ENHANCEMENT OF NONLINEARITY AND PARAMETRIC SONAR USING BUBBLES

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Abstract: *The potential of nonlinear pulsating bubbles to improve the performance of parametric sonar has been known for some years, but exploitation of this phenomenon and its theoretical modelling is still under investigation. This paper focuses on some theoretical aspects of determining sonar parameters through modelling nonlinearly pulsating gas bubbles in water. Existing expressions for these parameters were developed when bubble theories had newly appeared, and since then they were not always been updated with all the new modelling aspects. Without models, which encompass the relevant physics, it is not possible to estimate sonar performance and its feasibility in bubbly waters. For example, analytical formulae used today for the prediction of the coefficient of nonlinearity are based on the Rayleigh equation, which later was improved with damping effects. These formulae result in lower (and for some frequencies unphysical) predictions for the coefficient of nonlinearity. Analytical formulae from this work were used to estimate the parametric output. The sonar drive frequency was 920 kHz and the difference frequency 50 kHz. Tank experiments were undertaken in fresh water with low void gas bubbles to compare with this modelling approach.*

Keywords: *nonlinear bubble pulsations, parametric sonar, coefficient of nonlinearity*

1. INTRODUCTION

A parametric sonar uses the nonlinear propagation of two acoustic waves of different primary circular frequencies ω_1 and ω_2 to generate a difference frequency in addition to harmonics and combination frequencies. The difference frequency ω_{1-2} has a narrower beam and wider bandwidth than would result from a conventional source of the same size. However the difference frequency signal has a relatively low level. This can be mitigated by enhancing the nonlinearity of the medium, for example, through the use of nonlinear pulsating bubbles. A given bubble may scatter and absorb the incident acoustic waves and, if the bubble pulsates nonlinearly, it can reradiate sound at primary frequencies and at their combinations. A possible way to achieve parametric sonar enhancement at sea is to exploit the nonlinear emitted difference frequency of bubbles pulsating in the upper water layers. The greatest degree of nonlinearity tends to be generated by bubbles which are at, or near, resonance [1]. Even if there are resonant bubbles present, the population as a whole might not generate sufficient nonlinearity if there are large numbers of off-resonant bubbles [2]. When bubbles are present randomly over the propagation path, the scattered sound consists of coherent and incoherent parts [3]. The coherent part is the sound that is spatially and temporally correlated with the incident waves. In this paper coherent scattering is considered under dual excitation, and analytical expressions for the component of the radial displacement at the difference frequency are formulated. By assuming that the bubbles in a small region act coherently, the effective nonlinearity parameter for the medium is evaluated in terms of the bubble size distribution. In addition expressions for the attenuation and speed of sound of the primaries are obtained. These, together with the difference frequency attenuation and speed of sound, are crucial to the performance of parametric sonar. Preliminary results demonstrating the effect of bubbles on a short range parametric source are also presented.

2. BUBBLE DYNAMICS

When noninertial bubble pulsations in a liquid of low viscosity are considered, and the vapour pressure is not taken into account, these pulsations can be described with the Rayleigh-Plesset equation in dimensionless form [4]:

$$\begin{aligned}&\rho_0 R_0^2(\ddot{x}+\ddot{x}x)+1.5\rho_0 R_0^2\dot{x}^2+(3\kappa\left(p_0+2\sigma/R_0\right)-2\sigma/R_0)x\\&\quad-\left(1.5\kappa\left(3\kappa+1\right)\left(p_0+2\sigma/R_0\right)-2\sigma/R_0\right)x^2+4\eta\left(\dot{x}+x\dot{x}\right)=-P(t).\end{aligned} \tag{1}$$

The variable x denotes the bubble radius displacement normalised to the equilibrium radius R_0, such that the instantaneous bubble radius is $R=R_0(1+x)$. The surface tension is σ, the equilibrium density of the liquid is ρ_0, the shear viscosity at the bubble wall is η, whilst κ the gas polytropic index of the gas, and P(t) is the applied acoustic field which for the dual excitation considered in this paper the form: $P(t)=P_1\cos(\omega_1 t)+P_2\cos(\omega_2 t)$. Analytical expressions for x can be found by the asymptotic expansion method [5]. If the amplitude-phase representation $x_{oi}\cos(\omega_i t+\phi_i)$ is used for the harmonic components 'i' of x, then the following formulae give the amplitudes of the first (x_{o1}), second (x_{o2}) and difference frequency (x_{o1-2}) harmonics and their phases (ϕ_1, ϕ_2 and ϕ_{1-2} respectively) [4]:

