Institute of Sound and Vibration Research

An Investigation into Spatial Sampling Criteria for use in Vibroacoustic Reciprocity

K.R. Holland and F.J. Fahy

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INSTITUTE OF SOUND AND VIBRATION RESEARCH

FLUID DYNAMICS AND ACOUSTICS GROUP

An Investigation into Spatial Sampling Criteria for use in Vibroacoustic Reciprocity

by

K R Holland and F J Fahy

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Approved: Group Chairman, P A Nelson
Professor of Acoustics

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

c speed of sound (m/s)
D bending stiffness (Nm)
f frequency (Hz)
G Green function
h thickness (m)
$\boldsymbol{i}$ position of point on plate
k wavenumber (1/m)
m mass per unit area (kg/m^2)
p acoustic pressure (N/m^2)
$\boldsymbol{r}$ position of acoustic field point
S surface area (m^2)
u velocity (m/s)
ϕ angle (radians)
ν Poisson's ratio
θ angle (radians)
ρ density (kg/m^3)
ω radial frequency (radians/s)
η loss factor

An Investigation into Spatial Sampling Criteria for use in Vibroacoustic Reciprocity

1 INTRODUCTION

The practical exploitation of the principle of vibroacoustic reciprocity to many noise control problems relies on the division of a vibrating surface into finite elemental areas, on which the surface vibration and blocked acoustic pressures may be measured. In order to do this, a decision has to be made as to what size these elemental areas should be. Traditionally, the Nyquist sampling criterion has been applied to a vibrating surface, taking into account *acoustic wavelengths only*; accelerometers are placed at points on the surface with a maximum spacing determined by half of the acoustic wavelength of the highest frequency of interest, thus

$$d \leq \frac{\pi}{k_a(max)} \quad , \tag{1}$$

where d is the spacing between vibration measurement points and $k_a(max) = 2\pi f(max) / c_o$, is the acoustic wavenumber of the highest frequency of interest. If this spacing criterion is exceeded, then acoustic spatial aliasing may occur. An assumption inherent in the division of a vibrating surface into elemental areas is that the surface vibration is considered uniform over an element. Sampling the surface vibration field according to the acoustic Nyquist criterion will allow this requirement to be satisfied only for surfaces bearing vibrational waves which propagate at speeds higher than the acoustic sound speed. Spatial aliasing of the *vibration* field may occur if the surface vibration field contains wavenumber components which exceed the acoustic wavenumber.

This report is concerned with the results of a computer simulation of the radiation of sound from a plate vibrating in an otherwise rigid plane. Estimations of the radiated field, calculated using point sampled and area-integrated velocities, are compared to an analytic solution of the Rayleigh integral. Results of an experimental investigation into the radiation of sound from a plate are also presented.

2 DETAILS OF SIMULATION

The simulated rectangular plate is modelled as being simply supported in an otherwise rigid, infinite plane surface; the geometry used is shown in figure 1. The plate has dimensions of 0·48m × 0·7m × 1mm thick and has the properties of aluminium.

Initial investigations are carried out with a random vibration field consisting of the summation of the first 50 modes of vibration in the two planes of the plate (2500 modes in all); all of the modes having the same amplitude, but with uniformly distributed, random phase. Further investigations are carried out with the vibration field generated by a point force acting on the plate. Each simulation is carried out 50 times with randomly chosen far-field positions and randomly chosen modal phase or forcing positions. Simple statistical analysis is performed on the differences between the analytical results and those of the point sampled and area-integrated estimations to enable comparisons to be drawn between the performance of the two techniques.

