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The Selection of Robust and Efficient Transducer Locations for Active Sound Control

T. Bravo and S.J. Elliott

ISVR Technical Memorandum 843

September 1999

University of Southampton

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UNIVERSITY OF SOUTHAMPTON
INSTITUTE OF SOUND AND VIBRATION RESEARCH
SIGNAL PROCESSING & CONTROL GROUP

The Selection of Robust and Efficient

Transducer Locations for Active Sound Control

by

T Bravo and S J Elliott

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Authorised for issue by
Prof S J Elliott
Group Chairman

Contents

1. Introduction

Active Noise Control is a technique for controlling unwanted sound by means of the introduction of a secondary field. Under the assumption of linearity, both fields interfere destructively resulting in the overall reduction of the primary noise.

The sound pressure level inside propeller aircraft and the helicopters shows some low frequency components corresponding to the blade passing frequency of the main rotor and its harmonics. To reduce the low frequency noise, Active Noise Control methods have been used. The final attenuation values rely on how well both fields match, in the time and in the space. The reconstruction in time has been studied in the past and depends mainly on the electronic controller of the ANC system. This work deals with the reconstruction in the space, with is primarily determined by the distribution of the error sensors and the secondary sources [1, 2, 3].

The difficulty of choosing transducer locations in an ANC system is a result of the huge number of possible configurations, which differ in the number and locations of the transducers and in the overall attenuation levels which they are able to give. The cost related to the implementation and measure of all the combinations is very high and it is necessary to use an optimisation process before the physical installation. In order to solve this combinatorial optimisation problem, traditional approaches are not efficient. The searching spaces are too large for enumerative methods, and algorithms based in hill-climbing are local and run the risk to be trapped in a suboptimal solution, specially when the function is very noisy.

Several optimisation approaches have been used until now: integer programming methods combined with heuristic techniques [4], sequential quadratic programming algorithms for constrained optimisation [5], diffuse approximation methods [6, 7], multiple linear regression with subset selection methods [8], genetic algorithms [9, 10] and simulated annealing techniques [11, 12].

The objective function used by these methods in the evaluation module is normally the sum of square sound pressure values in a discrete number of points inside the enclosure. The attenuation for every different combination is calculated from the response of the secondary sources using the nominal plant at these points and from the primary field at the same sensors, but it is not clear what happens when the response of the physical plant varies from the nominal conditions and how this affects the performance of the ANC system.

In this work, the simulated annealing method has been used to select the best transducer positions in an enclosure for harmonic sound fields using 16 loudspeakers and 32 microphones, paying special attention to guaranteeing that the geometrical configuration of transducers is still efficient when the physical plant varies from the nominal state. It has been shown previously [13] that this technique constitutes a very good approximation in terms of performance with a minimum number of evaluations, although it is quite sensitive to the tuning of some characteristic parameters.

The rest of the report is as follows: Section 2 expounds briefly the theoretical model for active control in an enclosure used in the computer simulations, Section 3 explains the basic characteristics of the simulated annealing algorithm and the necessary modifications introduced to fit in with the particular problem, and Section 4 describes the laboratory built for the transfer function measurements used in the cost function. Six different perturbed transfer functions have been determined in the enclosure and a new objective function has been defined based on these measurements. The results obtained with this new cost function for the optimal positions of transducers and those obtained with the nominal plant by limiting the total control effort are compared in Sections 5 and 6. Finally, some conclusions are discussed in the last section.

2. Global control of enclosed sound fields

As the control of low frequency harmonic noise in a rectangular enclosure is the aim of this work, we follow the modal analysis to describe the interior sound field [14]. If the work is carried out for harmonic sound fields, the complex pressure amplitude can be expressed as a sum of modal contributions for N normal modes. Assuming that L sensors positions and M secondary sources have been determined previously, the sound field at the microphones positions can be written as the contribution due to the primary source plus the contributions due to the secondary sources

$$\mathbf{e} = \mathbf{d} + \mathbf{C}\mathbf{u} \tag{1}$$

where $\mathbf{e}$ is a Lx1 complex vector of error signals, $\mathbf{d}$ is a Lx1 complex vector of error signals due to the primary sources, $\mathbf{C}$ is a LxM complex transfer matrix from the secondary sources to the error sensors an $\mathbf{u}$ is a Mx1 complex vector of secondary source strengths. The objective here is to achieve global control over the whole enclosure, so an appropriate quadratic cost function has to be minimised. An approximation to the total time averaged acoustic potential energy defined as the sum of the modules squared pressures has been used in this work, as

$$J = \mathbf{e}^{\mathrm{H}}\mathbf{e} \tag{2}$$

and substitution into the previous expression gives

$$J = \mathbf{u}^{H}\mathbf{C}^{H}\mathbf{C}\mathbf{u} + \mathbf{u}^{H}\mathbf{C}^{H}\mathbf{d} + \mathbf{d}^{H}\mathbf{C}\mathbf{u} + \mathbf{d}^{H}\mathbf{d} \tag{3}$$

The matrix $\mathbf{C}^{H}\mathbf{C}$ is assumed to be positive definite, which generally true, and if the number of error sensors is greater than the number of loudspeakers, the cost function reaches a global minimum when the secondary source strengths vector takes the value

$$\mathbf{u}_{o} = -(\mathbf{C}^{H}\mathbf{C})^{-1}\mathbf{C}^{H}\mathbf{d} \tag{4}$$

The corresponding minimum value of the objective function is

$$J_{o} = \mathbf{d}^{H}\left[\mathbf{I} - (\mathbf{C}^{H}\mathbf{C})^{-1}\mathbf{C}^{H}\right]\mathbf{d} \tag{5}$$

and the attenuation for a fixed transducer configuration is taken as

$$Attenuation\,(dB) = 10\log_{10}\left(\frac{\mathbf{d}^{H}\mathbf{d}}{J_{o}}\right) = 10\log_{10}\left(\frac{\mathbf{d}^{H}\mathbf{d}}{\mathbf{d}^{H}\left[\mathbf{I} - (\mathbf{C}^{H}\mathbf{C})^{-1}\mathbf{C}^{H}\right]\mathbf{d}}\right) \tag{6}$$

which gives an idea of the noise reduction in the microphones positions before and after the ANC system.

The control effort is defined to be the sum of the modulus squared secondary source strengths, which is equal to

$$P = \mathbf{u}^{H}\mathbf{u} \tag{7}$$

and if the secondary sources are optimally adjusted as in equation (4), the control effort is equal to

$$P = \mathbf{d}^{H}\mathbf{C}(\mathbf{C}^{H}\mathbf{C})^{-2}\mathbf{C}^{H}\mathbf{d} \tag{8}$$

3. The experimental enclosure

To help understand the main characteristics of the sound field inside moving vehicles, a wooden laboratory mock-up of an aircraft interior has been constructed, as shown in Figure 1. The internal dimensions of the enclosure are L_x = 2.1m, L_y = 6.0m and L_z = 2.1m. The floor of the enclosure is covered by a thin layer of sound absorption material and there is an air gap between the material and the plywood panels.

A multichannel active control system has been developed for in-flight experiments to control propeller-induce passenger noise [15, 16]. It has 32 error signal inputs from electret microphones and 16 secondary control signal outputs to 200 mm loudspeakers. The error sensors were uniformly distributed in a 8x4 grid inside the enclosure at head height. The primary acoustic field is a pure tone a 88 Hz, generated by a 300 mm diameter loudspeaker driven from a oscillator which also provided the reference signal for the control system.