$$x_{o1,o2}=P_{1,2}\Big/\left[\rho_0 R_0^2\left(\left(\omega_0^2-\omega_{1,2}^2\right)^2+\delta_v^2\omega_0^2\right)^{1/2}\right] \tag{2}$$

$$\begin{aligned}&x_{o1-2}=\left(\left(\omega_0^2-\omega_{1-2}^2\right)^2+\delta_v^2\omega_0^2\right)^{-1/2}x_{o1}x_{o2}\\&\times\left(\left(0.5\left(\omega_1^2+\omega_2^2\right)-1.5\omega_1\omega_2+\left(1.5\kappa\left(3\kappa+1\right)\left(p_0+2\sigma/R_0\right)-2\sigma/R_0\right)/\rho_0 R_0^2\right)^2+0.5\delta_v^2\omega_{1-2}^2\right)^{1/2}\end{aligned} \tag{3}$$

$\tan\phi_{1,2}=\delta_{\mathrm{v}}\omega_{1,2}/(\omega_{1,2}^2-\omega_0^2)$ and $\tan\phi_{1-2}=-(1+\xi)/(1-\xi)$ where (4)

$$\xi = \frac{0.5(\omega_1 - \omega_2)^2 \delta_v^2}{(\omega_0^2 - \omega_{1-2}^2)\left(0.5(\omega_1^2 + \omega_2^2) - 1.5\omega_1\omega_2 + (1.5\kappa(3\kappa+1)(p_0 + 2\sigma/R_0) - 2\sigma/R_0)/\rho_0 R_0^2\right)} . \qquad (5)$$

These solutions are expressed in terms of the bubble eigenfrequency ω_0 and the viscous damping coefficient δ_v where

$$\omega_0^2 = 3\kappa(p_0 + 2\sigma/R_0) - 2\sigma/R_0 \big/ \rho_0 R_0^2 \quad \text{and} \quad \delta_v = 4\eta\big/\rho_0 R_0^2 . \qquad (6)$$

The main limitation of applying the expressions (2)-(5) to derive effective medium properties is the lack of radiation damping and thermal damping. In this paper the thermal effects are neglected whilst a frequency dependant radiation damping factor $\delta_{rad} = \omega^2 R_0 / c_0$ was added to δ_v to account for radiation losses [6].

3. EFFECTIVE MEDIUM PROPERTIES

The effective coefficient of nonlinearity (β_{eff}) is a measure of difference frequency generation when a two phase medium is considered for parametric propagation. In bubble-free conditions, the nonlinearity of pure water (β_w) is the relevant material property. Pulsating bubbles are another source for nonlinearity (β_{bub}). Assuming no phasing effects, the bubbly medium nonlinearity is the sum of these coefficients: $\beta_{eff} = \beta_w + \beta_{bub}$.

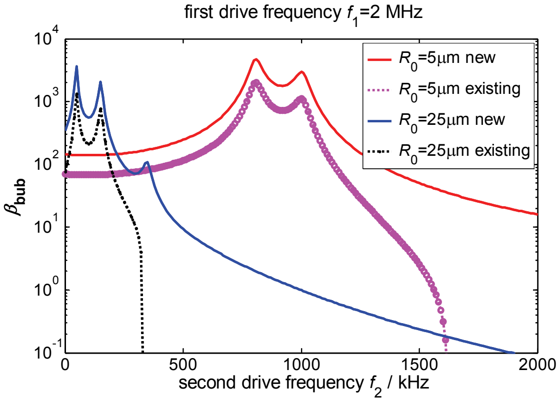
If the medium is assumed to contain $N(R_0)dR_0$ bubbles per cubic metre having radii in the range from R_0 to R_0+dR_0 then, in the quasi linear regime (where the first order terms are assumed to be unaffected by the nonlinear generation) the nonlinearity coefficient is given by [4]:

$$\beta_{bub} = \rho_0^2 c_0^4 \int \left(N(R_0) \left|\ddot{V}_-\right| / P_1 P_2 \right) dR_0 . \qquad (7)$$

