isvr Institute of Sound and Vibration Research

A Guide to the Exploitation of Vibroacoustic Reciprocity in Noise Control Technology

K.R. Holland and F.J. Fahy

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INSTITUTE OF SOUND AND VIBRATION RESEARCH

FLUID DYNAMICS AND ACOUSTICS GROUP

A Guide to the Exploitation of Vibroacoustic Reciprocity in Noise Control Technology

by

K R Holland and F J Fahy

ISVR Technical Report No. 264

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Approved: Group Chairman, P A Nelson
Professor of Acoustics

CONTENTS

		Page
	List of Illustrations	iii
	List of Symbols	iv
	PREFACE	1
1	INTRODUCTION	1
2	VIBROACOUSTIC RECIPROCITY	3
	2.1 Basic Acoustic Reciprocity	3
	2.2 Harmonically Vibrating Surfaces	3
	2.3 Harmonic Force Inputs	5
	2.4 Incoherent or Partially Coherent Inputs	6
	2.5 Non-Harmonic Inputs	7
3	SPATIAL SAMPLING	7
	3.1 Acoustic Field Considerations	7
	3.2 Vibration Field Considerations	8
	3.3 Area-Integrated Measurements	9
4	MEASUREMENT TECHNIQUES	10
	4.1 Dual-Channel Techniques	11
	4.2 Frequency Resolution	12
	4.3 Signal-to-Noise Ratio	13
	4.4 Repeatability	13
	4.5 Measurement of Green Functions	14
	4.6 Measurement of Velocities and Forces	15
5	APPLICATION EXAMPLES	15
	Ex1 Radiation of Sound by a Thick Plate	15
	Ex2 Radiation of Sound by a Thin Plate	16
6	VOLUME VELOCITY SOURCES	17
	6.1 Design Considerations	17
	6.2 Source Calibration	21
	6.2.1 Free-Field Calibration	21
	6.2.2 Half-Space Calibration	22
	6.2.3 Near-Field Calibration	22
7	REFERENCES	22
8	BIBLIOGRAPHY	23

LIST OF ILLUSTRATIONS

		page
Figure 1	Engine Noise Tests	2
Figure 2	Vehicle Interior Noise	2
Figure 3	Machine Operator Noise	2
Figure 4	Acoustic Reciprocity	3
Figure 5	Equivalence of a Vibrating Element and a Source on a Rigid Boundary	4
Figure 6	Application of Reciprocity to Applied Forces	5
Figure 7	Distribution of Vibration of Part of a Thin Aluminium Plate	9
Figure 8	Influence of Frequency Resolution on Coherence	12
Figure 9	Influence of Frequency Resolution Bias Error on Repeatability	14
Figure 10	An Approximate Pulsating Sphere Monopole Source	19
Figure 11	Omnidirectional Source using a Conventional Drive-Unit and a Tube	20
Figure 12	Omnidirectional Source using a Conventional Drive-Unit and an Inverse Horn	20

LIST OF SYMBOLS

a radius (m)
c speed of sound (m/s), compliance (m/N)
d sample spacing (m)
D bending stiffness per unit area
e_r phase reference signal (Volts or Amps)
E Young's modulus (N/m^2)
f frequency (Hz)
F force (N)
G Green function (N/m^2 per m^3/s)
h plate thickness (m)
k acoustic wavenumber (1/m)
m mass per unit area (kg/m^2), mass of moving parts (kg)
N number of sources
p acoustic pressure (N/m^2)
q volume velocity (m^3/s)
S surface area (m^2)
T transfer function
u normal velocity (m/s)
U transfer function
V volume (m^3)
W_s sound power output (Watts)
Z_r normalised radiation impedance
x position of acoustic field point
λ wavelength (m)
ν Poisson's ratio
θ angle (radians)
ρ density (kg/m^3)
ω radial frequency (radians/s)

A GUIDE TO THE EXPLOITATION OF VIBROACOUSTIC RECIPROCITY IN NOISE CONTROL TECHNOLOGY

PREFACE

The principles of acoustic and vibroacoustic reciprocity have been tried, tested and proven to be valid for a wide range of engineering noise control applications. However, a general lack of an understanding of the fundamental principles, their correct application and the limitations and strengths of reciprocity techniques has led to a degree of scepticism in industry about their applicability to real engineering situations. A number of 'failures', which can be largely attributed to poor practice, have helped to reinforce this doubt. It is the aim of this document to set out guide-lines for the successful exploitation of vibroacoustic reciprocity, thereby de-mystifying the technique and promoting its successful use. It is hoped that this document may be used by engineers in the field as a guide to good practice.

1 INTRODUCTION

There are many situations where a number of noise and / or vibrational sources contribute to a total noise problem via a number of different transmission paths. Noise control technology is concerned primarily with isolating the contribution of each separate source (and / or path) so that remedial treatment may be applied in a cost-effective manner to those sources which contribute most strongly. Vibroacoustics, in the context of noise control, is the study of the radiation of sound from vibrating structures, and can be applied to noise control problems whenever the sources of noise are of this type. Traditional vibroacoustic noise control techniques involve the masking of all but one of the possible noise sources, for example by using lead sheet or temporarily disconnecting joints, thereby isolating a source of interest and allowing an estimate to be made of its contribution to the total noise. For many structures, or structural assemblies, such masking techniques are difficult, expensive, time-consuming or impractical and alternative techniques are required. For applications in which the noise level at a specific point or number of points is important, vibroacoustic reciprocity offers the potential of a more cost-effective means of estimating the contributions of individual input forces and vibrating structures to the total noise problem. Examples of this type of application include standard engine noise tests, where the sound pressure level at a point 1m from the block is required (figure 1), vehicle interior noise, where the sound pressure levels at the driver's or passenger's ears are important (figure 2), or machine operator noise exposure (figure 3). Vibroacoustic reciprocity may also be used to determine the transfer functions required for inverse source detection methods when direct measurement is either impossible or impractical.

