THE INFLUENCE OF RADIATION DAMPING ON THROUGH-RESONANCE VARIATION IN THE SCATTERING CROSS-SECTION OF GAS BUBBLES

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When a gas bubble in water undergoes forced pulsations, sound is radiated at the forcing frequency, and the scattering cross-section exhibits a resonance peak when the forcing frequency passes through the bubble's natural frequency. At resonance, the amplitude of the scattered spherical wave is determined by the amount of damping associated with the bubble dynamics. In his 1967 article, 'Sound propagation in the presence of bladder fish', Weston describes a model for the through-resonance frequency dependence of the scattering and extinction cross-sections, based on the work of Andreeva (1964). In Weston's model, if all damping terms other than radiation damping are omitted, the resonance peak is skewed, with a tendency for the scattering cross-section to increase with increasing frequency through resonance. In 1977, Medwin published 'Acoustical determination of bubble-size spectra', based on Eller (1970), in which he describes a similar model, according to which the predicted resonance peak is also skewed, but in the opposite direction to that predicted by Weston. If Medwin's model turns out to be valid, this would have little impact, as his curves are already in widespread use. However, if the Andreeva-Weston model is correct, a small adjustment becomes necessary to Medwin's curves. A possible experiment designed to establish the true frequency dependence is described, involving the ensonification of a single spherical bubble with a broadband pulse, through the bubble's resonance frequency. If the radiation damping can be separated form other effects, the correct frequency dependence can be established by measuring the spectrum of the scattered sound.

Keywords: *bubbles, scattering cross-section, extinction cross-section, radiation damping*

1. INTRODUCTION

Bubbles play an important role in the generation, absorption and scattering of underwater sound [1]. The acoustic properties of bubbles are generally well understood, to the extent that acoustical measurements are sometimes used to determine characteristics of bubble clouds such as their size distribution [2]. Such acoustical characterisation of bubble properties requires a firm foundation in theory. The purpose of this article is to point out a discrepancy in two published theories relating to radiation damping, to discuss the consequences of the discrepancy and to suggest possible ways of resolving it. The two models are described in Sec. 2, including a discussion of the discrepancy between them and its consequences. A possible experiment, designed to establish which of the two models is correct, is described in Sec. 3, followed by brief conclusions (Sec. 4).

2. THE SCATTERING MODELS

In this section two models of scattering from gas bubbles are described. Only the final result is given for each, as the derivations can be found in the original source references. For both models the scattering cross-section σ_s of a bubble of radius a at acoustic frequency f is [2], [3]

$$\sigma_\mathrm{s} = \frac{4\pi a^2}{\left(f_\mathrm{R}{}^2/f^2 - 1\right)^2 + \delta^2}, \tag{1}$$

where, following Medwin's notation, f_R is the bubble resonance frequency [4]. The damping parameter δ can be written

$$\delta \equiv \delta_\mathrm{tot} = \delta_\mathrm{rad} + \delta_\mathrm{other}, \tag{2}$$

where δ_rad is the contribution from radiation damping and δ_other includes all other contributions, such as viscous and thermal losses. The subscript 'tot' is introduced in order to distinguish unambiguously between radiation damping and total damping.

2.1. Wildt-Devin-Eller-Medwin (WDEM) model

Wildt [5] derives Eq. (1), with radiation damping term

$$\delta_\mathrm{rad} = ka, \tag{3}$$

and shows further that the extinction cross-section σ_e is related to σ_s via

$$\sigma_e = \sigma_s \frac{\delta_{tot}}{ka}, \tag{4}$$

where k is the acoustic wave number in water. Wildt's model is adapted by Medwin [2] to incorporate improved models of thermal and viscous damping terms derived by Devin [6], and extended to off-resonance conditions by Eller [7]. We refer to the complete model henceforth as the Wildt-Devin-Eller-Medwin (WDEM) model. The resulting damping term δ_{tot} is plotted vs bubble radius in Fig. 1 (left).