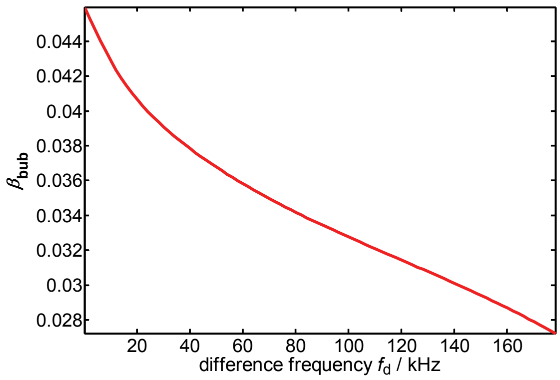
For this derivation multiple scattering effects are assumed to be negligible [7] and the density of the medium of propagation (ρ) is set equal to ρ_0. The nonlinearly pulsating bubbles act as acoustic sources, and the parameter β_{bub} is defined in terms of the second time derivative of the amplitude of the bubble volume pulsations emitting at difference frequency ($\left|\ddot{V}_-\right|$) and the amplitudes of the primary waves P_1 and P_2 [4]. The difference frequency source strength can be expressed in terms of volume changes. As a result, many works in underwater acoustics [8,9] (and other applications [10,11]) use the component of volume displacement at the difference frequency (V_-) as extracted from solutions of nonlinear equations which have the pulsating volume as variable. The approach of converting a bubble dynamics equation into volume notation is correct but existing expressions are based in the following approach: First the Rayleigh equation is linearised up to the second order [12]. Then a linear damping term that is proportional to $\dot{V}$ (which is the volume analogue term for the $\dot{x}$) is added to the linearised expression to compensate for losses. During this procedure, the nonlinear product damping term ($\dot{x}x$) is lost, a result that does not happen during the procedure outlined in section 2 [4].

In terms of the variables given by equations (2) and (3) $\left|\ddot{V}_-\right|$ is given by:

$$\left|\ddot{V}_-\right| = 4\pi R_0^3 (x_{o1-2} + x_{o1}x_{o2}) / P_1 P_2 \qquad (8)$$



(a)



(b)

Fig.1: The ordinate axis shows the coefficient of nonlinearity for fresh water with gas bubbles at 3 m depth. (a) For f_1 fixed at 2 MHz, and for f_2 on the abscissa varying from 0 to 2 MHZ, and for two monodisperse bubble populations (R_0 =5 μm or 25 μm) having void fractions of 10^{-6}, $\beta_{\rm bub}$ is calculated using the new eqn. (8) (solid line) or existing eqn. (17) of ref. [14]. (b) For varying f_1 and f_2 (such $f_{\rm d} = |f_1 - f_2|$ and centre frequency ($f_{\rm c} = (f_1 + f_2)/2$) always equal to 920 kHz), and for the bubble distribution used for the experiments [16] with void fraction 10^{-5}, $\beta_{\rm bub}$ is plotted from eqn. (8).

In Fig. 1(*a*), the values for $\beta_{\rm bub}$ are derived by substitution of equation (8) into (7) and are compared with those obtained from equation (17) of reference [6] with the same damping effects included ($\delta = \delta_{\rm v} + \delta_{\rm rad}$). As shown, $\beta_{\rm bub}$ based on the new expression for V_- never predicts zeros for bubble nonlinearity, and consequently these do not appear in the emitted pressure field when it is inserted into the general equation for a monopole source [13]. The predictions for $\beta_{\rm bub}$ are dependent on the drive frequencies and their combinations. In practice the centre frequency ($f_{\rm c}$) of the transducer is used to generate a limited range of difference frequencies ($f_{\rm d}$). Fig. 1(b) shows the predictions of the nonlinearity coefficient for the frequency combinations that could be realised with the parametric source used in our experiments. Expressing the volume variations in terms of radius harmonic components enables formulae for the phase speeds $c_{1,2}$, and the extra attenuations $\alpha_{1,2}$ of the primaries due to the bubbles to be derived [4]. The derivation is based on the definition of sound speed in terms of bulk modulus;