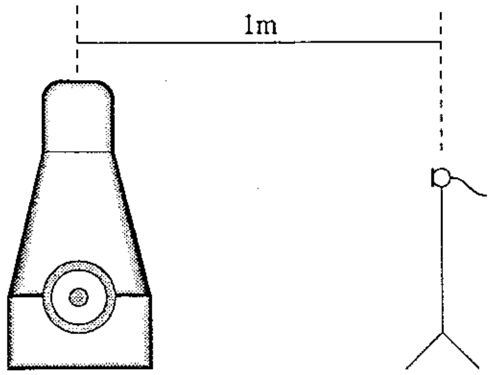

Figure 1 Engine Noise Tests

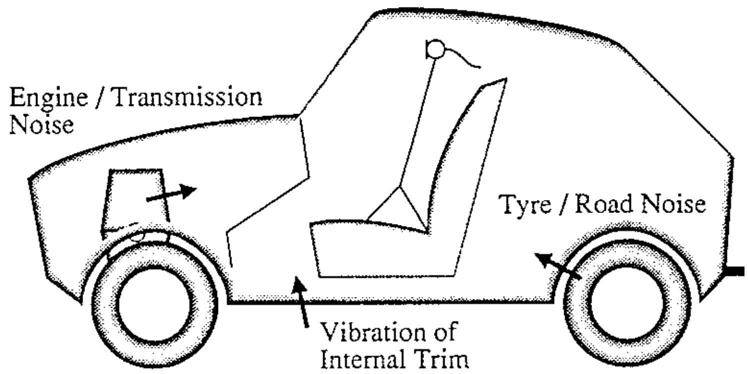

Figure 2 Vehicle Interior Noise

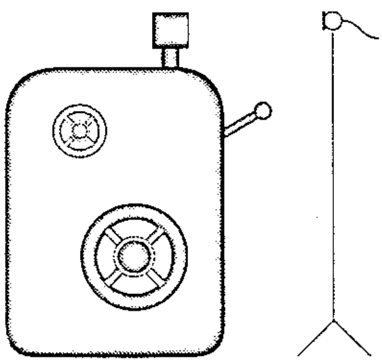

Figure 3 Machine Operator Noise

2 VIBROACOUSTIC RECIPROCITY

2.1 Basic Acoustic Reciprocity

The theory of acoustic reciprocity, first proposed by Helmholtz and later presented by Lord Rayleigh in his book "The Theory of Sound", states that:–

The acoustic pressure at a point in a fluid due to a harmonic point monopole acoustic source at another point in the otherwise still fluid is independent of an interchange of the positions of source and receiver, irrespective of the presence of any arbitrary, linearly reacting boundaries.

The basic reciprocal relationship is illustrated in figure 4, where q_1 is the volume velocity of a point monopole source at position 1 and p_1 is the acoustic pressure at position 1 etc.

$$\frac{p_1}{q_1} = \frac{p_2}{q_2}$$

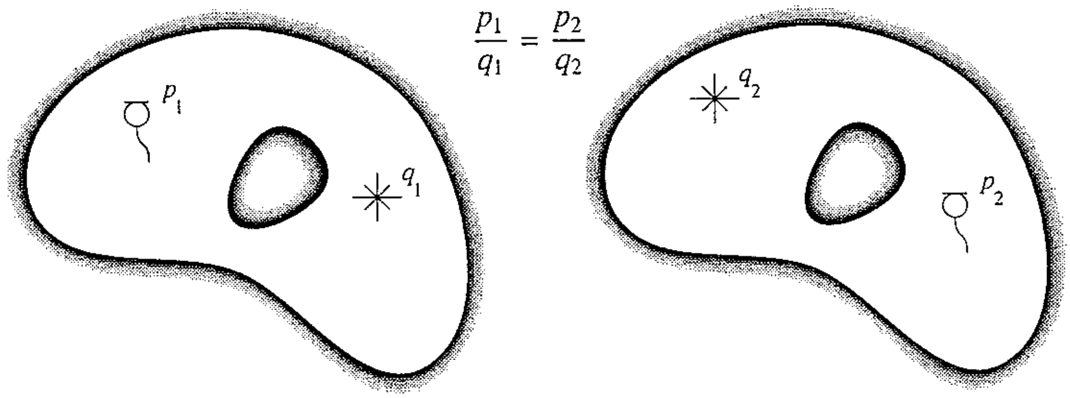

Figure 4 Acoustic Reciprocity

2.2 Harmonically Vibrating Surfaces

By the principle of superposition, the pressure field generated by any surface vibrating at a frequency f, can be estimated by summing the field generated by all of the elemental areas representing the surface, thus:

$$p(x, f) \approx \sum_i S_i u_i(f)\, G(x \mid i, f) \quad , \tag{1}$$

where $u_i(f)$ is the normal velocity of a surface element i having an area S_i, the product $S_i u_i(f)$ is known as the ***volume velocity*** of the element, and $G(x \mid i, f)$ is the Green function representing the pressure at the acoustic field point x due to unit volume velocity of surface element i. In the limit of vanishingly small elements, the summation becomes an integral and the relationship is exact.

