35 CONTINUE
      RETURN
   36 DELXJ(1,1) = PM(N,1,1)
      DELXJ(2,2) = PM(N,2,2)
      DELXJ(1,2) = PM(N,1,2)
      DELXJ(2,1) = PM(N,2,1)
      RETURN
      END
      SUBROUTINE CHECK (DELXJ,VALUE)                              CHECK
      REAL EM(6),V(6),XMU(6),BETA(6),ALPHA(6),RHO(6),HH(6),
     1 FREQ1,FREINC
      REAL ABS,SQRT,XLMDA(6)
      COMPLEX       XK,A(6),R(6),S(6),T(6),E(6,4,4),
     1 PM(6,4,4),CINV(6,4,4),XJ(4,4),VALUE,DELXJ(2,2),
     2 T6,T7,G(4,4)
      COMPLEX  CB1,CB1TRL,CINCR,WLNGTH,CV1,CV2,CMPLX,XN,
     1 DELTA1,DELTA2,DELTAN,DELTAP,FREQ,CVP,CVN
      COMPLEX  CSQRT,CCOS,CSIN,CRH,CSH,SRH,SSH
      COMMON ALPHA,BETA,XK,XMU,XLMDA
      COMMON CB1,DELTA1,DELTA2,CV1,CV2,CB1TRL,CINCR,DELTAN,DELTAP,
     1 CVN,CVP,WLNGTH,WSWTCH,NTRLS,ICYCLE,ISWTCH
      VALUE = DELXJ(1,1)*DELXJ(2,2) - DELXJ(1,2)*DELXJ(2,1)
      RETURN
C     DEBUG TRACE, SUBTRACE, INIT
C    1           (EM,V,XLMDA,XMU,BETA,ALPHA,XK,A,R,S,T,E,PM,CINV,
C    1XJ,CRH,CSH,SRH,SSH,TEST,VALUE,HH,I,J,K,K1)
      END
      FUNCTION XTERPL(DELTA1,DELTA2,CV1,CV2)                      XTERPL
```

```
      SLOPE=(DELTA2-DELTA1)/(CV2-CV1)
      XTERPL=CV1-DELTA1/SLOPE
      RETURN
      END
```

A.4.3 Sample input to RAY.FOR

The following input (data) file was generated by the progam PREP.FOR

Note that it is formatted by the program PREP.FOR

```
LCPC DATA E1=11200,E2=34000,E3=1340;H1=0.1,H2=0.2,NU1=NU3=0.25,NU2=0.4 0.
3 11200. .2534000. .40 1340. .25
2. 2. 2.
.10 .20
3.50 .00 .40 .00 .10 .01 .02 .10
```

A.4.4 Sample output from RAY.FOR

An abbreviated sample output from RAY.FOR is shown in Table A.3. The velocity, wavelength, frequency and wavenumber are complex. The counter registration is real.

LCPC DATA E1=11200,E2=34000,E3=1340;H1=0.1,H2=0.2,NU1=NU3=0.25,NU2=0.4
THE PROBLEM PARAMETERS ARE
LAYER 1 HAS MODULUS 11200. POISSON'S RATIO .250
DENSITY 2. AND THICKNESS .10 UNITS
LAYER 2 HAS MODULUS 34000. POISSON'S RATIO .400
DENSITY 2. AND THICKNESS .20 UNITS
LAYER 3 HAS MODULUS 1340. POISSON'S RATIO .250
DENSITY 2. AND IS SEMI-INFINITE.

VELOCITY	WAVELENGTH	FREQUENCY	WAVE NUMBER	COUNTER
4.246 .000	3.500 .000	1.213 .000	.286 .000	42
3.421 .001	2.593 .001	1.319 .000	.386 .000	34
3.020 .001	2.059 .000	1.467 .000	.486 .000	30
2.764 .001	1.707 .000	1.619 .000	.586 .000	27
.694 .000	1.458 .001	.476 .000	.686 .000	13
.761 .001	1.273 .001	.598 .000	.786 -.001	12
.821 .001	1.129 .001	.727 .000	.886 -.001	12
.874 .001	1.014 .001	.861 .000	.986 -.001	13
.920 .001	.921 .001	.999 .000	1.086 -.001	13
1.029 .001	.843 .001	1.221 .000	1.186 -.001	14

Table A.3: Sample output from program RAY.FOR

A.5 Inverse programmes APP_S.FOR and APP_H.FOR

Inverse programmes APP_S.FOR and APP_H.FOR were written in order to facilitate matching results of measurements with those expected from ideal layered structures. The ideal structures are composed of a single layer overlying a semi-infinite medium. The program APP_S.FOR is intended for a structure in which the stiffness of the materials increases with the depth. The program APP_H.FOR is for the opposite case, in which the stiffness of the materials decreases with the depth.

Both programmes are intended to be used interactively. The limiting velocities β_1 and β_2 are supplied at a prompt from the program. Then a point on the plot of reciprocal wavelength versus frequency is entered in response to a prompt. The point is selected, subjectively, to indicate the curvature of the plotted experimental points.

Both programmes utilize an approximation to the frequency equation of the system. The approximation is based on expanding the transcendental functions to their first terms. While not consistently so, it appears that these programmes yield thicknesses of the surface layer which are too small by a factor of two. This error could possibly be decreased if additional terms were used in the approximation.

A.5.1 APP_S.FOR