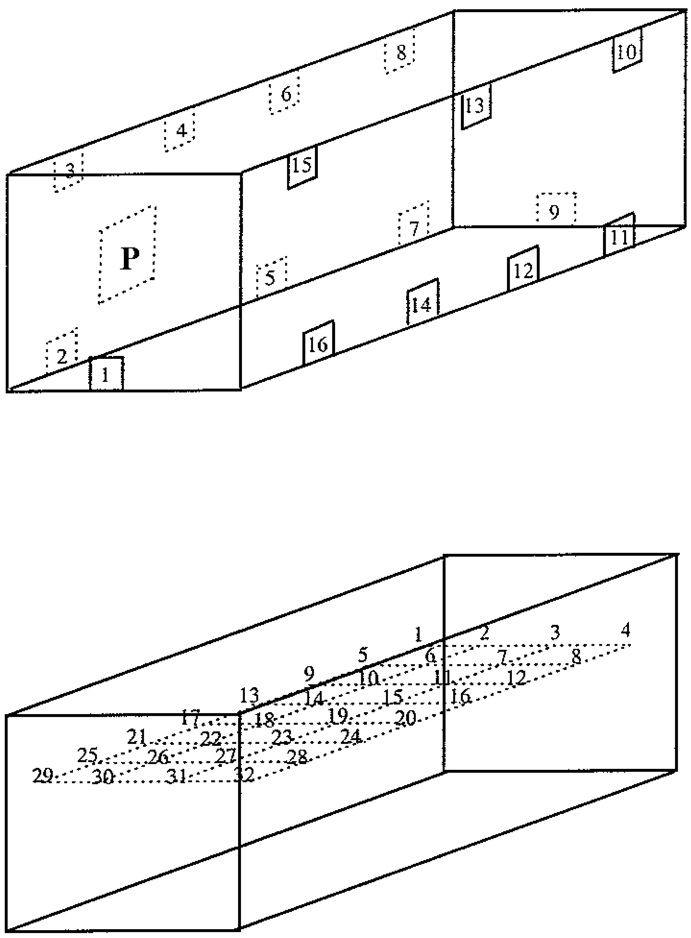

Figure 1. Loudspeaker and microphone positions in the laboratory enclosure

The electronic controller implements the Multiple Channel LMS Algorithm. This algorithm minimises a cost function taking into account the sum of the mean-square signals form the error sensors and the sum of the mean-square signals fed to the secondary sources. In order to perform the simulations, it is necessary to used the control system to measure the transfer functions from each secondary source to each error sensor and to measure the primary field at the error sensor positions. The controller generates a pure tone at the first harmonic of the propeller blade passing frequency, 88 Hz. The vector **d** and the matrix **C** in equation (1) have dimensions Lx1 and LxM respectively. These data have been used in the simulations implemented from the theoretical expressions in Section 2.

4. The simulated annealing algorithm

The simulated annealing method is a search algorithm based on the properties of a system with a large number of atoms that simulated what happens in the limit of low temperature. Since the number of atoms is very high, the system will be governed by the laws of statistical mechanics, and each configuration will be scaled by the Boltzmann probability factor, exp(- E / K_B T), where E is the particular configuration energy, K_B is the Boltzmann's constant, and T is the temperature of the system [17]. The algorithm which implements this ideas is the Metropolis algorithm. Basically, when the system evolves into a new configuration and changes its energy by an amount $\Delta E \leq 0$, the new state is accepted unconditionally, but if ΔE>0, the case is treated probabilistically and the configuration is accepted with probability exp(- ΔE / K_B T). The system is able to go uphill as well as downhill, but the probability of going uphill decreases as the temperature of the system decreases. This kind of probability distribution prevents the system to fall into a local optimum. By repeating this steps a number of times, the process of cooling a solid until it reaches the limit of low temperature is simulated.

In order to implement this algorithm it is necessary to have a description of the possible configurations taken by the system, a random change generator, a cost function to be minimised and an annealing schedule. For the particular problem we are dealing with, finding the best transducer positions from a fixed number, the optimisation problem has been coded as a finite-length string over a binary alphabet. Each possible transducer position is indicated by an element in the string, with length equal to the total number of transducers, and every element taking a value of 0 or 1 indicating the absence or presence of a transducer. To generate all the combinations to be presented to the system, an initial string is generated randomly filling up the elements with 1's and 0's until the required number of transducers are present. In order to maintain this number constant over the whole process, a mutation of one gene in a string must be accompanied by another mutation of a different gene with the opposite value.

The cost function used by the algorithm to be maximised is the reduction of the sum of the square signals at the microphone positions, and this is indicated by the attenuation in equation (6). The annealing schedule is normally obtained by trial and error and depends on the size of the problem.

The appropriate selection of the parameters (temperature, length of time to allow the system to evolve,...) is very important for the success of the method. If the system is cooled very quickly, it does not collapses into the lowest energy state but into a local optimum. The problem of find 12 best microphones from 32 by means of the simulated annealing method with different values for the parameters is illustrated below. The effects of different initial temperatures and different cooling coefficients are shown in Figure 2 and 3, respectively. In both cases the number of iterations at each temperature and the number of successful changes before continuing has been fixed at 100 and 20 respectively.

5. Optimal loudspeaker positions in an enclosure

5.1. Optimal loudspeaker positions without restrictions

From a theoretical point of view, once the transfer functions between each loudspeaker an each sensor, and the primary field at the microphones positions have been determined, it is possible to calculated the attenuation values for all the combinations of secondary sources. However, in practical situations, this number of possible combinations is so large that only can be solved exactly for problems with small number of transducers.

We are now faced with the selection of the best M secondary sources, when M varies from 1 to 16, from a total number of 16. In this case it is feasible to calculated the attenuation values provided for each combination, a total number of about 66000, resulting from ${}_{16}C_1$, ${}_{16}C_2$,..., ${}_{16}C_{16}$. The results for the number of combinations giving specific attenuation levels are shown in Table 1.

The maximum attenuation, the control effort values and the selected combination for every set of loudspeakers from 1 to 16 are shown in Table 2. It can be seen that for some combinations of loudspeakers, the corresponding value of the control effort for the maximum attenuation is very high. The maximum attenuation values found by exhaustive search are presented in Figure 4, and it observed that 10 appropriately placed secondary sources can give an attenuation level which is almost as good as the value provide by 16 loudspeakers.

The aim of this work is to used the simulated annealing method to obtain an approximate solution avoiding the prohibitive computing effort required by the exhaustive method. The same problem of finding M loudspeakers from 16 has been treated by the simulated annealing method and the results obtained are shown and compared with the results obtained by exhaustive search. It can be seen that the simulated annealing method is able to find almost the exact solution in this problem searching only in a very small number of possible combinations.

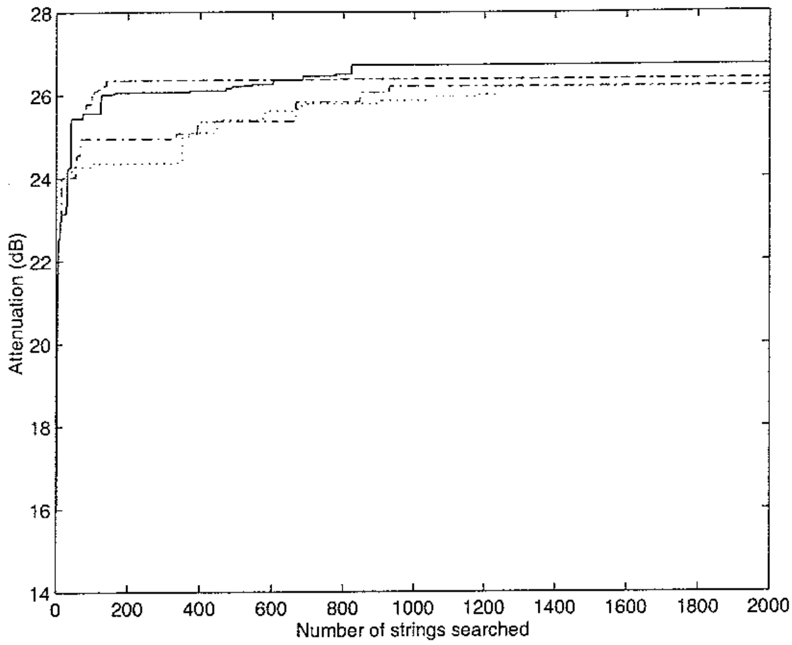

Figure 2. Effects of initial temperature on the performance of the simulated annealing method for finding 12 microphones positions from 32 (cooling coefficient=0.95). Initial temperature: __ 1; -·0.98; --0.95; ···0.9.