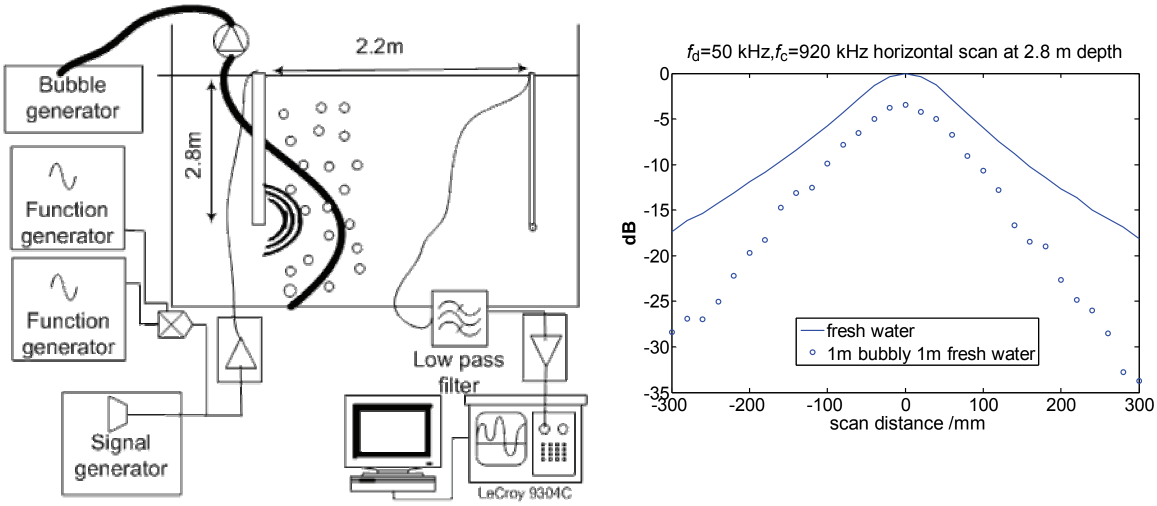
$$1/c_{1,2}^2 = 1/c_0^2 + \rho_0 \int N(R_0) 4\pi R_0^3 (x_{o1,o2}/P_{1,2}) \cos\phi_{1,2} dR_0 \tag{9}$$

$$\alpha_{1,2}^2 = -\omega_{1,2}^2 \rho_0 \int N(R_0) 4\pi R_0^3 (x_{o1,o2}/P_{1,2}) \sin\phi_{1,2} dR_0 \tag{10}$$

Equations (9) and (10) give results which are identical to the expressions obtained when derived from the complex form (see for example equation 36 of reference [6]).

4. EXPERIMENTAL RESULTS & DISCUSSION

Preliminary experiments were carried out in a laboratory tank with fresh water with dimensions: 8 m x 8 m x 5 m x 5 m (Fig. 2(a)). An air compressor (bubble generator) was used to generate bubbly water that was pumped via a hose at the bottom of the tank; this generated a low void fraction (~10^{-5}) air bubble plume [14]. The hose end was modified so that the bubble cloud extended from in front of the transducer to a distance halfway to the hydrophone (approx 1 m). The hydrophone was mounted on two stacked linear translation stages (not shown in Fig. 2(a)) driven by stepper motors in order to perform vertical and horizontal field scans. A conventional source of 2.5 cm radius and 920 kHz resonance frequency was used to generate a f_d =50 kHz parametric beam.



(a) *(b)*

Fig. 2: (a) Experimental set up: the hydrophone was aligned at a 2.2 m from the source at a depth of approximately 2.8 m; when the bubble generator was on the bubbles formed an approximately 1 meter thick layer in front of the transducer. (b) Measured beam profile for f_d= 50 kHz in fresh water (solid line) and with bubbles for the first half of propagation path (Fig. 2(a)). Errors due to noise are approximately ± 1 dB.

Fig. 2(b) shows the beam profile (normalised with respect to the broadside value for bubble free water) for a difference frequency of 50 kHz, as measured from a horizontal scan starting -30 cm off axis and ending 30 cm of axis, in the tank without bubbles (solid line) and with bubbles (circles). The attenuation of the primary beams (not shown) was small (1.3 dB for the whole propagation length) as expected from the bubble distribution [14]: given that whilst there were measurable numbers of bubbles resonant at 50 kHz the resonant bubble radius for the primaries was ~ 4 μm and only a few bubbles of that size was likely to be present [14].

For this system where the hydrophone was in the nearfield of the parametric generation region; narrowing of the beam was observed. As the distribution and phasing of the secondary sources are determining the beam profile and the primaries were only slightly attenuated by the bubble cloud, the difference frequency sources nearest to the hydrophone were hardly affected. In contrast 50 kHz generated nearer to the transducer would have been attenuated by the bubble cloud. The change in beam profile can be attributed to this change in the significance of the secondary sources. Asymmetry of the beam profile is attributed mainly to phase changes caused by the cloud and to fluctuation of the cloud in terms of its density and position. For this preliminary scan the cloud drift caused a change of approximately 1 dB over the period of measurement.

5. CONCLUSIONS

This paper has presented a new theoretical approach to predicting the bubble mediated contribution to nonlinearity for parametric sonar. A preliminary comparison with experiment has been made. Further data is required to validate this theory.

6. ACKNOWLEDGEMENTS

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