```
c This is an inverse program. The limiting velocities are read. Then
c pairs of reciprocal wavelength and frequency are used interactively to
c calculate the thickness of the surface layer. The calculation is the
c result of a quadratic: two values of the thickness are obtained.
c
c This program is an approximation to Lee's determinant for the
c "soft over hard" case, using a first order expansion of the transcendental
c functions. It is based on equation (21) in the condensed version of
c A.W.Lee's paper, A_W_LEE.DOC. The full paper is A.W.Lee, "The effect of
c geological structure upon microseismic disturbance", Monthly Notices of the
c Royal Astronomical Society, Geophysical Supplement, Vol 3, 1932,
c pp. 83 - 105   It solves for <h> in terms of the
c measured quantities, c the reciprocal wavelength and the corresponding
c frequency. The coefficients "aa", "bb", "cc" are derived in the
c Mathematica notebook LEE_EQS.nb, where they are expressed in Fortran
c form using LEE_EQS.nb

      implicit real (i-k)
 201    format(3g16.3)
  10    write(*,*) "  Enter beta1 and beta2 "
      read(*,*) beta1, beta2
      alpha1=1.732*beta1
      alpha2=1.732*beta2
  20    write(*,*) "Enter reciprocal wavelength, frequency"
      read(*,*) rlngth,freq
      write(*,201) rlngth,freq
      mu1=1.0
```

```
      mu2=mu1*(beta2/beta1)**2
      c=freq/rlngth
      omega= 6.28*freq
      k=6.28*rlngth
      ka1=omega/alpha1
      kb1=omega/beta1
      ka2=omega/alpha2
      kb2=omega/beta2
c        write(*,*) "to here1"
      write(*,*) "omega=",omega,"   c=",c
      Rarg= +(omega**2/alpha1**2) - omega**2/c**2
      if(Rarg.lt.0) write(*,*)"Rarg.lt.0"
      R=Sqrt(+(omega**2/alpha1**2) - omega**2/c**2)
      write(*,*) "R=", R
c       write(*,*) "beta1 = ",beta1,"beta2 = ",beta2
      Sarg= +(omega**2/beta1**2) - omega**2/c**2
      S=Sqrt(+(omega**2/beta1**2) - omega**2/c**2)
      write(*,*) "S=", S
      write(*,*) "  alpha1=",alpha1,"  alpha2=",alpha2
      r2arg= -omega**2/alpha2**2 + omega**2/c**2
 22     r2=Sqrt(-omega**2/alpha2**2 + omega**2/c**2)
c       write(*,*) " r2=",r2
      s2arg= -omega**2/beta2**2 + omega**2/c**2
      if(s2arg) 23,23,24
 23     write(*,*) "s2arg .lt. 0; c must be less than beta2"
      goto 20  ! The solution failed. Try again, using a smaller value of c.
 24   continue
      s2=Sqrt(-omega**2/beta2**2 + omega**2/c**2)
c       write(*,*) " s2=",s2


c now define the Love parameters
c        write(*,*) "to here2"