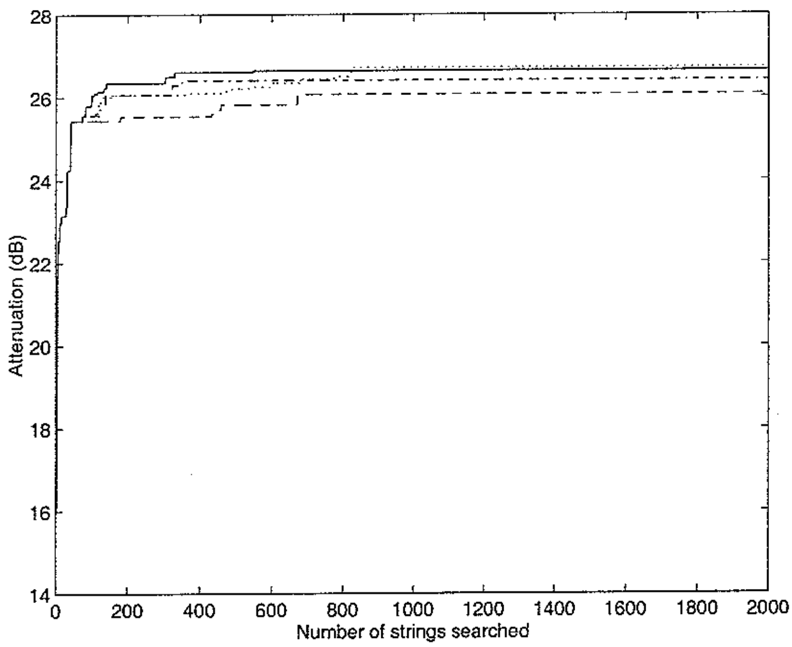

Figure 3. Effects of cooling coefficient on the performance of the simulated annealing method for finding 12 microphones positions from 32 (initial temperature=1). Cooling coefficient: ···0.98; __ 0.95; -·0.92; --0.88.

Table 1. Number of loudspeakers combinations giving specified attenuation level when the total number of secondary sources is restricted to 1, 2, ...16 at 88 Hz excitation

Att.	Total number of loudspeakers															
dB	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	1	11	21	21	10	1
27	0	0	0	0	0	0	0	0	0	22	104	175	142	55	5	0
26	0	0	0	0	0	0	0	3	60	252	476	447	188	30	1	0
25	0	0	0	0	0	0	4	63	332	692	649	291	72	6	0	0
24	0	0	0	0	0	1	29	203	559	739	588	275	61	5	0	0
23	0	0	0	0	0	13	114	392	726	862	637	239	39	2	0	0
22	0	0	0	0	2	26	221	747	1209	1137	549	130	16	1	0	0
21	0	0	0	0	11	117	444	933	1213	806	301	75	12	0	0	0
20	0	0	0	1	29	200	517	763	730	536	276	81	6	0	0	0
19	0	0	0	7	64	189	445	735	812	647	306	60	3	0	0	0
18	0	0	1	18	55	131	372	835	1079	707	198	20	0	0	0	0
17	0	0	1	0	17	153	554	993	995	513	126	8	0	0	0	0
16	0	0	0	2	49	344	939	1286	904	346	65	4	0	0	0	0
15	0	0	0	17	130	408	772	963	769	307	48	2	0	0	0	0
14	0	0	3	22	100	378	875	1123	683	162	13	1	0	0	0	0
13	0	0	1	19	146	480	880	881	424	120	17	0	0	0	0	0
12	0	0	0	14	109	465	950	828	382	86	9	1	0	0	0	0
11	0	0	1	34	219	662	982	740	268	46	4	0	0	0	0	0
10	0	0	3	56	292	802	1050	661	186	16	1	0	0	0	0	0
9	0	1	15	87	431	986	975	382	62	9	0	0	0	0	0	0
8	0	0	10	163	671	1016	629	188	32	3	0	0	0	0	0	0
7	0	1	40	291	685	695	344	91	15	0	0	0	0	0	0	0
6	0	9	90	304	496	432	204	53	0	0	0	0	0	0	0	0
5	0	13	101	266	348	271	100	7	0	0	0	0	0	0	0	0
4	2	20	62	163	222	137	35	0	0	0	0	0	0	0	0	0
3	1	18	86	174	181	87	5	0	0	0	0	0	0	0	0	0
2	2	11	68	140	105	15	0	0	0	0	0	0	0	0	0	0
1	3	38	77	42	6	0	0	0	0	0	0	0	0	0	0	0
0	8	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2. Best attenuation values for the problem of finding M loudspeakers positions, where M varies from 1 to 16 , with the corresponding control effort values and the selected configurations

Total No. of loudspeakers	Attenuation (dB)	Total control effort	Loudspeaker positions
1	4.3316	0.7273	0000000000000010
2	8.6616	0.9969	0010000000000010
3	17.5176	52.0880	1110000000000000
4	19.9093	279.1099	1100000000010001
5	22.2851	60.8382	1110000000010100
6	23.5659	47.1862	1100001000011100
7	25.0834	3.1494	0111101010100000
8	25.6616	7.3498	1101101010010100
9	26.4259	15.9127	1101101010011100
10	27.32.3	14.6926	1101101010111100
11	27.7639	16.0691	1111101010111100
12	27.8991	15.9173	1111101110111100
13	28.0146	14.6884	1111111110111100
14	28.1744	22.3177	1111111110111101
15	28.2454	36.7473	1111110111111111
16	28.2827	25.5081	1111111111111111

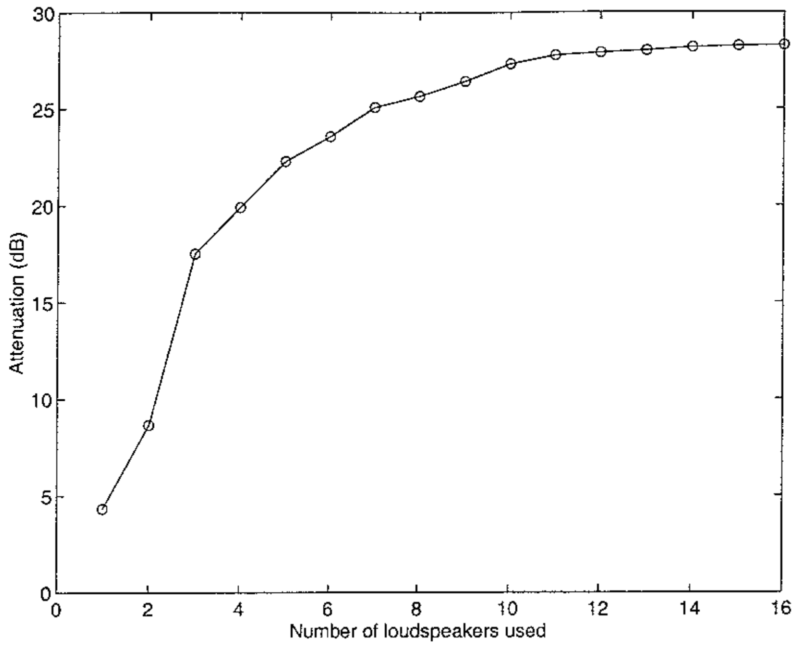

Figure 4. Maximum attenuation values obtained from an exhaustive search at a primary source excitation frequency of 88 Hz while the number of loudspeakers are limited to 1, 2, ...16

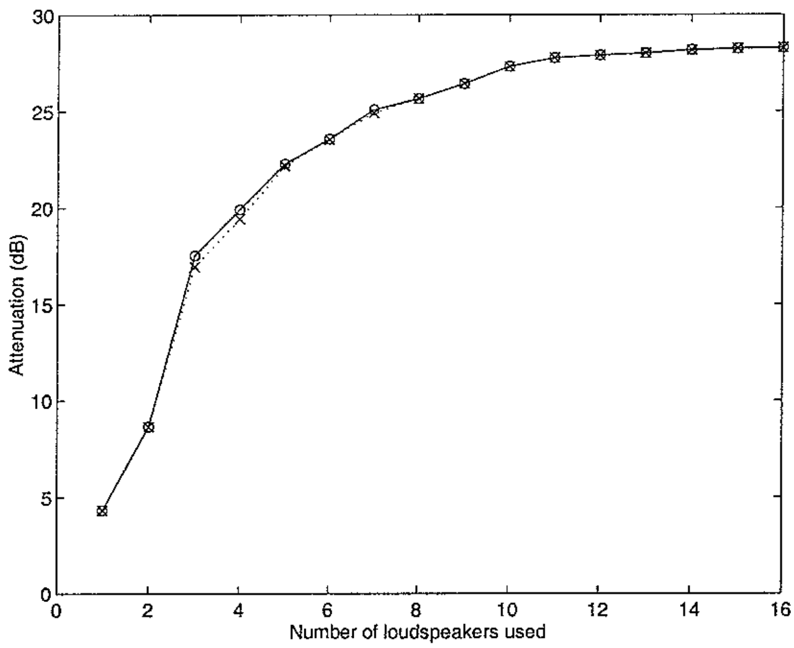

Figure 5. Maximum attenuation values obtained at a primary source excitation frequency of 88 Hz while the number of loudspeakers are limited to 1, 2, ...16. o- exhaustive search; x·· simulated annealing algorithm

5.2. Optimal loudspeakers positions with constraint on total control effort

It has been demonstrated in previous studies [18] that loudspeakers positions which give good attenuation levels and low control effort values will generally be robust to uncertainties in the plant under control. To analyse the effect of the effort control on the performance of the system, the control effort values for a number of possible loudspeakers configurations have been studied. The ordered attenuation levels for the best 100 cases and their corresponding effort values for the problem of choosing 9 and 10 loudspeakers from 16 have been plotted in Figures 6 and 7 as an example. It can be seen clearly that there is a constant background corresponding to a low effort control value for the majority of loudspeakers configurations, but for a few particular combinations the effort control values are much higher.