The necessary Green functions can be calculated for systems with simple geometry; for example, in the case of the vibration of part of an (otherwise rigid) infinite plane surface radiating into free space, the Green functions between the vibrating elements and an acoustic field point are merely twice the free-space Green functions.

$$G(x \mid i, f) = \frac{e^{-jk(x-i)}}{|x-i|} \quad .$$

so given a knowledge of the vibration of the surface at a frequency f, the pressure at any acoustic field point x can be calculated using a numerical approximation to the Rayleigh integral,

$$p(x,f) \approx \frac{j\rho ck}{2\pi} \sum_i S_i u_i(f) \frac{e^{-jk(x-i)}}{|x-i|} \ , \qquad (2)$$

where ρ and c are the density and speed of sound of the fluid, k ($= 2\pi f / c$) is the acoustic wavenumber and $|x - i|$ is the distance from surface element i to the acoustic field point x. In general however, the Green functions for surfaces having arbitrary geometry and / or in the presence of other boundaries cannot be calculated easily; these may be measured by invoking the principle of ***vibroacoustic reciprocity***:–

The vibration of a small element of a surface can be represented by an equivalent monopole acoustic source acting at the otherwise rigid surface, and the reciprocal relationship between source and receiver still applies (fig 5).

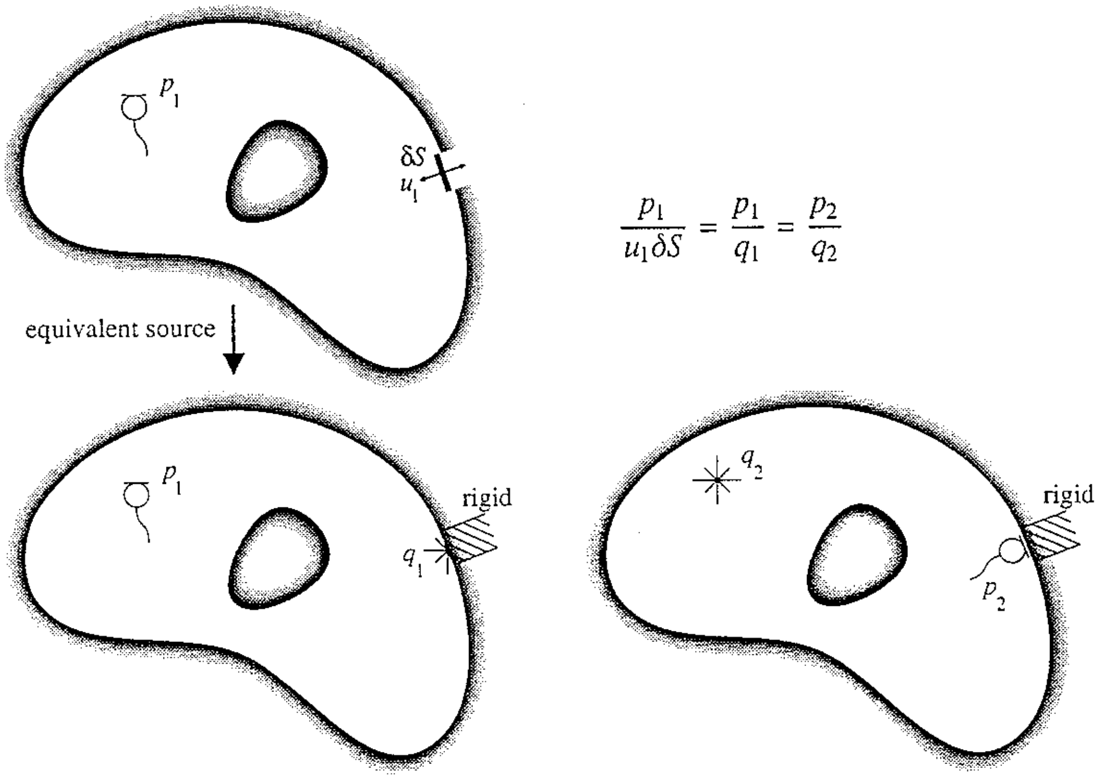

Figure 5 Equivalence of a Vibrating Element and a Source on a Rigid Boundary

To estimate the acoustic pressure at an acoustic field point using vibroacoustic reciprocity, the vibrating surface is divided into a number of small areas and an acoustic monopole source is located at the acoustic field point. Measurements are then taken of the acoustic pressure on each of the elemental areas in the absence of any vibration of the surface (the ***blocked*** pressure measured on the surface in a ***rigid*** state). Using superposition, a summation over the surface, of the volume velocities of the surface elements multiplied by the blocked acoustic pressures, measured on the surface over which the externally generated vibration acts, yields an estimate of the acoustic pressure at the acoustic field point, thus:

$$p(x,f) \approx \frac{1}{q(f)} \sum_i S_i u_i(f) \ p(i \mid x, f) \ . \qquad (3)$$

where $p(i \mid x, f)$ are the measured blocked pressures on surface element i due to a monopole source at the acoustic field point x and $q(f)$ is the volume velocity of the monopole source at frequency f. Note that by acoustic reciprocity, the function $p(i \mid x, f)$, after division by $q(f)$, is identical to the Green function $G(x \mid i, f)$ in equation (1).