      X=c**2*mu2/(beta2**2*mu1) - 2*(-1 + mu2/mu1)
      Y=c**2/beta1**2 + 2*(-1 + mu2/mu1)
      Z=-(c**2/beta1**2) + c**2*mu2/(beta2**2*mu1) - 2*(-1 + mu2/mu1)
      W=2*(-1 + mu2/mu1)
c now define the terms of the quadratic (=zero) aa*h^2+bb*h+cc=0
c from Mathematica notebook LEE/APP_S.MA
       aa=-4*k**2 + 4*r2*s2 + 16*r2*s2*W/3 + 4*r2*s2*W**2/3 +
     -    16*k**2*X/3 - 4*k**2*X**2/3 - 4*r2*s2*Y - 8*r2*s2*W*Y/3 +
     -    r2*s2*Y**2 - 4*k**2*Z + 8*k**2*X*Z/3 - k**2*Z**2
       bb=
     -  2*r2*W - 2*s2*W/3 + 2*r2*X - 2*s2*X/3 - 2*r2*Y -
     -    2*s2*Y + r2*X*Y + s2*X*Y - 2*r2*Z - 2*s2*Z - r2*W*Z - s2*W*Z

c      cc=4-4*r2*s2/k**2-4*r2*s2*W/k**2-r2*s2*W**2/k**2-4*X+X**2
      cc=4-4*r2*s2*W/k**2
      write(*,*) "beta1 = ",beta1,"  beta2 = ",beta2
```

```
      write(*,*) "          aa             bb             cc    "
      write(*,201) aa,bb,cc
      if((bb**2-4*aa*cc).lt. 0) write(*,*) "(bb**2-4*aa*cc) .lt. 0"
      if((bb**2-4*aa*cc).lt. 0)goto 20
      h=(-bb+sqrt(bb**2-4*aa*cc))/(2*aa)
      h1= (-bb-sqrt(bb**2-4*aa*cc))/(2*aa)
      write(*,*) "  Thickness(1)     Thickness(2)       S*h "
      write(*,201) h,h1,S*h
      goto 20
      end
```

A.5.2 APP_H.FOR

```
c The following file is an approximation to Lee's determinant for the
c "hard over soft" case, using a first order expansion of the transcendental
c functions. It is based on equation (21) in the condensed version of
c A.W.Lee's paper, A_W_LEE.DOC. It solves for <h> in terms of the
c measured quantities,  the reciprocal wavelength and the corresponding
c frequency. The coefficients "aa", "bb", "cc" are derived in the
c Mathematica notebook LEE/APP_H.MA, where they are expressed in Fortran
c form.
c
        implicit real(i-n)
  201    format(3g16.3)
  202   format(2g10.1)
  203   format('  alpha1=',f8.1,'  alpha2=',f8.1)
  204   format('  beta1 = ', f8.1, '   beta2 = ', f8.1)
  205   format('  omega=  ',f8.1,'    c= ',f8.1)
 10     write(*,*) "  Enter beta1 and beta2 "
        read(*,*) beta1, beta2
        alpha1=1.732*beta1
        alpha2=1.732*beta2
 20     write(*,*) "Enter reciprocal wavelength, frequency"
        read(*,*) rlngth,freq
        write(*,201) rlngth,freq
        mu1=1.0
        mu2=mu1*(beta2/beta1)**2
        c=freq/rlngth
        omega= 6.28*freq
        k=6.28*rlngth
        ka1=omega/alpha1
        kb1=omega/beta1
        ka2=omega/alpha2
        kb2=omega/beta2
c        write(*,*) "to here1"
        write(*,205) omega,   c
        Rarg= -(omega**2/alpha1**2) + omega**2/c**2
        if(Rarg.lt.0) write(*,*)"Rarg.lt.0 c must be less than alpha1"
        if(Rarg.lt.0) goto 20
        R=Sqrt(-(omega**2/alpha1**2) + omega**2/c**2)
```

```
c       write(*,*) "R=", R
c       write(*,202) "beta1 = ",beta1,"beta2 = ",beta2
        Sarg= -(omega**2/beta1**2) + omega**2/c**2
        if(Sarg.lt.0) write(*,*)"Rarg.lt.0 c must be less than beta1 "
        if(Sarg.lt.0) goto 20
        S=Sqrt(-(omega**2/beta1**2) + omega**2/c**2)
        write(*,203) alpha1,alpha2
        r2arg= omega**2/alpha2**2 - omega**2/c**2
        if(r2arg) 21,21,22
 21     write(*,*) "r2arg .lt. 0; c must exceed alpha2"
        goto 20
 22     r2=Sqrt(omega**2/alpha2**2 - omega**2/c**2)
c       write(*,*) " r2=",r2
        s2arg= omega**2/beta2**2 - omega**2/c**2
        s2=Sqrt(omega**2/beta2**2 - omega**2/c**2)
c       write(*,*) " s2=",s2
c  now define the Love parameters
c        write(*,*) "to here"