To avoid the selection of these solutions by the searching algorithm, a modified cost function has been used that includes a factor proportional to the effort control, as:

$$J = \mathbf{e}^{\mathrm{H}}\mathbf{e} + \beta\, \mathbf{u}^{\mathrm{H}}\mathbf{u} \tag{9}$$

where β is a weighting factor that has been chosen here so that the best possible value for the attenuation is obtained with a reasonable value of the control effort background for M = 1, 2, ...16.

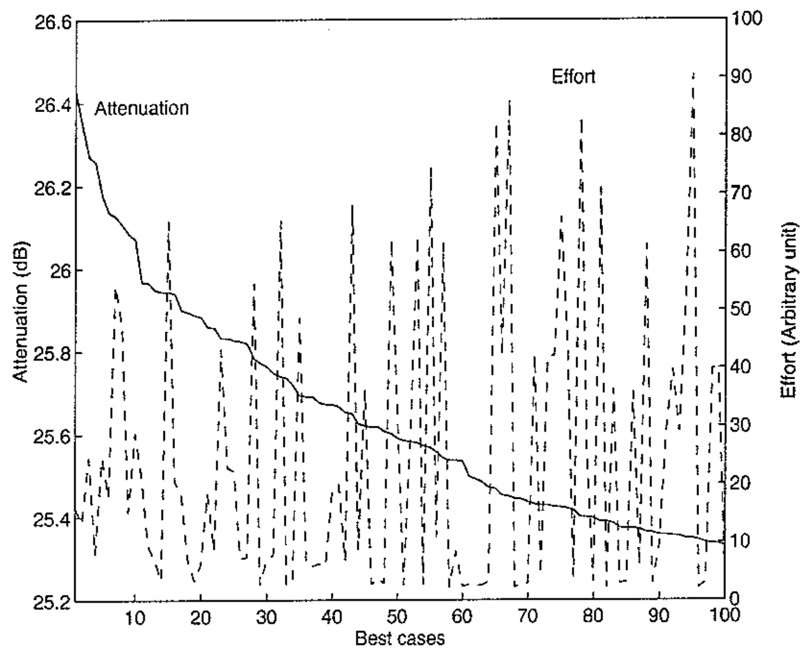

Figure 6. Attenuation and corresponding total control efforts plot for the best 100 combinations in attenuation found by exhaustive searching for the problem of finding optimal 9 secondary sources from 16 at 88 Hz primary excitation

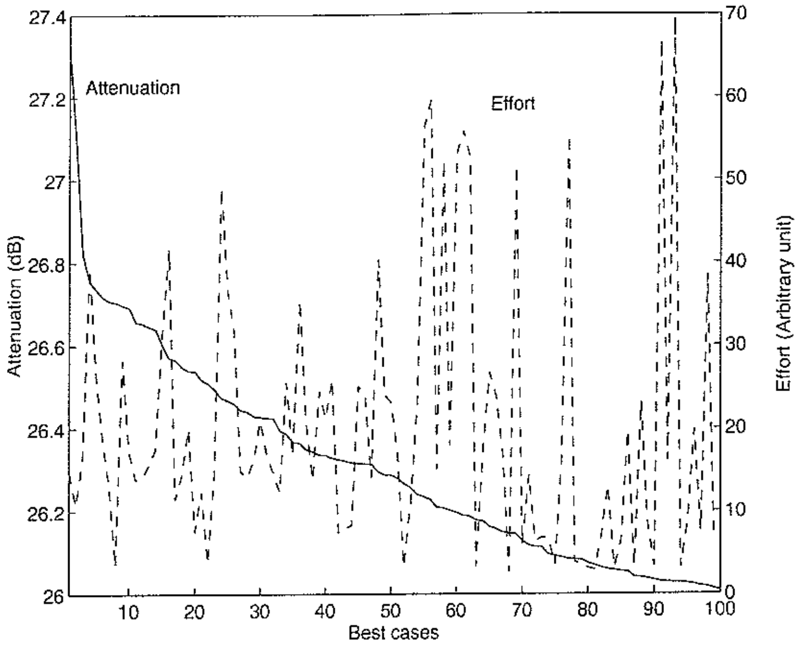

Figure 7. Attenuation and corresponding total control efforts plot for the best 100 combinations in attenuation found by exhaustive searching for the problem of finding optimal 10 secondary sources from 16 at 88 Hz primary excitation

The attenuation values obtained using this new objective function, the total control effort levels and the corresponding loudspeakers configuration selected appear in Table 3.

Table 3. Best attenuation values for the problem of finding M loudspeakers positions, where M varies from 1 to 16, with the corresponding control effort values and the selected configurations when there is a limit in the total control effort

Total No. of loudspeakers	Attenuation (dB)	Total control effort	Loudspeaker positions
1	4.3316	0.7273	0000000000000010
2	8.6616	0.9969	0010000000000010
3	14.0449	2.4439	0000000000010110
4	16.2093	2.1349	0000000000011110
5	17.6441	2.2034	0001000000011110
6	22.9562	3.9504	0101101010100000
7	24.6728	2.4578	0101101010010100
8	25.2143	2.1418	0101101010010110
9	25.5373	2.1894	0101111010010110
10	25.5890	2.1088	0101111010011110
11	25.8456	2.1322	0101111011011110
12	26.2341	2.2720	0101111011111110
13	25.9057	2.0924	0111111111011110
14	26.6072	2.5831	0111111111111110
15	27.3138	3.4531	0111111111111111
16	28.2827	25.5081	1111111111111111

These attenuation values are plotted in Figure 8. It can be seen that it is possible to choose loudspeakers configurations with low control effort values without affecting the system performance very much.

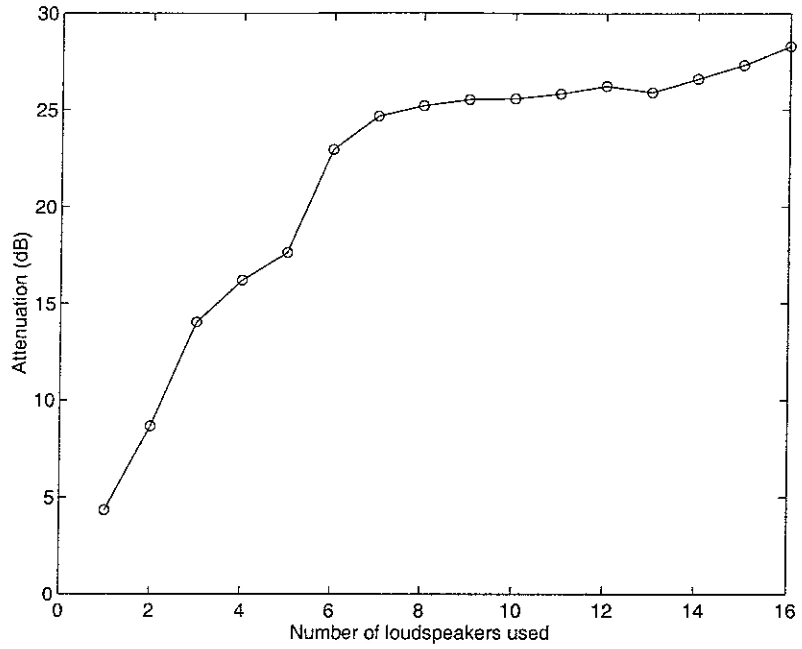

Figure 8. Maximum attenuation values obtained from an exhaustive search at a primary source excitation frequency of 88 Hz while the number of loudspeakers are limited to 1, 2, ...16 and there is a limit on the total control effort values

5.3. Optimal loudspeaker positions with structured uncertainties

To investigate the robustness of the control system when changes occur in the plant, six different perturbed plants have been measure in the laboratory, corresponding to six different situations that could happen in practice. All these different perturbed plants can be classified as structured uncertainties in the plant response because they occur as a result of physical changes in the system under control.