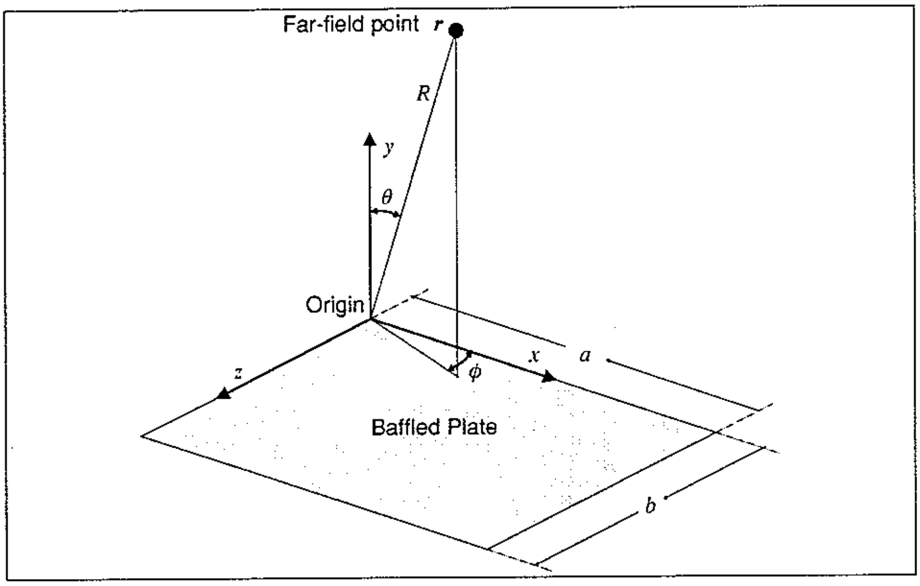

Figure 1 Geometry for Plate Simulation

2.1 Analytic Solution of the Rayleigh Integral

For modal vibration of a rectangular plate simply supported in an infinite plane baffle, the Rayleigh integral takes the form,

$$p(\boldsymbol{r}) = j\,\frac{\rho ck}{2\pi}\sum_p\sum_q u_{pq}\int_0^a\int_0^b \frac{\sin(p\pi x/a)\,\sin(q\pi z/b)\,e^{-jk|\boldsymbol{r}-\boldsymbol{i}|}}{|\boldsymbol{r}-\boldsymbol{i}|}\,\mathrm{d}x\,\mathrm{d}z\ , \qquad (2)$$

where u_{pq} is the complex vibrational velocity amplitude of mode pq, a and b are the dimensions of the plate along the x and z coordinates respectively, $\boldsymbol{r}$ is the position of the acoustic field point and $\boldsymbol{i}$ is the position of a point on the plate. An exact analytic solution of this integral does not exist, but assuming the point $\boldsymbol{r}$ at which the sound pressure is to be evaluated is a sufficient distance away from the plate such that $|\boldsymbol{r}-\boldsymbol{i}| \gg a$ and $|\boldsymbol{r}-\boldsymbol{i}| \gg b$, and using the polar geometry shown in figure 1, the equation can be written (Wallace [1]),

$$p(\boldsymbol{r}) = j\,\frac{\rho ck}{2\pi R}\sum_p\sum_q u_{pq}\,e^{-jkR}\int_0^a\int_0^b \sin\!\left(\frac{p\pi x}{a}\right)\sin\!\left(\frac{q\pi z}{b}\right)e^{j\left(\frac{\alpha x}{a}+\frac{\beta z}{b}\right)}\,\mathrm{d}x\,\mathrm{d}z\ , \qquad (3)$$

where $\alpha = ka\sin(\theta)\cos(\phi)$, $\beta = ka\sin(\theta)\sin(\phi)$ and R is the distance from the corner of the plate at the origin of the coordinate system (shown in figure 1) to the far-field point $\boldsymbol{r}$. Equation (3) has the solution,

$$p(\boldsymbol{r}) = j\,\frac{\rho ck}{2\pi R}\sum_p\sum_q u_{pq}\,e^{-jkR}\,\frac{ab}{pq\pi^2}\left\{\frac{(-1)^p\,e^{-j\alpha}-1}{(\alpha/p\pi)^2-1}\right\}\left\{\frac{(-1)^q\,e^{-j\beta}-1}{(\beta/q\pi)^2-1}\right\}\ . \qquad (4)$$

This geometric approximation corresponds physically to a 'warping' of the shape of the plate, the severity of which depends upon the validity of the far-field approximation, thus a small but significant error would exist in the calculated pressure. To correct for this error, the same geometry was used in the discrete sampling calculations, and the 'warping' of the plate was assumed not to influence the validity of the comparisons.

2.2 Point Sampling

To simulate point sampling of the vibration field, the plate is divided into small elements and the velocity at a point in the centre of each element (x_i, z_i) is calculated by summing the contributions of all of the modes:

$$u_i = \sum_p \sum_q u_{pq} \sin\left(\frac{p\pi x_i}{a}\right) \sin\left(\frac{q\pi z_i}{b}\right) \tag{5}$$

where u_i is the surface velocity at the centre of elemental area i. The radiated field is then calculated from these velocities via a discrete approximation to the Rayleigh integral,

$$p(r) = j\,\frac{\rho c k}{2\pi} \sum_i \frac{u_i\, S_i}{|r - i|}\, e^{-jk|r-i|} \ , \tag{6}$$

where $|\boldsymbol{r} - i|$ is the distance from the centre of the elemental area i to a far-field point $\boldsymbol{r}$ and S_i is the elemental area.