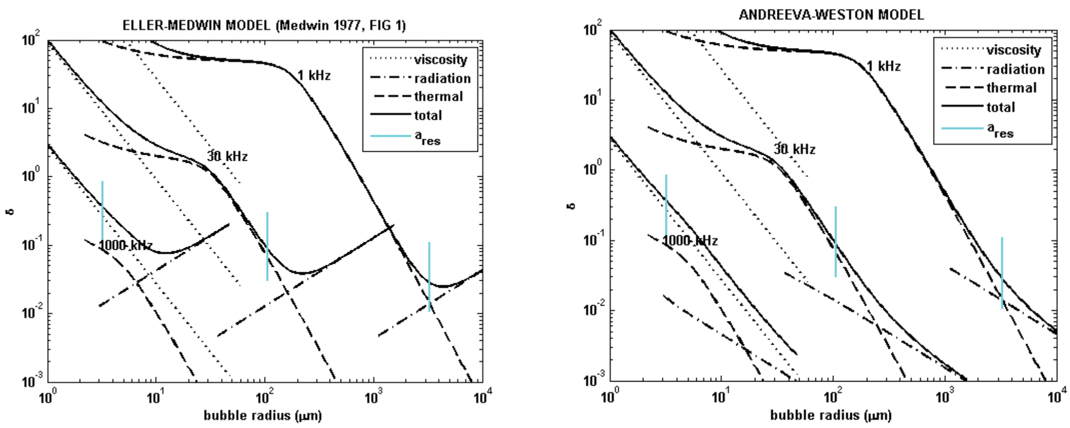

Fig.1: Theoretical damping parameter δ versus bubble radius for acoustic frequencies 1 kHz, 30 kHz and 1 MHz; the resonant bubble radius is marked for each frequency using a vertical grey (cyan) line; The contribution due to acoustic radiation is shown as a dash-dotted line (-.-). left: Wildt-Devin-Eller-Medwin; right: Andreeva-Weston.

2.2. Andreeva-Weston (AW) model

The scattering cross-section is given by Andreeva [8] in the same form as Eq. (1), but with the following expression for the radiation damping term

$$\delta_{rad} = \left(f_R^{\,2} / f^2\right) ka \,. \tag{5}$$

Weston [3] derives Eq. (1) for σ_s, and the following expression for σ_e

$$\sigma_e = \sigma_s \left(1 + \frac{\delta_{other}}{ka}\right), \tag{6}$$

but with δ_{rad} given by Eq. (5) instead of Eq. (3), in agreement with Andreeva. We refer to this model henceforth as the Andreeva-Weston (AW) model. Notice that, according to Eq. (5), the radiation damping term decreases linearly with increasing frequency (or bubble radius), exactly the opposite as predicted by WDEM. The value of δ_{tot} is plotted vs bubble radius in Fig. 1 (right), as calculated using the AW model, i.e., with Eq. (5) for radiation damping. It follows from Eq. (1) that σ_s is inversely proportional to δ^2 in the immediate vicinity of the resonance peak. Therefore, if the only damping arises through acoustic re-radiation, AW predicts an increase in σ_s with increasing frequency in this limited region, whereas the opposite dependence is predicted by WDEM. The overall effect is for the resonance peak to be skewed compared with expectation for a constant value of δ. The predicted skewness is the same for both models but in opposite directions.

2.3. The discrepancy and its consequences

A comparison between the two graphs of Fig. 1 shows that there is a large difference between the damping terms predicted by the two models for large bubbles (above resonance). Nevertheless, as pointed out by Weston [9], the impact is not as serious as it at first seems, because away from resonance, the effect of the radiation damping on both σ_s and on σ_e is small.

Even though the difference in magnitude turns out to be small, the difference in functional form is puzzling, and suggests that the through-resonance behaviour is not properly described by one of the two models. This difference should not be taken lightly. If sound is to be used as a measurement tool, the interaction of the sound with the observed object needs to be understood. If one is unaware of the discrepancy one might see nothing wrong in using δ_{rad} from one model, Eq. (5) say, to calculate extinction cross-section with the other, Eq. (4), leading to large errors in the prediction of σ_e.

3. WHICH MODEL IS CORRECT?

Having established that there is a discrepancy, the obvious question is "Which of the two models is correct?" A theoretical study of the difference between them is outside the present scope (see Ref. [10]). It is also desirable to confirm the theoretical findings by measurement, so the question becomes "Can we measure the variation of (say) σ_s through the resonance frequency". This is not a simple question to answer, because of the way the amplitude of the acoustic scattering cross-section peaks around the bubble resonance. Therefore, whilst the difference between Eqs. (3) and (5) would appear to become more pronounced the further one moves away from resonance, the amplitude of the acoustic scattering cross-section locally decreases away from the resonance to such an extent that the measurement itself becomes significantly more challenging. Furthermore, citing δ_{rad} on its own is in itself an artifice, because other loss mechanisms are also important close to resonance.

Hence, the experiment in question would have to select a bubble which, when insonified around resonance, produces dissipation mainly through radiation damping. For many sizes of air bubbles in water, resonance occurs roughly in the region where there is a balance between thermal and re-radiation losses. A large (e.g. millimetre-sized) bubble would be preferable to a smaller one because then the contribution from viscous damping would not be so great. It may also be possible to reduce the influence of thermal damping by carrying out the measurements under pressure.

The experiment would then be aimed at conducting measurements to examine the asymmetry in the bubble response around the resonance. This can be achieved using the apparatus shown in Fig. 2, which combines a number of transducers to elicit from the bubble several signals that characterise the bubble resonance. Measurements can be made both on freely rising bubbles, and on bubbles held stationary by 'tethering'. Of particular note for the purposes of this paper is the 'pump' transducer designed to drive the bubble into pulsation. By sending a pseudo-random noise sequence through this transducer, the through-resonance response can be obtained (see Fig. 10 of Ref. [11]) and examined for its asymmetry about the resonance, in keeping with the contribution of the radiation damping to the total losses.