This matrices are:

C_1: Occupants standing under the error microphones.

C_2: The same that C_1 but two persons walking along a central corridor

C_3: Occupants walking over the whole enclosure

C_4: Occupants walking along a central corridor

C_5: Occupants sitting down under the error microphones

C_6: The same that C_5 but two persons walking along a central corridor

A new objective function is then defined taking into account the transfer matrices and primary disturbances for the six perturbed plants, as:

$$J_1 = 10\log_{10}\left(\frac{\sum_{i=1}^{6} \mathbf{d}_i^H \mathbf{d}_i}{\sum_{i=1}^{6} \mathbf{e}_i^H \mathbf{e}_i}\right) \quad (10)$$

where $\mathbf{d}_i$ is the primary disturbance vector measure at the error sensor positions for the i-th condition and $\mathbf{e}_i$ is the corresponding minimum error.

Using this new cost function J_1, the exhaustive search method has again been used in the same problem, and the attenuation level and the corresponding control effort values have been studied. Figures 9 and 10 represent again the ordered attenuation levels for the best 100 cases together with the control effort. It is clear that the attenuation values with the perturbed plants are lower than those obtained using the nominal plant, but the most important different is the total control effort values, which have fallen by about one order of magnitude. So, the "best" loudspeakers position in this case are selected from those that give a low total control effort value.

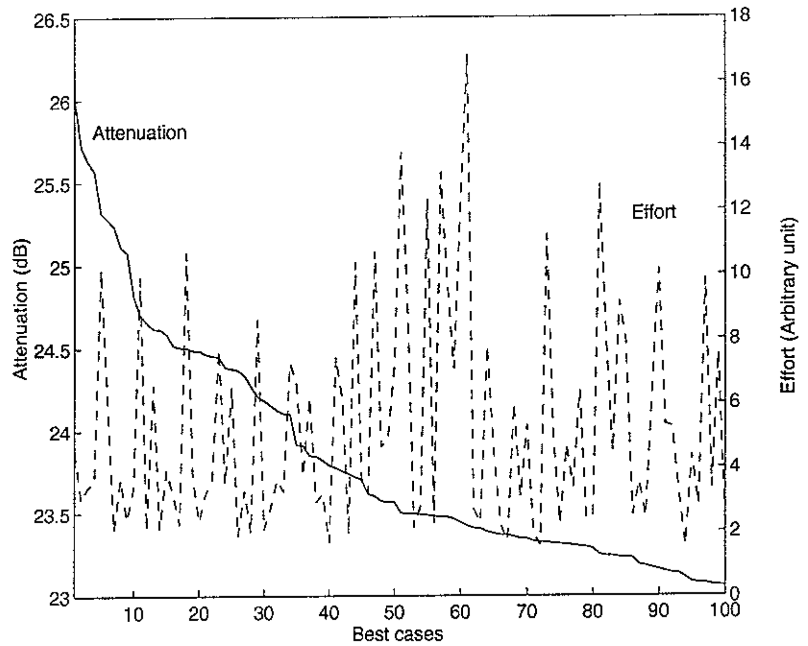

Figure 9. Attenuation and corresponding total control efforts plot for the best 100 combinations in attenuation found by exhaustive searching for the problem of finding optimal ten secondary sources from 16 at 88 Hz primary excitation using the objective function J_1

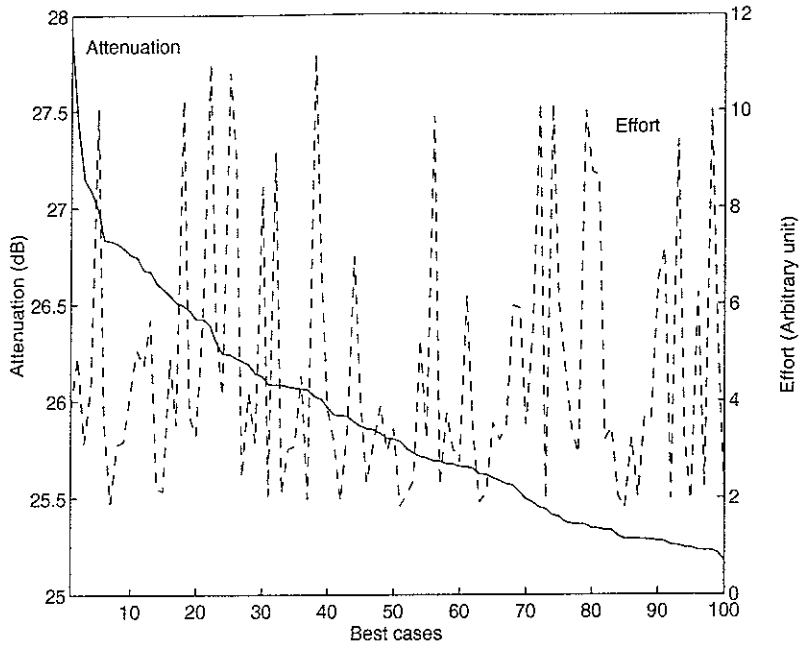

Figure 10. Attenuation and corresponding total control efforts plot for the best 100 combinations in attenuation found by exhaustive searching for the problem of finding optimal ten secondary sources from 16 at 88 Hz primary excitation using the objective function J_1

The attenuation values, control effort levels and particular configuration selected obtained by exhaustive search with this new cost function are presented in Table 4. These attenuation values are plotted in Figure 11.

Table 4. Best attenuation values for the problem of finding M loudspeakers positions, where M varies from 1 to 16, from a total number of 16, with the corresponding control effort values and the selected configurations using the cost function J_1

Total No. of	Attenuation	Total control	Loudspeaker
1	5.4302	0.8196	0000000000000010
2	8.1265	0.8081	0010000000000010
3	10.2569	1.1988	0010000000010010
4	13.7470	1.4347	0000000000011110
5	15.1177	1.6533	0101100000010100
6	17.8583	5.3215	1101100000010100
7	20.2873	12.0900	1100100010010101
8	23.6281	3.2146	1101101010010100
9	26.0160	4.5338	1101101010010101
10	27.9154	3.8425	1101111010010101
11	28.6594	4.6010	1101111010011101
12	29.5964	4.8088	1101111010011111
13	30.0594	4.8036	1111111010011111
14	30.5746	4.6565	1111111110011111
15	30.6256	4.4416	1111111110111111
16	30.6800	4.6515	1111111111111111

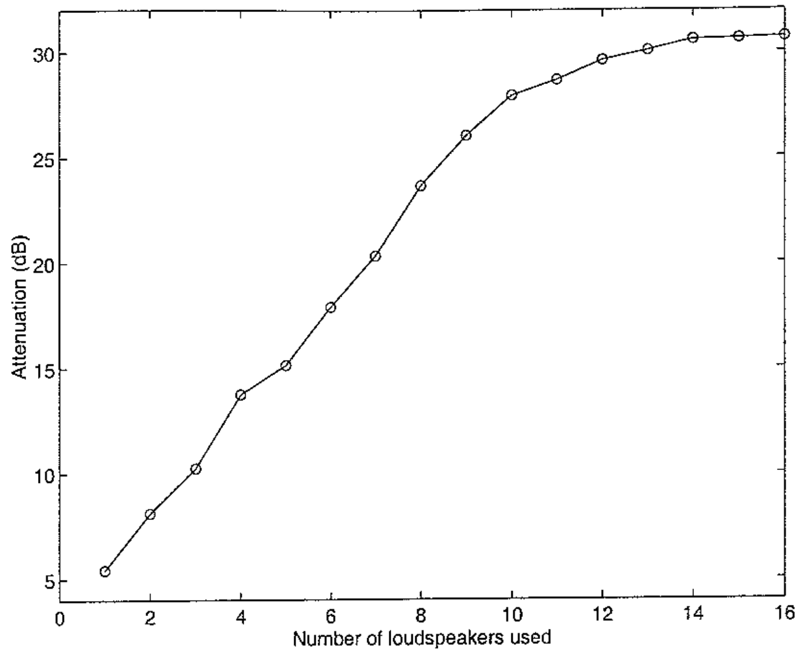

Figure 11. Maximum attenuation values obtained from an exhaustive search at a primary source excitation frequency of 88 Hz while the number of loudspeakers are limited to 1, 2, ...16 from a total of 16 and the cost function J_1 has been used

It can be seen that, although the shape of the graph is more similar to the shape of the figure obtained by limiting the total control effort than to the one obtained without any limit, the two figures are not the same. So, even thought the selected configuration have low control values, the attenuations obtained are much more smaller than those obtained by limiting the effort control.