        X=c**2*mu2/(beta2**2*mu1) - 2*(-1 + mu2/mu1)
        Y=c**2/beta1**2 + 2*(-1 + mu2/mu1)
        Z=-(c**2/beta1**2) + c**2*mu2/(beta2**2*mu1) - 2*(-1 + mu2/mu1)
        W=2*(-1 + mu2/mu1)
c now define the value of the determinant (=zero) aa*h^2+bb*h+cc=0
        aa=-4*k**2 + 4*r2*s2 + 16*r2*s2*W/3 + 4*r2*s2*W**2/3 +
     -    16*k**2*X/3 - 4*k**2*X**2/3 - 2*r2*s2*Y - 2*r2*s2*W*Y -
     -    4*k**2*Z + 8*k**2*X*Z/3 - k**2*Z**2
        bb=
     -  2*r2*W - 2*s2*W/3 + 2*r2*X + 10*s2*X/3 +
     -    4*s2*W*X/3 - 2*s2*Y - s2*X*Y - 2*r2*Z - 2*s2*Z - r2*W*Z -
     -    s2*W*Z
        cc=4 - 4*r2*s2/k**2 -
     -    4*r2*s2*W/k**2 - r2*s2*W**2/k**2 - X**2
        write(*,204) beta1, beta2
c       write(*,*) "X*Y+W*Z =",X*Y+W*Z ! a factor of <bb>
c       write(*,*) "cc=",cc
        write(*,*) "          aa            bb             cc   "
        write(*,201) aa,bb,cc
        h=(-bb+sqrt(bb**2-4*aa*cc))/(2*aa)
        h1= (-bb-sqrt(bb**2-4*aa*cc))/(2*aa)
        write(*,*) "   Thickness(1)     Thickness(2)       S*h "
        write(*,201) h,h1,S*h
        goto 20
        end
```

Appendix B

Errors in inverse

In this appendix a further evaluation is attempted of the approximate inverse programme APP_H.FOR (section A.5.2). Interpretations made with the aid of APP_H.FOR are given, followed by graphical presentation of the experimental results.

B.1 Experimental results.

Table B.1 and Table B.2 have been given previously.[1] They are presented again for convenience, followed by graphs of the experimental results. The graphs show also the approximations made and referred to in Table B.1 and Table B.2. The equation quoted by Cogill [3] is used with the results given in Cogill [2] to obtain a quadratic equation in terms of the pavement thickness h.