2.3 Area-Integrated Sampling

Considering the numerical approximation to the Rayleigh integral (equation 5), and the assumption of uniform velocity over an elemental area of the surface, the product of the surface normal velocity and the area of the element, $u_i S_i$, represents the volume velocity of the element. The following argument demonstrates that it is this volume velocity which is of importance in the determination of sound radiation.

Consider an infinite, plane vibrating surface bearing a transverse vibrational wave propagating in a direction x and having a wavenumber $k_s = \omega/c_s$, where c_s is the phase speed of the wave. The radiated sound field can be calculated from the solution of the acoustic wave equation as,

$$p = Ae^{j\left(k_s x + \sqrt{k_a^2 - k_s^2}\, y\right)} \ , \tag{7}$$

where k_a is the acoustic wavenumber ($= \omega/c_o$) and y is the direction normal to the surface. When $k_s < k_a$, the y-dependent part of the exponent is imaginary and sound is radiated away from the surface. When $k_s < k_a$, the y dependent part of the exponent is real and the sound decays exponentially away from the surface and is not propagated into the far field; for the infinite vibrating surface case, sound will only be radiated if the wavenumber of the surface vibration is less than the acoustic wavenumber. A plate of finite size can be considered to be a truncated infinite plate, and a particular mode of vibration can be reduced to an interference between waves propagating in either direction with wavenumbers of $\pm\, k_s$. The truncation of the surface corresponds to the multiplication of the infinite surface by a rectangular spatial window, which gives rise to an infinite number of vibrational wave components having wavenumbers with maximum amplitudes at $\pm\, k_s$ (the Fourier transform of a sinusoid multiplied by a rectangular window). Applying the above sound radiation argument to the finite surface reveals that sound is only radiated by those wavenumber components that are less than the acoustic wavenumber, thus when $k_s > k_a$, only the low amplitude wave components lie within this range and little sound is radiated, and when $k_s < k_a$, sound is radiated by the high amplitude wave components. As the size of the plate is increased, or k_s is increased, the widths, in wavenumber space, of the peaks at $\pm\, k_s$ reduce, the amplitudes of the components away from $\pm\, k_s$ reduce and the sound radiation tends towards that for the infinite plate.

This argument shows that all of the sound radiated by an arbitrarily vibrating surface is due to those waves having wavenumbers less than the acoustic wavenumber. If the volume velocity of an element of the surface having an area of $d \times d$, where d is the acoustic Nyquist sample spacing (equation 1), is evaluated by averaging the velocity over the area, the wave components with wavenumbers greater than the acoustic wavenumber will tend to be averaged out, leaving only those components that contribute significantly to the radiated sound. Thus the quantity of importance to sound radiation is the *instantaneous velocity integrated over an area* of $d \times d$. The errors in the estimate of the radiated sound when the vibration of a surface is sampled at points are thus due to the high wavenumber components aliasing to appear as low wavenumber, radiating components.

For simulation purposes, the volume velocity of an element of the surface of the plate can be calculated for each mode by integrating over the elemental area; thus

$$u_i\, S_i = \int_{x_i-d/2}^{x_i+d/2} \int_{z_i-d/2}^{z_i+d/2} u(x,z)\,\mathrm{d}x\,\mathrm{d}z$$

$$= \frac{ab}{\pi^2}\sum_p\sum_q u_{pq}\left\{\cos\left(\frac{p\pi}{a}\left(x_i+\frac{d}{2}\right)\right)-\cos\left(\frac{p\pi}{a}\left(x_i-\frac{d}{2}\right)\right)\right\}\left\{\cos\left(\frac{q\pi}{b}\left(z_i+\frac{d}{2}\right)\right)-\cos\left(\frac{q\pi}{b}\left(z_i-\frac{d}{2}\right)\right)\right\}. \quad (8)$$

The radiated field is then calculated as for the point sampled velocities (eq. (6)).