6. Optimal error sensor positions in an enclosure

6.1. Optimal error sensor positions without uncertainties

Until now, for the selection of the best control sources positions the attenuation value evaluated at all the 32 microphone positions have been maximised, as an approximation to the acoustical potential energy in the enclosure. The corresponding values for every particular secondary loudspeaker configuration have been calculated by the selection of the appropriate columns from the plant transfer function matrix. It has been shown that 10 well selected loudspeakers can give an attenuation value which is only 1 dB below than of the best achievable for 16 loudspeakers. With these 10 loudspeakers positions selected, it is necessary to investigate the best microphone positions from the possible 32. This is again a combinatorial optimisation problem, but in this case, the total number of possible combinations resulting from ${}_{32}C_1$, ${}_{32}C_2$,....,${}_{32}C_{32}$,about 4x10^9. This number is so high that it is impossible carry on the process for every set of microphones, so the simulated annealing method has been used.

In this case, with the 10 loudspeaker positions fixed, the attenuation values can be calculated now by selecting the appropriate rows from the plant transfer functions matrix, which correspond to the selected microphone positions. However, it is necessary to note that the reduction of the sound pressure level at some determined error sensor positions could produce an increase of the sound in others microphone positions, which result in an overall increased of the acoustic potential energy over the enclosure. To try to avoid this effect as much as possible, the input signal to the loudspeakers are calculated to minimise the sum of the square outputs of only the selected microphones, but the noise reduction is evaluated from all the microphone positions, and this is the final value which is taken into account for the simulated annealing algorithm.

If the system under control has a total of M loudspeakers and L microphones and we want to choose a subset of L_r microphones, the optimal strength vector which must be fed to the loudspeakers is expressed as [19]

$$\mathbf{u}_{or} = -(\mathbf{C}_r^H\mathbf{C}_r)^{-1}\mathbf{C}_r^H\mathbf{d}_r \qquad \textit{for } L_r > M \tag{11}$$

$$\mathbf{u}_{or} = -\mathbf{C}_r^{-1}\mathbf{d}_r \qquad \textit{for } L_r = M \tag{12}$$

$$\mathbf{u}_{or} = -\mathbf{C}_r^H(\mathbf{C}_r\mathbf{C}_r^H)^{-1}\mathbf{d}_r \qquad \textit{for } L_r < M \tag{13}$$

where $\mathbf{d}_r$ is a reduced vector which has dimensions L_rx1 and $\mathbf{C}_r$ is the matrix of the reduced transfer functions with dimensions L_rxM. The optimal reduced source strength vector $\mathbf{u}_{or}$ is then substituted into the equation

$$\mathbf{e} = \mathbf{d} + \mathbf{C}\mathbf{u}_{or} \tag{12}$$

so that the error signal is calculated in all the microphones positions, but with the secondary source strengths calculated from the reduced set of error sensors. The definition of the attenuation is expressed as in equation (6), and the objective is to get good attenuation levels over the whole enclosure. The attenuation and control effort shown previously for different numbers of secondary sources are now presented for different numbers of control microphones in Table 5.

Table 5. Best attenuation values for the problem of finding L microphone positions, where L varies from 1 to 32, from a total number of 32, with the corresponding control effort values and the selected configurations using the nominal plant

Total No. of microphones	Attenuation (dB)	Total control effort	Microphone positions
1	3.5432	0.4212	00000000100000000000000000000000
2	4.8607	0.5023	00000000100000000000000000100000
3	7.0302	2.0806	00001000000000000000000000100100
4	13.4682	1.9862	00001000010000000000000100000100
5	18.1538	2.4669	00010010000000100000000010000001000
6	22.1710	2.2062	00010000000101000000001010100000 0
7	24.0116	2.5041	00010000100110000000100001000001 0
8	24.3072	2.4960	00100000100110000000110001000000 1
9	24.9337	12.0551	10000000100100000010101001001001 0
10	26.2123	11.1665	10001001100000011100000000100000 11
11	26.4091	18.1395	01001001010000001110000001100100 01
12	26.6334	12.0510	10100001110001010101010010000000 1
13	26.8468	14.7523	00011010010010001001101010110001
14	26.9822	12.6838	00101001010010011010100110110001
15	27.0374	13.6393	00011010110010001101100110110001
16	27.1216	13.6981	00101001110010111010100110110010
17	27.1635	13.7981	00101001110010111010100110110101
18	27.1771	13.8942	10010010101111001011110110110001
19	27.2046	12.8350	00110101001110101110100111111001
20	27.2290	13.2460	01011101011100011010110111111001
21	27.2481	14.6857	01011110101101011011101101111001
22	27.2581	13.5175	00111011101111101011100101111011
23	27.2674	14.6966	01111011101101111011101101111001
24	27.2802	14.5469	11010110111110011110110111111011
25	27.2883	15.3328	11011110111110111110100111111101
26	27.2965	14.6970	11011110011110101111111111111101
27	27.3028	14.5412	11011110111110011111111111111011
28	27.3063	14.3741	11011111111110101111111111111101
29	27.3097	14.4149	11011111111110101111111111111111
30	27.3140	14.8273	11111111111110111111111111111101
31	27.3185	14.6608	11111111111111111111111111111101
32	27.3203	14.6926	11111111111111111111111111111111

This table shows that the attenuation values obtained with 12 well selected microphones provide only slightly lower levels of attenuation that those provided by 32 microphones, so the control of the primary sound at those 12 error sensors positions gives a reasonable attenuation over the enclosure. This result can be observed better in Figure 12, which shows the attenuation versus the total number of microphones used by the control system. It should be noted from Table 5 that the control effort required to minimise the sum of squared pressures at each of these numbers of microphones in not large.

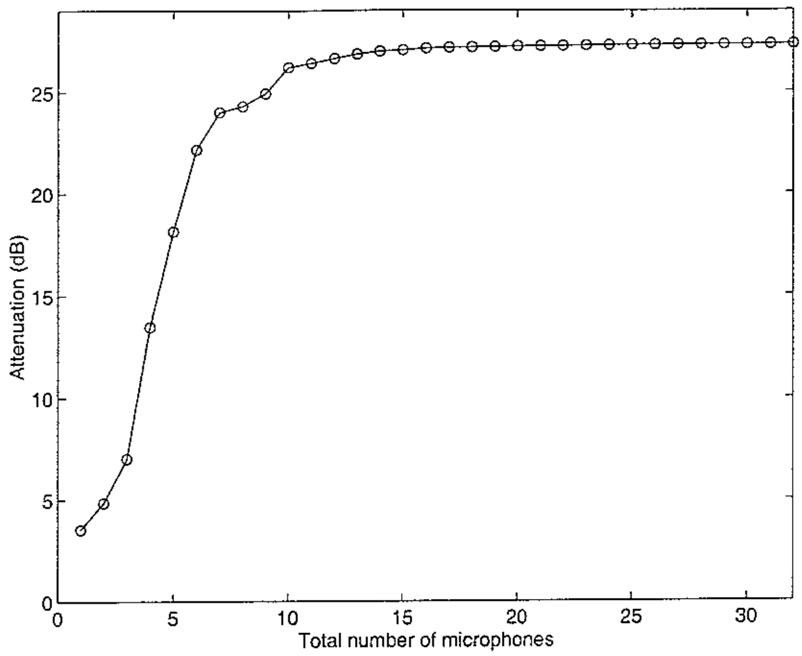

Figure 12. Maximum attenuation values obtained from an exhaustive search at a primary source excitation frequency of 88 Hz while the number of microphones are limited to 1, 2, ...32 from a total of 32 for the nominal plant

6.2. Optimal error sensor positions with structured uncertainties

To compare the results obtained in the last section with those obtained when changes appear in the nominal plant, the cost function J_1 has been used again, but using the reduced vector and matrices defined in equations (9), (10), (11) and (12) for the six perturbed plants measured in the enclosure. The same process has been repeated with the new objective function, selecting the best error sensor combinations, the attenuation levels and the corresponding total control effort values. The results obtained are presented in Table 6. It is observed that the same characteristics, such as the very high attenuation levels reached for 12 microphones instead of the total 32, occur with the nominal plant as well as with structures uncertainties present in the system. So, for the selection of the best error sensor positions, changes in the physical plant do not seem to have a large effect on the performance of the system.