In the case where some regions of the surface are 'actively' vibrating due to external sources, and other regions are 'passive' (but may be moving under the action of the sound field generated by the 'active' regions), the Green functions determined by reciprocity automatically account for any acoustically-induced movement of the passive regions. It should be stressed however, that all regions of the surface that are considered to be active must be blocked while the Green functions are being measured, and that the velocity of any regions that are considered to be passive should not be measured. The decision as to whether a region is active or passive is not always trivial; incorrectly treating an active region as passive will introduce errors in the estimated sound field; treating all surfaces as active will not introduce any error but will increase the number of measurements that have to be taken. If in doubt, all regions should be treated as active.

2.3 Harmonic Force Inputs

Vibroacoustic reciprocity may also be applied in situations where a structure radiates sound in response to applied harmonic forces. Traditionally, if a number of applied forces are present, for example at the engine mounts in a vehicle, the relative contributions of the forces to the acoustic pressure at a point are estimated by measuring each operating force input, removing each mount in turn and measuring the acoustic pressure with a shaker attached to the mounting point (in the absence of the other forces). This technique often proves time-consuming and difficult, as the mounting points may be relatively inaccessible. By exploiting the principle of vibroacoustic reciprocity, the estimates of the relative contributions of the applied forces to the acoustic pressure can be obtained without the need to attach a shaker.

A reciprocal relationship holds between the pressure at an acoustic field point due to an applied force, and the velocity at the position of the applied force due to an acoustic source at the acoustic field point (fig. 6).

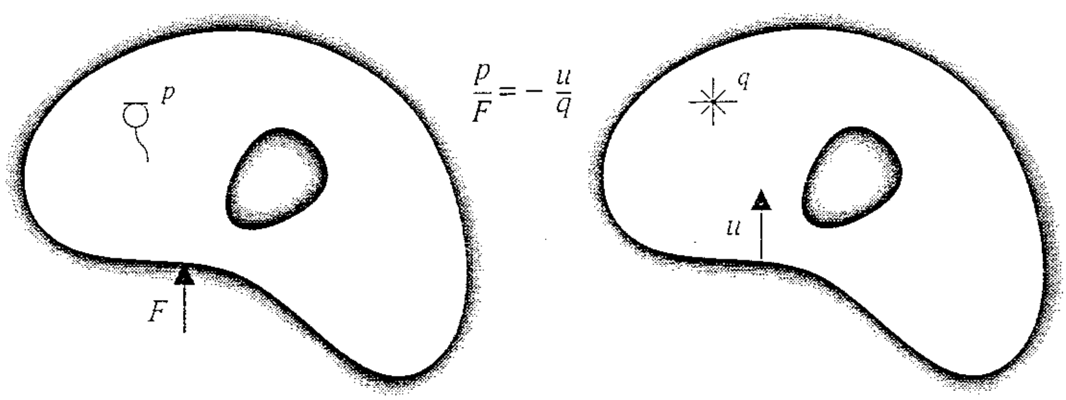

Figure 6 Application of Reciprocity to Applied Forces

This relationship holds if the velocity is measured in the same direction as the applied force, and the velocity is that present in the absence of any external constraints at the forcing point (the ***free*** velocity). To estimate the contribution of a force input to the pressure at an acoustic field point, an acoustic monopole source is placed at the acoustic field point and the free velocity at the forcing position is measured. The acoustic pressure results from a multiplication of this velocity by the measured force input and division by the volume velocity of the monopole source, thus:

$$p(\boldsymbol{x},f) \approx -\frac{u(i\,|\,\boldsymbol{x},f)}{q(f)}\,F_i(f) \ , \tag{4}$$

where $u(i\,|\,\boldsymbol{x},f)$ is the free velocity measured at forcing point i due to the monopole source at the acoustic field point $\boldsymbol{x}$, F_i is the force input at i and $q(f)$ is the volume velocity of the monopole source. This relationship can be extended by superposition to an array of point forces:

$$p(\boldsymbol{x},f) \approx -\frac{1}{q(f)}\sum_i u(i\,|\,\boldsymbol{x},f)\,F_i(f) \ , \tag{5}$$

to yield the total pressure at $\boldsymbol{x}$ due to a number of applied forces.

2.4 Incoherent or Partially Coherent Inputs

Equations (1), (2), (3) and (5), for systems with harmonic excitation, only strictly apply when the inputs forces and / or vibrations are fully coherent with each other; for harmonic excitation at a single frequency f, all input vibrations and / or forces must be coherent. Many practical vibroacoustic systems have inputs which, when measured over a narrow frequency band, are not fully coherent with each other – in these situations, more general formulations of the above equations are required, and as phase is now no longer unique, only the mean-squared pressure can be calculated.

Consider two partially coherent sources, i and j, contributing to an acoustic pressure p at an acoustic field point $\boldsymbol{x}$. The mean-square acoustic pressure in a narrow frequency band centred on f, will be the sum of the individual fields generated by the two sources plus an additional field due to the interference between the sources which will depend upon the degree of correlation between them. The mean-squared pressure at the acoustic field point due to N partially correlated sources can be written,

$$\frac{|p|^2}{2} = \frac{1}{2}\,\mathcal{R}e\left\{\sum_{i=1}^{N}\sum_{j=1}^{N} q_i(f)\,q_j^*(f)\,G(\boldsymbol{x}\,|\,i,f)\,G(\boldsymbol{x}\,|\,j,f)^*\right\} \ , \tag{6}$$

thus for partially correlated inputs, equations (1), (2), (3) and (5) can be replaced by the following,

$$\frac{|p(\boldsymbol{x},f)|^2}{2} \approx \frac{1}{2}\,\mathcal{R}e\left\{\sum_i\sum_j S_iS_j\,u_i(f)u_j^*(f)\,G(\boldsymbol{x}\,|\,i,f)\,G(\boldsymbol{x}\,|\,j,f)^*\right\} \ . \tag{7}$$