2.4 Modal Amplitudes due to a Point Force

The modal velocity amplitudes u_{pq} that result from a unit point force acting at a location $x = x_f$, $z = z_f$, can be calculated from consideration of the mode shapes and point impedance for the plate:

$$u_{pq}(\omega) = \frac{\sin\left(\frac{p\pi x_f}{a}\right)\sin\left(\frac{q\pi z_f}{b}\right)}{\eta\,\frac{\omega_o^2}{\omega}}\,, \quad (9)$$

where η is the loss factor for the plate material and ω_o is the resonance frequency of the mode given by

$$\omega_o = \sqrt{\frac{D}{m}}\left\{\left(\frac{p\pi}{a}\right)^2+\left(\frac{q\pi}{b}\right)^2\right\}, \quad (10)$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad \text{and} \quad m = \rho h\,, \quad (11)$$

are the bending stiffness and mass per unit area of the plate with E the Young's modulus, ν the Poisson's ratio, ρ the density and h the thickness of the plate.

3 RESULTS OF SIMULATION

Figure 2 shows the mean and standard deviation of the error in the estimation of the radiated field for point sampled velocities and equal amplitude / random phase plate vibration over a range of frequencies. Figure 3 is as figure 2 but for area-integrated velocities. Figures 4 and 5 are as figures 2 and 3 but with point force excitation of the plate.

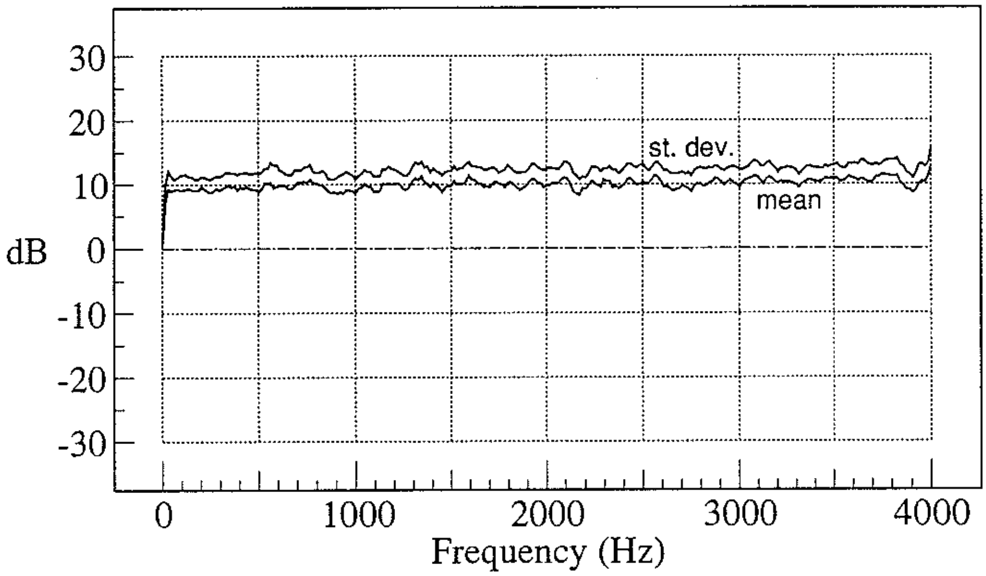

Figure 2 Statistics of Errors in Radiated Sound Field
Point Sampled Velocities – 2500 Equal Amplitude, Random Phase Modes