By extrapolating the data shown in Fig. B.1: $\alpha_1 = 1732m/s$, $\beta_1 = 1000m/s$, $\alpha_2 = 346m/s$, $\beta_2 = 200m/s$. At reciprocal wavelength = 1.0, the frequency is 470Hz. The approximation leads to a calculated thickness of 0.19m. The measured overall thickness of the surface slab is 0.22m. The results are shown in Table B.1.

The system analysed next consisted of 150 mm compacted thickness of limestone quarry scalpings, and a surface course of 60 mm consolidated thickness of

[1] Cogill, W.H., "Single-layered inverse: First-order approximation". ISVR Technical Memorandum No. 833, January 1999. pp. 10,11. Table 1, Table 2.

Reference to original data	Data		Approximations		Thickness	
	Surf. velocity β_1 (m/s)	Lower velocity β_2 (m/s)	Recip. w'lngth (1/m)	Freq-uency (Hz)	Calc. (m)	Meas. (m)
Goulburn	1000	200	1	470	0.19	0.22
COMBA2	1600	500	1	900	0.10	0.1
240A2	2000	300	0.35	400	0.11	0.1
480A2	850	80	0.33	55	0.24	0.24 [2]

Table B.1: **Approximate layer thicknesses, calculated from the results of the measurements of the surface phase velocity at various frequencies.**

Reference to original data	Data		Approximations		Thickness	
	Surf. velocity β_1 (m/s)	Lower velocity β_2 (m/s)	Recip. w'lngth (1/m)	Freq-uency (Hz)	Calc. (m)	Meas. (m)
10_25	3040	920	0.37	7900	0.13	0.15
1_05	1220	284	0.27	2500	0.12	0.15
1_10	1830	850	0.118	1700	0.19	0.15
20_25	1530	289	0.15	1200	0.19	0.15

Table B.2: **Approximate layer thicknesses, calculated from the results of the measurements of the surface phase velocity at various frequencies: Prepared base course 0.15m in thickness.**

28 mm nominal size dense imestone bitumen macadam base course and 25 mm compacted thickness of 10 mm nominal size dense limestone bitumen macadam wearing course. From results of measurements made on this system, (COMBA2) $\alpha_1 = 2771m/s$, $\beta_1 = 1600m/s$, $\alpha_2 = 866m/s$, $\beta_2 = 500m/s$. At a value of the reciprocal wavelength = 1.0, the frequency is 900Hz. The approximation leads to a calculated thickness of 0.10m.