Figure 2 also shows various other transducers that can be deployed simultaneously. Of particular relevance to the current study are the high frequency projector and receiver ('Panametrics V302'). These have been used to monitor the through-resonance behaviour of the combination-frequency $f_i \pm f_p$, where f_i is the 'imaging frequency' 1.134 MHz and f_p is the 'pump frequency'. The results for rising bubbles are shown in Fig. 3. As before, the asymmetry around resonance of these responses could be used to discriminate the true frequency dependence of the acoustic scattering cross-section. Improved statistics can be expected using tethered bubbles [11].

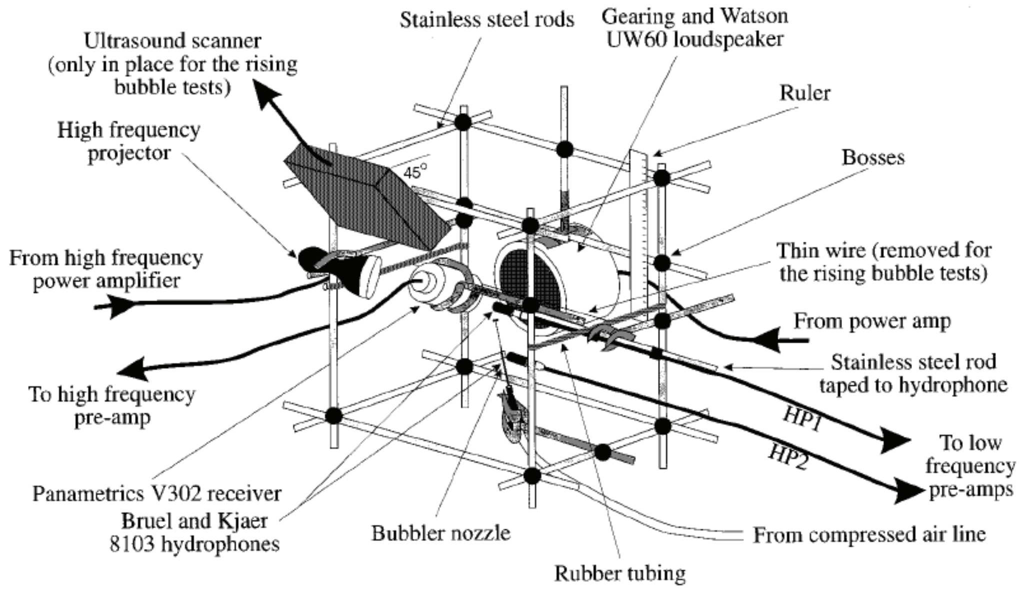

Fig.2: Apparatus used by Leighton et al. [11] for characterisation of the resonance.

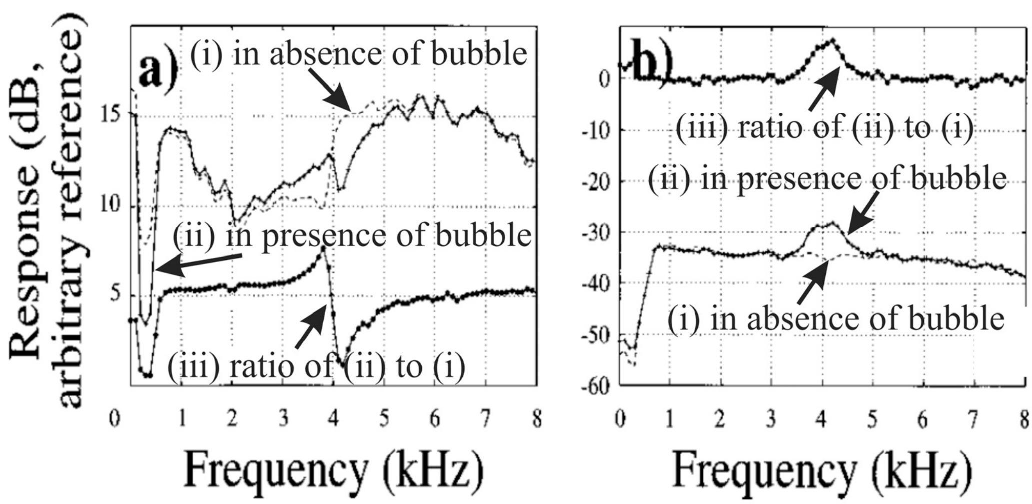

Fig.3: Plots with 98 Hz resolution of (a) the modulus of voltage transfer function and (b) coherence, calculated from the (a) signal from the B&K8103 hydrophone and (b) the heterodyned signal from the Panametric V302 receiver shown in Fig. 2. A rising stream of bubbles of radius 800 μm was subjected to broadband insonification by a band limited 1–8 kHz pseudo-random noise sequence.

4. CONCLUSIONS

A discrepancy is found between two models, published by Medwin [2] and Weston [3], and referred to in the present text, respectively, as WDEM and AW, that describe the scattering cross-section of gas bubbles in water. These two models make almost identical predictions but exhibit an unexplained difference in behaviour for small departures from resonance. The difference is negligible both far from resonance and exactly at resonance.

Because the difference in the models' predictions is small, a measurement able to distinguish between them is a challenging one. A possible experiment is described that involves scattering of pseudo-random noise pulses from tethered bubbles.

5. ACKNOWLEDGEMENTS

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