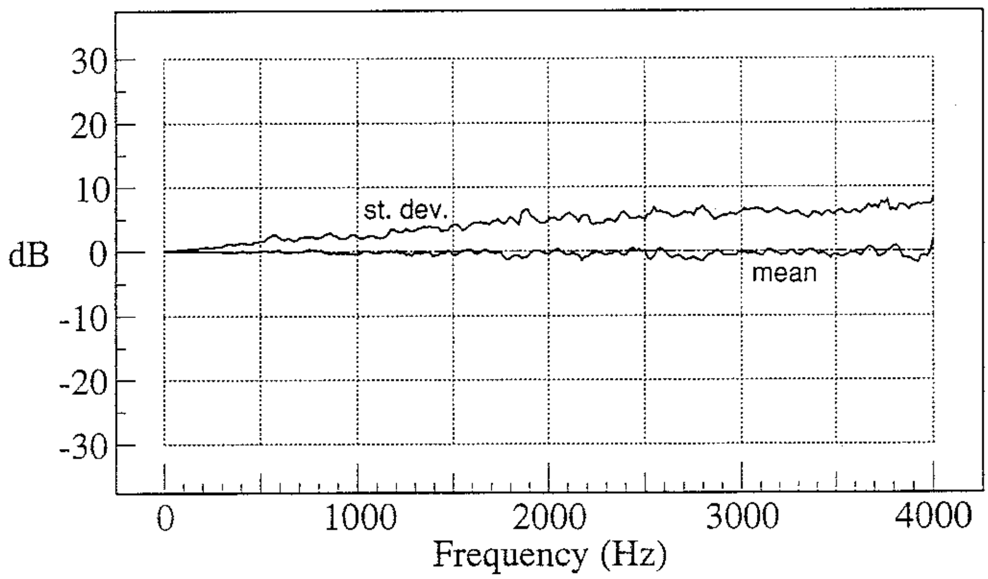

Figure 3 Statistics of Errors in Radiated Sound Field
Area-Integrated Velocities – 2500 Equal Amplitude, Random Phase Modes

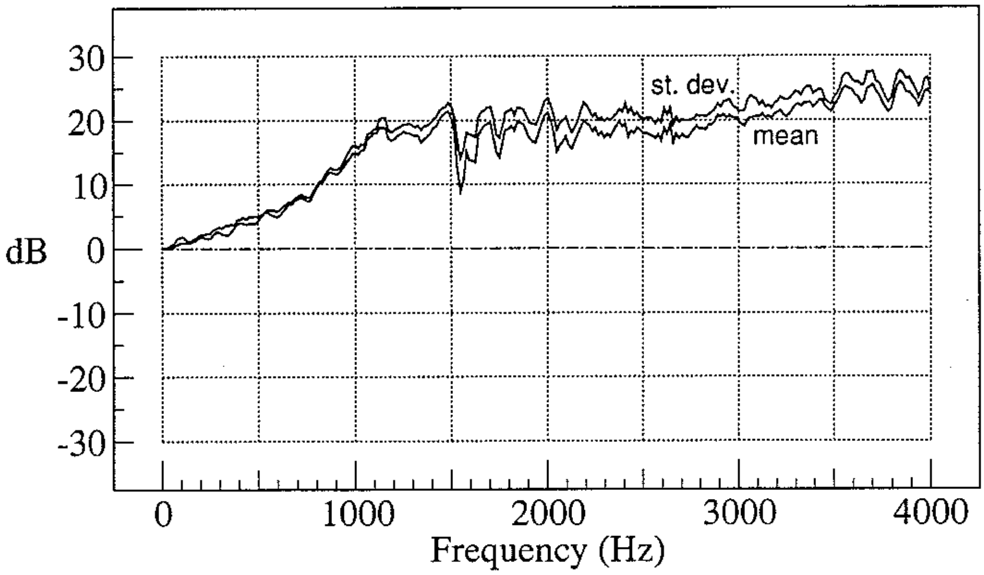

Figure 4 Statistics of Errors in Radiated Sound Field
Point Sampled Velocities – Point Force Excitation