Similar conclusions can be extracted from Figure 13, which shows the attenuation values achieved at all 32 microphones with different number of error microphones. In this case, the slope of the curve when the number of error sensors is not very high is slightly smaller than the case with no uncertainties, but the reduction of the primary sound field is very similar and the shapes of both graphs are very alike.

Table 6. Best attenuation values for the problem of finding L microphones positions, where L varies from 1 to 32, from a total number of 32, with the corresponding control effort values and the selected configurations using the cost function J_1

Total No. of	Attenuation	Total control	Microphone
1	3.6129	0.3440	00000000100000000000000000000000
2	5.9602	0.5439	00000000100000001000000000000000
3	8.7700	1.3048	00000000000010000000000100000100
4	12.6603	1.1961	00001000000000000000001101000000
5	15.5159	1.4883	10000000001000000000100000000011
6	17.1758	1.3765	00011000001000000000100100001000
7	20.4118	1.7963	00101000001000000000101000100000 1
8	21.8303	1.8559	00001001000000101000101000100000 1
9	23.6332	1.9812	00000101000000101000100101001001 0
10	25.8027	3.7154	00001001000000101000101010001011 0
11	26.8823	4.1300	01000000010100011000101010001101 0
12	27.0834	4.1384	01000000101000101001101010001010 1
13	27.2204	3.9483	11000000010100011010101010001010 1
14	27.3182	3.7932	00011000001011011000110010011100 1
15	27.3971	3.9704	11000000111000011010101010011101 0
16	27.4748	3.8369	11000000010100111010101011011100 1
17	27.5545	3.8238	00101100000111011001110011011100 1
18	27.6283	3.5601	01011000101110110101100110111010
19	27.7114	3.9430	10110000110110110011100110111011
20	27.7571	3.9860	10010100111110110011100111111001
21	27.7967	4.0225	10110000111110110011100111111101
22	27.8145	3.7990	10110101110110111011110110111001
23	27.8354	3.8518	10110100111110111011110111111001
24	27.8529	3.9809	10111011110110110011110111111101
25	27.8700	3.9213	11110010110111110111101111111011
26	27.8818	3.8426	10111111110110111011110111111011
27	27.8951	3.8715	11111010111111110111101111111011
28	27.9017	3.8383	11110101110111111111111111111011
29	27.9067	3.8152	11111011110111111111111111111011
30	27.9107	3.8641	11111101110111111111111111111111
31	27.9145	3.8510	11111111110111111111111111111111
32	27.9154	3.8425	11111111111111111111111111111111

6.3 Error sensor selection with control effort weighting factor

To investigate the performance of different error microphones selections with the control weighting factor β, a number of possible configurations have been generated randomly and their corresponding attenuation values have been classified depending on the total control effort values.

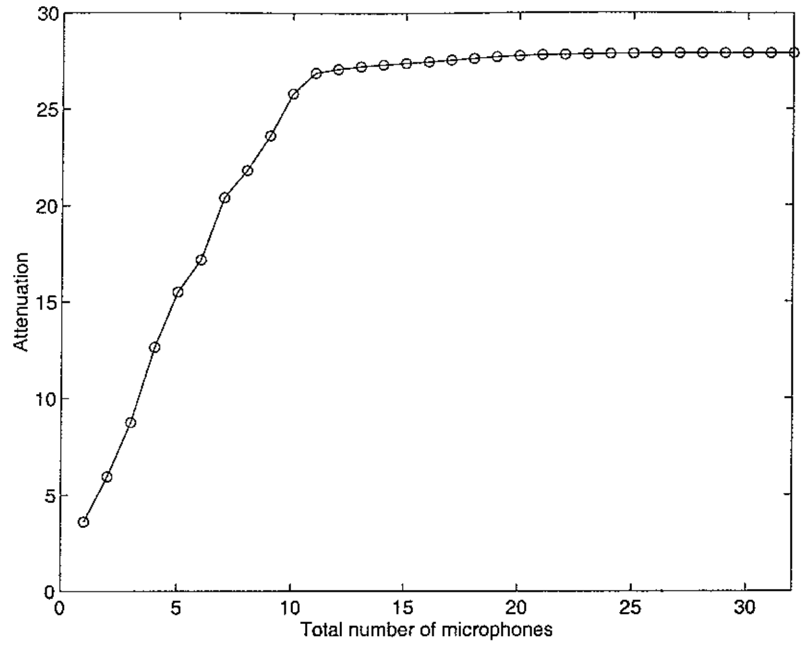

Figure 13. Maximum attenuation values obtained from an exhaustive search at a primary source excitation frequency of 88 Hz while the number of microphones are limited to 1, 2, ...32 from a total of 32 and the cost function J_1 has been used

In Section 5.2 the effect of limiting the total control effort in the system under control has been investigated. If the cost function is changed to include a control effort factor, so that it becomes $\mathbf{e}^{H}\mathbf{e} + \beta\,\mathbf{u}^{H}\mathbf{u}$, the total control effort can now be expressed as

$$\mathbf{u}_o^{H}\mathbf{u}_o = -\mathbf{d}^{H}\mathbf{C}(\mathbf{C}^{H}\mathbf{C} + \beta\,\mathbf{I})^{-2}\mathbf{C}^{H}\mathbf{d} \tag{15}$$

The value of the control effort monotonically decreases as the β value increases, because the other terms are fixed positive real number. This is illustrated for a particular case in Figure 14.

In this section, the effect of the effort weighting factor on two different cost function has been analysed. The first cost function was the same as that used in Section 5 for the selection of the best error microphones positions by calculating the reduction of the sound pressure level measured in all the error microphones present in the system. The second cost function was the cost function experienced by the control system. This was calculated by optimally adjusting the strength secondary sources vector as indicated in equations (11), (12) and (13) but, instead of use this vector for the calculus of the attenuation in all the microphones, it is substitute into the expression

$$\mathbf{e}_r = \mathbf{d}_r + \mathbf{C}_r\mathbf{u}_{or} \tag{16}$$

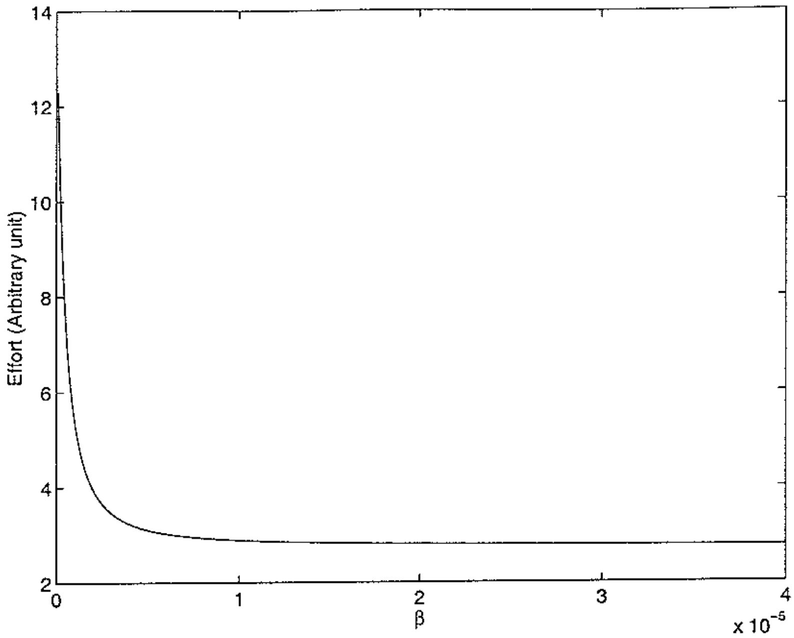

Figure 14. A particular example showing the quadratic variation of the total control effort with the weighting factor β

using only the reduced matrices, as defined in Section 6.1.