$$\frac{|p(\boldsymbol{x},f)|^2}{2} \approx \frac{(\rho ck)^2}{8\pi^2}\,\mathcal{R}e\left\{\sum_i\sum_j S_iS_j\,u_i(f)u_j^*(f)\,\frac{e^{-jk(|\boldsymbol{x}-i|-|\boldsymbol{x}-j|)}}{|\boldsymbol{x}-i|\,|\boldsymbol{x}-j|}\right\} \ . \tag{8}$$

$$\frac{|p(\boldsymbol{x},f)|^2}{2} \approx \frac{1}{2|q(f)|^2}\,\mathcal{R}e\left\{\sum_i\sum_j S_iS_j\,u_i(f)u_j^*(f)\,p(i\,|\,\boldsymbol{x},f)\,p(j\,|\,\boldsymbol{x},f)^*\right\} \ . \tag{9}$$

and,

$$\frac{|p(x,f)|^2}{2} \approx \frac{1}{2|q(f)|^2} \Re e \left\{ \sum_i \sum_j u(i \mid x, f)\, u(j \mid x, f)^* \, F_i(f)\, F_j^*(f) \right\} , \tag{10}$$

where each summation is taken over all inputs. The quantity $u_i(f)\, u_j(f)^*$ represents the cross-spectrum between velocities i and j. Clearly, for a system having N inputs, N^2 cross-spectrum measurements are required if the inputs are uncorrelated compared to N measurements for correlated inputs. In practice, due to the reciprocal relationship between $u_i(f)\, u_j(f)^*$ and $u_j(f)\, u_i(f)^*$, only $(N^2 + N)/2$ measurements need be taken.

2.5 Non-Harmonic Inputs

The above reciprocal relationships also hold for the Fourier components of non-harmonic, time stationary vibration or force inputs; the associated functions may, in these cases, be determined using FFT analysis.

3 SPATIAL SAMPLING

The practical exploitation of the principle of vibroacoustic reciprocity in dealing with noise control problems where the input is a vibrating surface, relies on the division of the surface into finite-sized elemental areas, within which the surface vibration and blocked acoustic pressures may be measured at representative points. In order to do this, a decision has to be made as to what size these elemental areas should be. From a practical point of view, larger areas means that fewer measurements have to be taken; however, if the areas are too large, errors in the predicted pressure field will become significant. The following section explains the acoustic and structural considerations that have to be taken into account before this decision can be made.

3.1 Acoustic Field Considerations

Traditionally, the 'Nyquist' sampling criterion has been applied to vibrating surfaces, taking into account acoustic wavelengths only; accelerometers are placed at points on the surface with a maximum spacing determined by half of the acoustic wavelength of the highest frequency of interest, thus:

$$d \leq \frac{\lambda_{(\min)}}{2} = \frac{c}{2 f_{(\max)}} , \tag{11}$$

where d is the spacing between vibration measurement points, c is the acoustic sound speed and $\lambda_{(\min)}$ is the wavelength of the highest frequency of interest, $f_{(\max)}$. If this spacing criterion is violated, then acoustic spatial aliasing may occur at high frequencies.

This spacing requirement may be relaxed in some circumstances, depending upon the orientation of the vibrating surface relative to the acoustic field point and the presence of other boundaries. The acoustic phase error due to the path length difference between adjacent sampling points on a surface to the acoustic field point must be less than π radians if acoustic spatial aliasing is to be avoided. Generally, the spacing requirement of a pair of adjacent sampling points can be determined from,

$$\Delta r \leq \frac{c}{2 f_{(\max)}} . \tag{12}$$

where Δr is the path length difference between the two adjacent sampling points to the acoustic field point. The spacing requirement is therefore a function of the distance from the acoustic field point to the surface, the angle between them and the presence of any boundaries. If the acoustic field point is a sufficient distance away, such that $r \gg d$, where r is the distance from the surface to the point, and no other boundaries are present (free-field conditions), this relationship simplifies to,

$$d \le \frac{c}{2 f_{(\max)} \sin(\theta)} , \tag{13}$$

where θ is the angle between a normal to the surface (at a point between the two adjacent sample points) and the acoustic field point. When $\theta = 0°$, the path length difference is zero and there is no restriction on the sample spacing. When $\theta = 90°$, the sample spacing becomes that of the acoustic Nyquist criterion given in equation (11). Under free-field radiation conditions therefore, the vibrating surfaces 'facing' the acoustic field point may be sampled with wider spacing than those that 'face away' from it. Regardless of the geometry, the spacing requirements given in equations (12) and (13) are never less than the acoustic Nyquist criterion given in equation (11). Under reverberant acoustic conditions, or for acoustic field points 'close' to a vibrating surface, *considering the acoustic field only*, the acoustic Nyquist criterion should be regarded as the maximum 'safe' sample spacing.

3.2 Vibration Field Considerations

An assumption inherent in the use of point velocity measurements in vibroacoustics is that the surface vibration is considered uniform over an elemental area. Sampling a surface vibration field according to the acoustic criteria above will allow this requirement to be satisfied only for surfaces bearing vibrational waves which propagate at speeds higher than the acoustic sound speed (the wavelengths in the vibration field are longer than in the acoustic field). If the surface vibration field propagates slower than the acoustic sound speed (shorter vibration field wavelengths), then spatial aliasing of the vibration field will occur if acoustic sampling criteria are used.