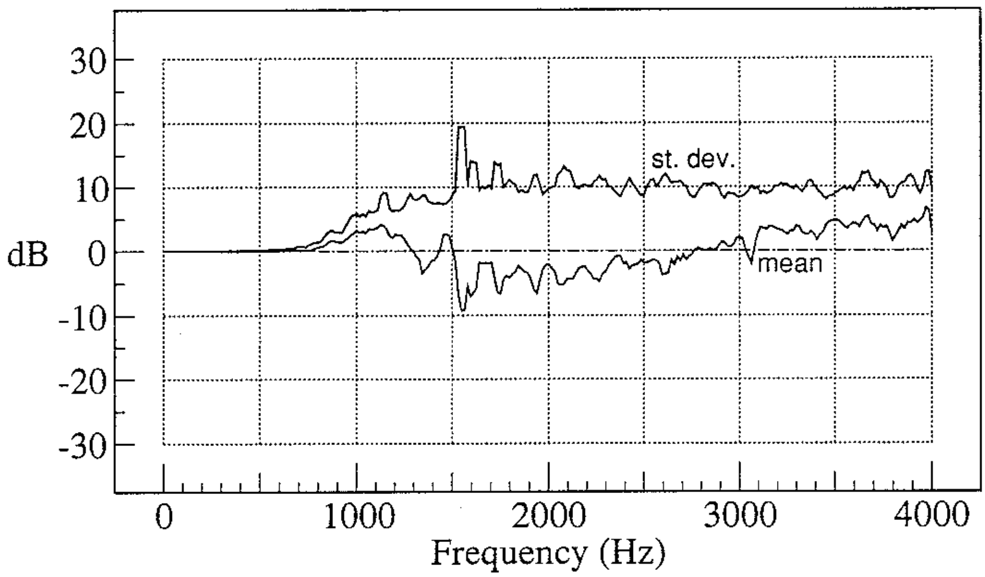

Figure 5 Statistics of Errors in Radiated Sound Field
Area-Integrated Velocities – Point Force Excitation

4 EXPERIMENTAL INVESTIGATION

4.1 Area-Integrating Volume Velocity Transducer.

Experimental verification of the results of the simulation described in section 2 was made possible by the development of a prototype area-integrating volume velocity transducer [2,3,4] at ISVR. The transducer (VVT) consists of a microphone mounted in the wall of a square-section tube. One end of the tube is anechoically terminated and the other end is open. When the open tube-end is brought close to a vibrating surface, an acoustic wave propagates along the tube towards the microphone in response to the velocity of the surface. If the microphone is mounted a sufficient distance from the open tube-end, only plane-waves, generated by the area-integrated velocity of the surface, will reach the microphone; the output from the microphone is then a measure of the instantaneous normal volume velocity of the surface, integrated over the area beneath the tube-end.

4.2 Experimental Procedure.

The objective of the experiment was to predict the sound field radiated by a vibrating surface via measurements of the vibration of the surface and a number of Green functions from the surface to an acoustic field point. The pressure at the acoustic field point is then estimated by a vector summation of the contributions from the elements of the surface, thus:

$$p(r) \approx \sum_i S_i u_i(f)\, G(\boldsymbol{r} \mid i) \ , \tag{12}$$

where $G(\boldsymbol{r} \mid i)$ is the Green function representing the pressure at the acoustic field point $\boldsymbol{r}$ due to unit volume velocity of surface element i and is equivalent to the free-space Green function in equation (6). The quantities $S_i u_i$ were determined by two methods, first using traditional point accelerometer measurements and then by direct measurement of the volume velocities using the VVT. The two estimates were then compared to a direct measurement of the pressure at the acoustic field point.

The apparatus used for verification consists of a very heavy wooden box having an aluminium plate of 1mm thickness and dimensions of 0·48m × 0·48m as its top surface. The surface of the plate was divided into 36 elements having sides of 80mm to match the 'footprint' of the VVT. For practical reasons, the plate is clamped along all four edges unlike the simply supported plate in the simulations. Excitation of the plate was via a powerful loudspeaker mounted inside the box which was fed with a white noise signal. The box was mounted on the floor of a small, otherwise empty room with a reverberation time of about 1·5 seconds.

The 36 Green functions were estimated by placing a calibrated point monopole source at the acoustic field point (near the corner of the room about 1m from the floor), and measuring the transfer functions between the acoustic pressure induced by the source on the surface of the plate at the centre of each element and the electrical drive signal to the source. According to the principle of reciprocity [5], the transfer functions measured in this way are equivalent to the required Green functions. The loudspeaker in the box was then operated and the vibration of each of the 36 elements of the plate were measured (relative to the electrical drive signal to the loudspeaker), first using an accelerometer placed at the centre of each element and then using the VVT. Finally the monopole source was replaced by a microphone having the same physical dimensions and the pressure at the acoustic field point induced by the vibration of the plate was measured directly for comparison with the indirect estimates.

4.3 Experimental Results.

Figure 6 shows a comparison between the directly measured pressure and that estimated using the point velocity measurements (accelerometer), and figure 7 shows the same but using the are-integrated velocity measurements (VVT).