Using the nominal plant without any restrictions on effort and the best 10 secondary source positions found by exhaustive searched, several sets of 12 error microphone positions have been generated and studied. The most relevant result are presented in Figures 15, 16 and 17.

In figure 15 the attenuation values for both cost functions have been plot against the control effort weighting factor for a particular error sensors configuration with a high value of the attenuation but a low value of the effort control. The same form of graph is presented in Figure 16 but, in this case, for a different set of error microphones for which the control effort value is quite high. Figure 17 represents the same configurations in Figure 16, but a logarithm scale has been used for the control effort weighting factor.

It can be observed that, for some configurations with high control effort values, the attenuation levels obtained using the cost function with the 32 microphones reaches a maximum level for a particular β value and then starts to decreases when β becomes smaller. This behaviour is not observed for the cost function used by the electronic control system, which continues increasing while the β value decreases.

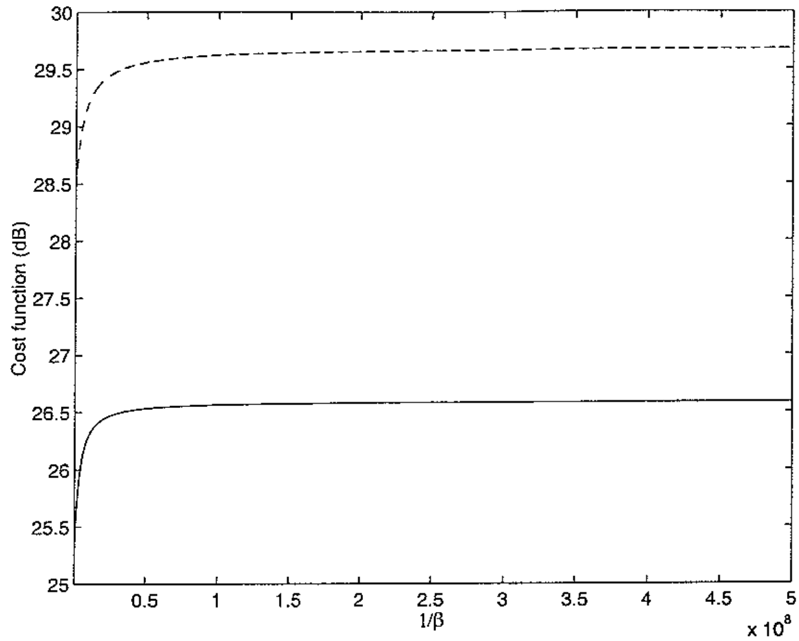

Figure 15. Attenuation values for the objective function using the best 10 loudspeakers positions and 12 selected error microphones positions for the nominal plant when evaluated at 32 error sensors (continues line), and when evaluated at the error microphones (dashed line) versus the control effort weighting factor for a configuration with low level of the total control effort

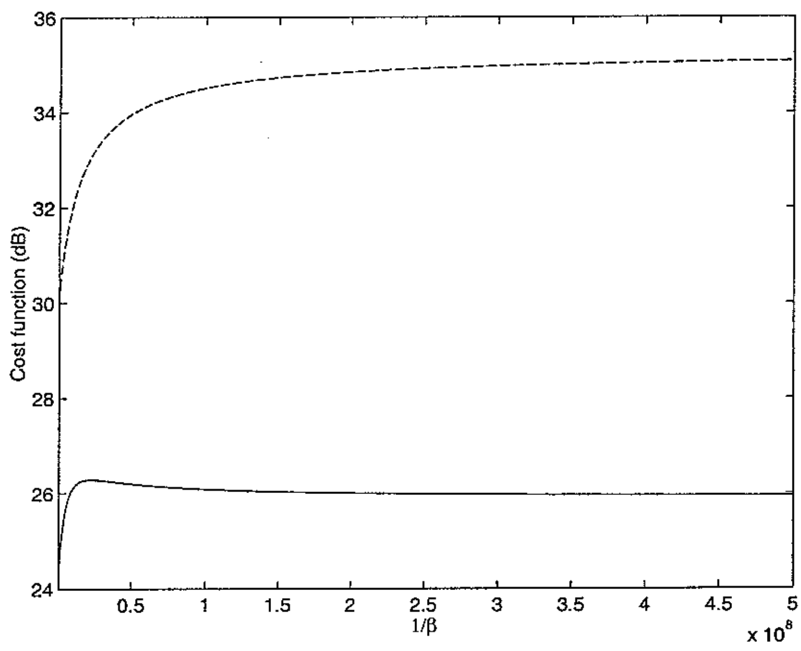

Figure 16. Attenuation values for the objective function using the best 10 loudspeakers positions and 12 selected error microphones positions for the nominal plant when evaluated at 32 error sensors (continues line), and when evaluated at the error microphones (dashed line) versus the control effort weighting factor for a configuration with high level of the total control effort

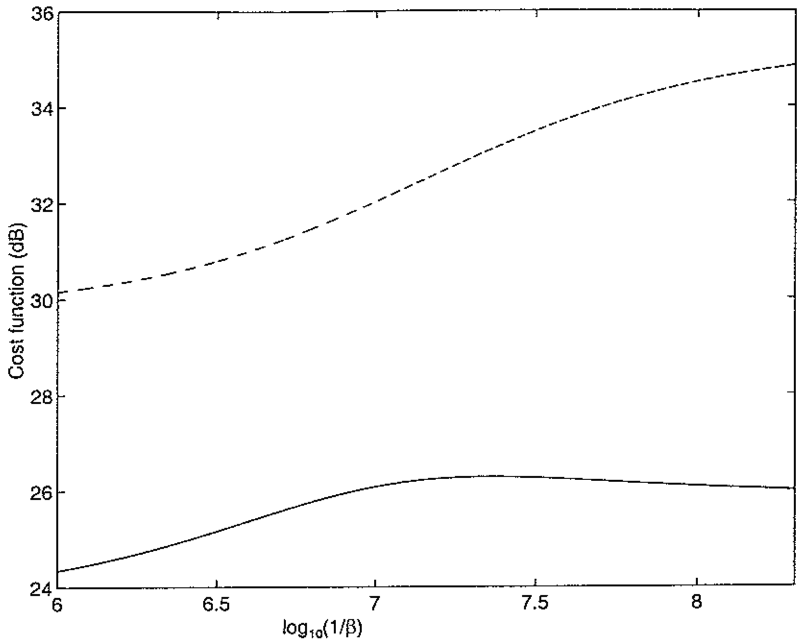

Figure 17. As Figure 16 but the ordinate axis is represented in a logarithm scale

7. Summary and conclusions

The optimal locations for the acoustic secondary sources and the error sensors in an active control system for a rectangular enclosure has been investigated in this work using the simulated annealing algorithm. Attention has been focusing not only in the reduction of the primary sound field in the enclosure provides by the possible combinations, but their robustness, that is, the selection of those positions which are most resistant to plant variations caused, for example, by the presence or absence of objects inside the enclosure.

For the nominal plant, the best M loudspeaker positions, where M varies form 1 to 16, have been found by exhaustive search, and the result have been compared with those obtained by the simulated annealing method. This algorithm is able to find a good approximate solution by searching only a small number of combinations. In both cases, the results indicate that the attenuation values obtained with 10 well chosen loudspeakers are less than 1 dB below the values obtained by using all 16 loudspeakers. A constrained searching process has also been carried on limiting the total control effort values below a particular level. It can be seen than it is possible to select configurations with low control effort values without drastically affecting the system performance. To investigate the robustness of the configurations, six perturbed transfer function have been measured in the enclosure, and an "average" cost function has been used in the optimisation process. In this case about 12 loudspeakers are required to obtain attenuation values within 1 dB of those which can be obtained with 16 loudspeakers, but the levels of control effort were intrinsically lower than when only a single transfer function was used in the selection process.

The same process has been developed for the selection of the best microphones positions. Again, it has been found that the attenuation values provided for 12 microphones are almost as good as those provided for 32, and in this cases similar results are obtained using the nominal plant and using the averaged results from the perturbed plant. The selection the control effort weighting factor has been studied, comparing the cost function experienced by the control system and the approximation to the acoustic potential energy in the enclosure.

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