Considering that only the supersonic wave components of a vibration field are efficient radiators of sound [*Fahy*, 1], the aliasing of any subsonic vibration components, due to sampling according to the acoustic Nyquist criterion, will result in an over-estimation of the radiation of the surface.

By way of example, figure 7 shows the distribution of the amplitude of the normal vibration velocity of a 1mm thick uniform aluminium plate measured at 25 points covering an 80mm × 80mm area. The area has a centre with coordinates x = 360mm, y = 200mm on a plate of dimensions 480mm × 480mm; the plate was excited via a loudspeaker source.

Figure 7 shows that variations in surface normal velocity of the order of 10dB exist within the area, and considering that the sample spacing, according to the acoustic Nyquist criterion, would be 94mm at this frequency, it would be unwise to expect any single measurement point within the area to be representative of the vibration field. For a thin plate of uniform thickness, the wavelength of vibrational bending waves is given by,

$$\lambda_b = 2\pi\, \omega^{-1/2} \{D/m\}^{1/4} . \tag{14}$$

where m is the mass per unit area of the plate and D is the bending stiffness per unit area. If the plate is homogeneous,

$$D = \frac{Eh^3}{12(1-\nu^2)} \; ,$$

where E is the Young's Modulus, ν is the Poisson's ratio of the plate material and h is the thickness of the plate. For a 1mm thick aluminium plate, the bending wavelength at 1800Hz is approximately 75mm, which is considerably shorter than the acoustic wavelength of 190mm on which the Nyquist sampling criterion is based. For vibrating surfaces of this nature, the sample spacing needs to be smaller than that required to satisfy the acoustic Nyquist criterion if errors in the estimate of the radiated pressure are to be avoided. Many practical vibrating surfaces, for example, the cover of part of an engine, have stiffeners and edges which can give rise to localised vibration patterns containing short wavelength components; decisions have to be made as to whether the vibration measurement positions should be on the stiffeners or between. In practice, it is likely that neither position would yield a representative measure of the vibration field.

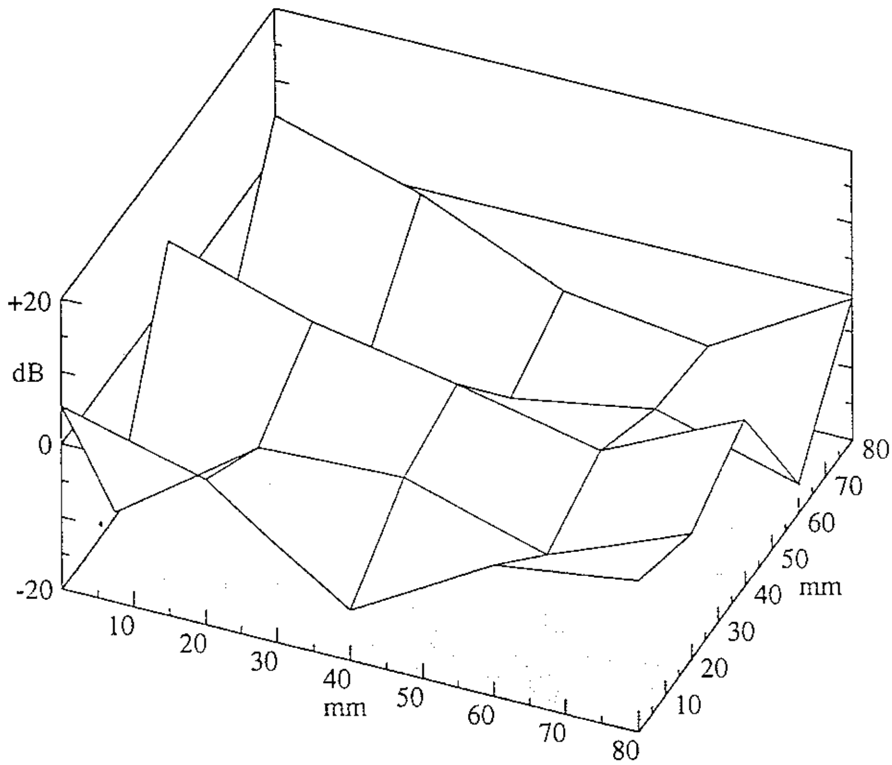

Figure 7 Distribution of Vibration of part of a thin aluminium plate; f = 1800Hz

Clearly, decisions concerning the spacing and positioning of point vibration measurements can only be made after careful consideration of the nature of the vibrating field. If the nature of the vibration field is not known, experiments can be carried out in which measurements of the vibration at two or more positions on the surface are compared as the spacing between the positions is reduced, but these are time-consuming and inefficient procedures.

3.3 Area-Integrated Measurements

If a vibrating surface is divided into elemental areas $d \times d$, where d is defined using the acoustic Nyquist criterion, the blocked pressure measured at a point within an element can be considered representative of the pressure over the entire element. With a vibration field known to consist of only supersonic wave components, the velocity can also be considered uniform over an element and the vibration field can be characterised by point vibration measurements at the same spacing. If the vibration field is either unknown, or is known to contain subsonic wave components, use of the acoustic Nyquist criterion may lead to spatial aliasing of the vibration field and subsequent over-estimation of the radiated acoustic pressure. Considering equation (3), the vibration quantities of interest ($S_i u_i(f)$) for the estimation of the radiated sound field are the volume velocities of the elements. An elemental volume velocity can be defined as the integral of the velocity over the surface of the element, thus integration over an element defined using the acoustic Nyquist criterion will act as a spatial filter and 'average out' any subsonic wave components; only wave components with wavelengths greater than $2d$ will contribute significantly to the elemental volume velocity and hence the estimate of sound radiation.