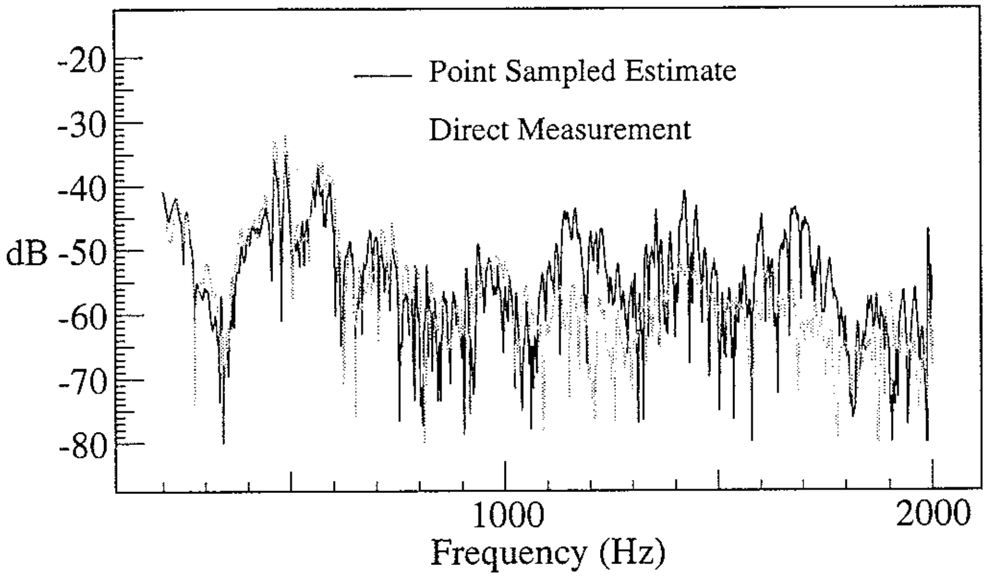

Figure 6 Measured Estimation of the Sound Radiated by a Rectangular Plate to a Point in a Reverberant Room using Point Sampling of the Velocity Field

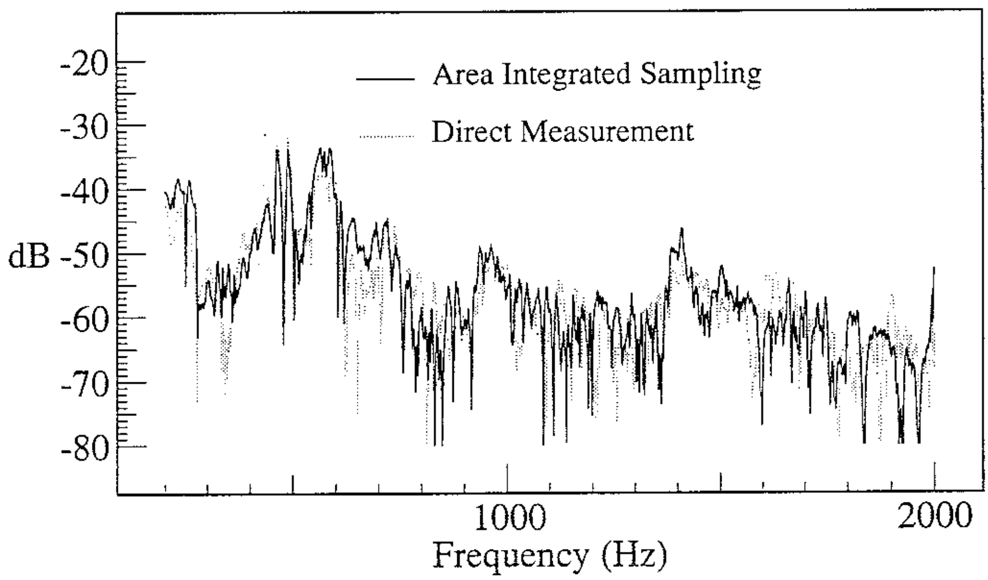

Figure 7 Measured Estimation of the Sound Radiated by a Rectangular Plate to a Point in a Reverberant Room using Area-Integrated Sampling of the Velocity Field

A comparison between figures 6 and 7 shows that the radiated field is generally overestimated at high frequencies when point velocity measurements are used in equation (12), whereas the use of area-integrated measurements yields acceptable results at all frequencies. These results are in accordance with those of the simulation.

5 CONCLUSIONS

A computer simulation of the estimation of the sound radiated by a vibrating plate is described. The results show that the radiated field will generally be overestimated if the vibration field is sampled at discrete points, using an accelerometer for example. It is shown that this problem can be overcome by replacing the point velocity measurements with area-integrated measurements.

The results of the computer simulation are verified experimentally using a prototype area-integrating volume velocity transducer.

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