Finally, field measurements were performed on the surface of a base course composed of unbound gravel laid on a prepared subgrade. Table B.2 shows the values of the approximate thicknesses calculated from the results obtained[3]. The approximation is valid only at low frequencies, although complete measurements must be performed in order to estimate the limiting velocities of propagation in the media composing the structure.

The following figures show the results of experimental measurements on pavements having a variety of compositions. The approximations, indicated in Tables B.1 and B.2, are shown in the figures.

B.2 Constructed pavements

The results shown in Figures B.1 to B.4 were obtained on the surfaces of constructed pavements. The results of the interpretations are summarized in Table B.1.

B.3 Prepared basecourses

The results shown in Figures B.5 to B.8 were obtained on the surfaces of prepared basecourses. The results of the interpretations are summarized in Table B.2.

[3]Poisson's ratio = 0.05 for unbound gravel.

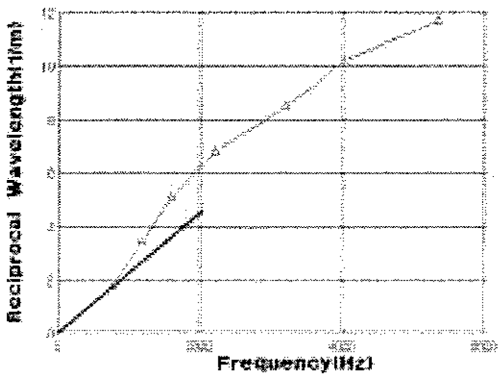

Figure B.1: Results of measurements : Freeway pavement.

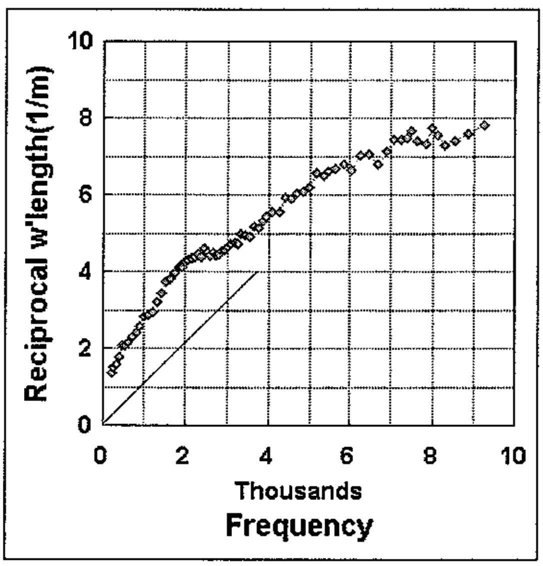

Figure B.2: Results of measurements : Parking area: Combined results

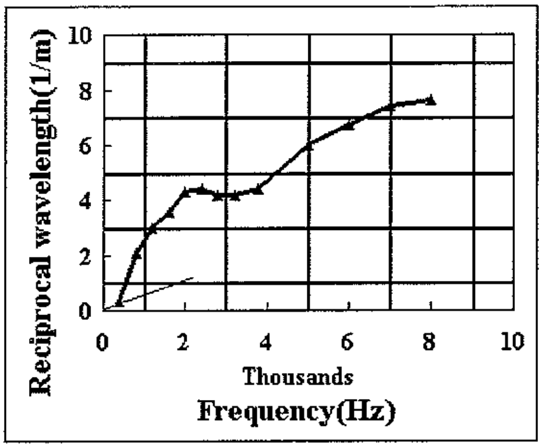

Figure B.3: Results of measurements : Parking area: 240mm spacing.

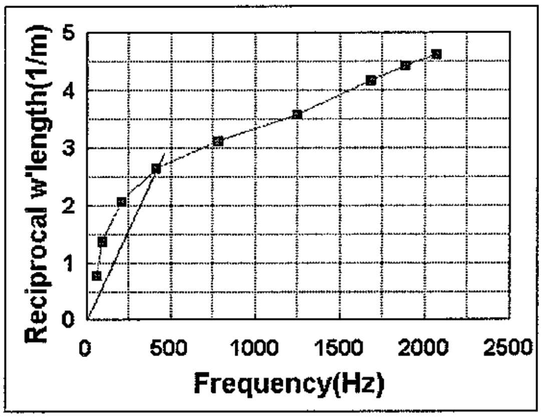

Figure B.4: Results of measurements : Parking area: 480mm spacing.

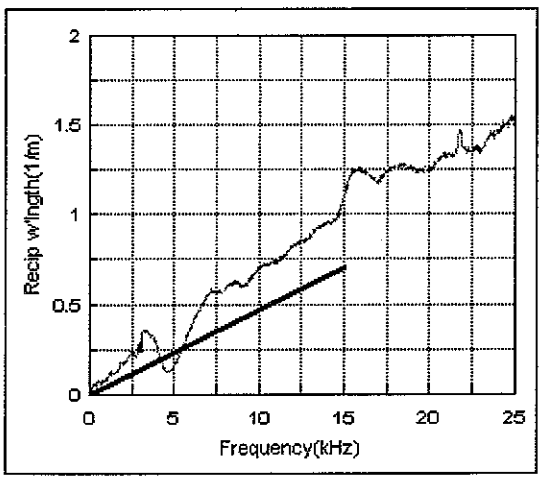

Figure B.5: Results of measurements: 10_25. Hammer 1, 0.25 feet spacing

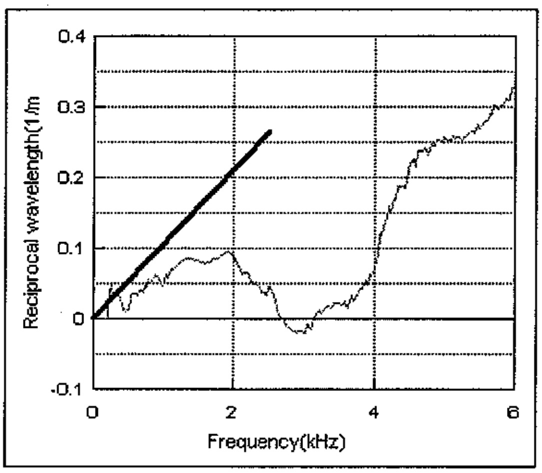

Figure B.6: Results of measurements: 1_05.Hammer 1, 0.5 feet spacing

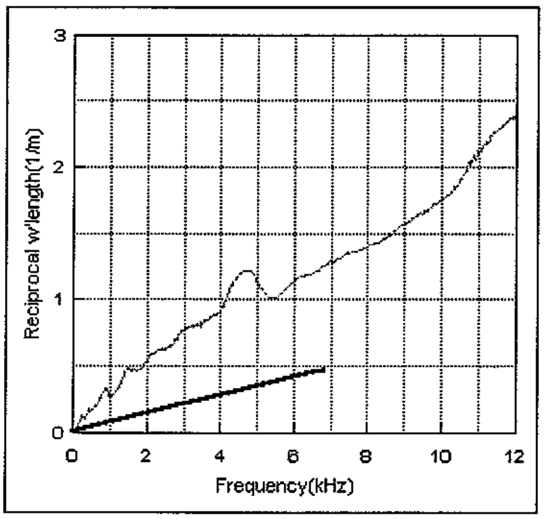

Figure B.7: Results of measurements: 1_10. Hammer 1, 1.0 foot spacing

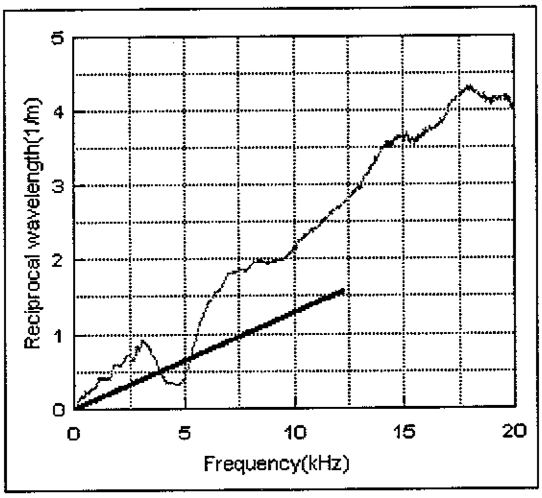

Figure B.8: Results of measurements: 20_25. Hammer 2, 0.25 feet spacing