The above argument demonstrates that vibroacoustic reciprocity may be successfully applied to *any* vibrating surface using elemental areas defined using the acoustic Nyquist criterion, provided measurements of the volume velocity of the elemental areas are considered, rather than point vibration measurements. Computer simulations [2] and experiments using a prototype area-integrating velocity transducer [3, 4] have demonstrated considerable improvements in the accuracy of estimates of the sound field radiated by vibrating plates when elemental volume velocities are used in equation (1) in place of point sampled vibrations.

4 MEASUREMENT TECHNIQUES

In order to apply the principles of vibroacoustic reciprocity to practical noise control problems, techniques for the accurate measurement of the required velocity distributions, force inputs and Green functions need to be developed. This section is intended as a guide to good measurement practice only; the exact techniques used will depend largely on available equipment and resources.

In general, the pressure, velocity and force quantities of interest are complex, possessing both amplitude and phase. If only one excitation input is to be considered - a single point force for example, then only the amplitude or modulus of the quantities is required for accurate estimation of the acoustic pressure. However, where more than one input is considered - a vibrating surface divided into elements for example, the quantities in equations (3) and (5) need to be summed with due regard for any phase interference between the contributions of the various inputs to the acoustic pressure. To take phase interference into account, the measured quantities need to be complex, with phase relative to an arbitrary reference, and vector summations are required in equations (3) and (5).

For all but the simplest of vibroacoustic systems (or those where the noise is tonal in nature), the acoustic pressure at a single frequency is of limited interest: the large number of structural and acoustic modes present in typical systems gives rise to considerable variations in noise level with frequency. It is therefore usually desirable for measurements to be taken over the range of frequency of interest. The techniques available can be broadly classified into broad-band and narrow-band techniques.

Broad-band techniques involve splitting the frequency range into a number of bands, usually one octave, one third of an octave or one twelfth of an octave wide, by passing the measured signals through a set of filters. These techniques can only be used when phase information is not required; for example, for single input systems or systems where a number of inputs can be considered to be uncorrelated over each frequency band.

Narrow-band techniques rely on the assumption that any signal can be decomposed into an infinite number of discrete frequencies. As each frequency component has a particular amplitude and phase, it can be treated as a single frequency in its own right, and calculations can be performed on the measured frequency-domain data in the same way as for harmonic excitation. In practice, this decomposition can only be carried out approximately by assuming that the signal is periodic, giving rise to a finite number of discrete frequencies. With the advent of modern digital signal processing techniques, Fourier analysis, in the form of the Fast Fourier Transform (FFT) algorithm, is the most widely used narrow-band technique, but single-tone and swept-tone techniques are still used. Signal analysers based on the FFT are often dual- or multi-channel allowing complex measurements to be taken directly.

4.1 Dual-Channel Techniques

Dual-channel analysers have the capability of simultaneously measuring two signals, which can then be compared. One signal is often considered to be the input to the system being measured, and the other the output. After transformation of the two signals to the frequency domain using FFT techniques, the division of output / input yields the complex frequency response function of the system. The same technique can be applied to any two signals which can be considered to be correlated; for example, the output from an accelerometer placed at a point on a vibrating surface could be compared to the output from another - the resultant complex transfer function would represent the difference in amplitude and phase of the two accelerometer signals as a function of frequency.

A number of estimates of a measurement can be made and averaged to improve the accuracy of the required transfer function, especially in the presence of background noise. Given a sufficient number of averaged estimates, a useful quantity known as ***coherence*** can be derived from the two input signals. Coherence is defined as the modulus of the cross spectrum of the two signals squared divided by the product of the two autospectra, and it expresses the degree of linear relationship between the two signals. For a single estimate, or the average of a number of identical estimates, the coherence will be equal to one; however, if differences exist between individual estimates, the coherence will be less than one. A coherence of less than one can result from the presence of noise in the signals, insufficient frequency resolution, a long time delay between the two signals or nonlinearity in the system being measured.

The coherence between the two signals being measured proves to be a very effective indicator of the accuracy of a transfer function measurement. Higher confidence can be had in a measurement for which the coherence is close to unity than one for which it is lower.

Most dual-channel analysers will calculate the coherence function during a measurement and allow it to be displayed.

4.2 Frequency Resolution

Frequency resolution can be important in the accurate measurement of complex vibroacoustic systems; the variation in the velocities, forces and pressures with frequency can be quite rapid, especially if damping is light. A coarse frequency resolution can lead to important 'peaks' or 'dips' at certain frequencies being missed out, giving rise to bias errors in the estimated quantities. Frequency resolution should be chosen to adequately resolve the sharpest peaks and dips in the measurements; this can be checked by trial-and-error or, preferably, by monitoring the coherence (see section 4.1) between the two measured signals. If a measurement is made with inadequate frequency resolution, the coherence function will show sharp dips at the frequencies where the peaks and dips in the transfer function occur. Figure 8 shows the coherence function for two measurements of the same system; one with adequate, and one with inadequate frequency resolution. Note that the dips in coherence coincide with the sharp peaks and dips in the transfer function.

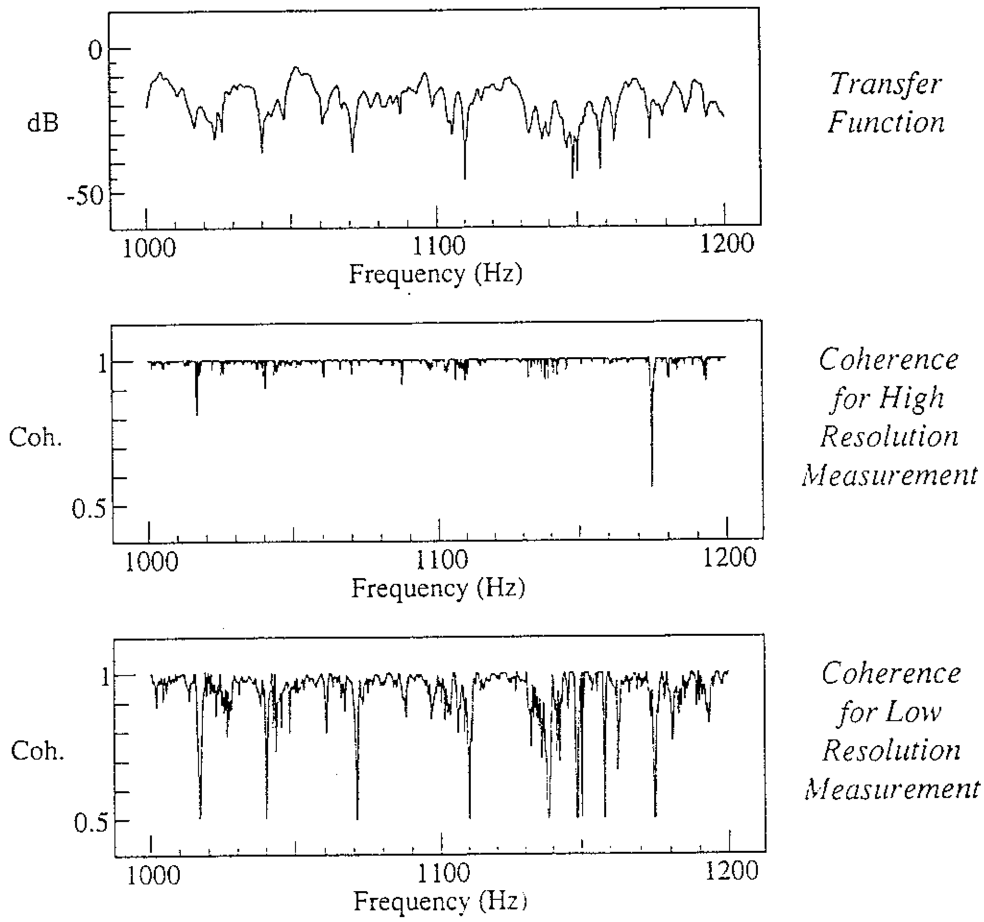

Figure 8 Influence of Frequency Resolution on Coherence

Regardless of the technique used, the frequency resolution is equal to the reciprocal of the length of the time window (T) used to make the measurement:

$$\Delta f = \frac{1}{T} \ . \tag{15}$$

where Δf is the spacing between individually resolved frequencies.

4.3 Signal-to-Noise Ratio

In practice, background noise of some form is always present in a measurement. It may be electrical 'hum' or 'hiss' from electronic components and transducers or external noise sources such as wind, traffic etc. The presence of noise in a signal being measured gives rise to errors in the measurement which depend, to a large extent, on the relative levels of the signal and the noise at each frequency. The signal-to-noise ratio can be estimated in many cases by taking background level measurements in the absence of the source of interest; the background level should be lower than the measured signal at all of the frequencies of interest. In situations where this is impractical, monitoring the coherence (see section 4.1) between the two measured signals will give an indication of the presence of excessive noise, provided the noise present in the two signals is uncorrelated; the coherence will be seen to drop at those frequencies where the signal-to-noise ratio is poor. Unless the noise present in the two signals is correlated, averaging a large number of estimates will improve the accuracy of the measurement. It is possible, using averaging under favourable conditions, for accurate measurements to be taken when the signal-to-noise ratio is less than one – the coherence in these cases will remain poor.

4.4 Repeatability

Measurements are generally only valid if the physical properties of system being measured can be considered to be time invariant. If this is not the case, a measurement can only be considered valid at the time the measurement was taken, and does not describe the system in any general sense. When applying vibroacoustic reciprocity, it is *very* important that the properties of the system being measured do not vary with time. The technique requires that the blocked pressure (or free velocity) measurements be taken at a different time than the velocity (or point force) measurements. If conditions change between, or during the two measurements, errors in the calculated sound pressure can occur which are larger than the actual difference in the pressure due to the change in conditions.

The most common cause of time variance is changes in temperature. The resonance frequencies of many vibroacoustic systems, particularly those with light damping, change as a function of temperature, so a measurement taken at a certain frequency may coincide with a resonant peak at one temperature and a dip at another. Multiplication of one measurement by another taken at a different temperature will lead to errors in the predicted sound field. In this case, the bias errors due to inadequate frequency resolution (section 4.2) will be compounded. Figure 9 shows the repeatability of a measured transfer function taken with adequate and inadequate frequency resolution. The repeated measurements were taken one day after the originals when the temperature was approximately 5 degrees different. The sharp peaks and dips 'missed' by the low resolution measurement have moved very slightly in frequency with the change in temperature.

To check for time invariance, it is desirable to repeat one or more of the measurements over the time-span of the whole measurement and to compare the results of these with those of the original measurements. If significant time variance is observed, the cause should be pinpointed and steps should be taken to stabilise the system.

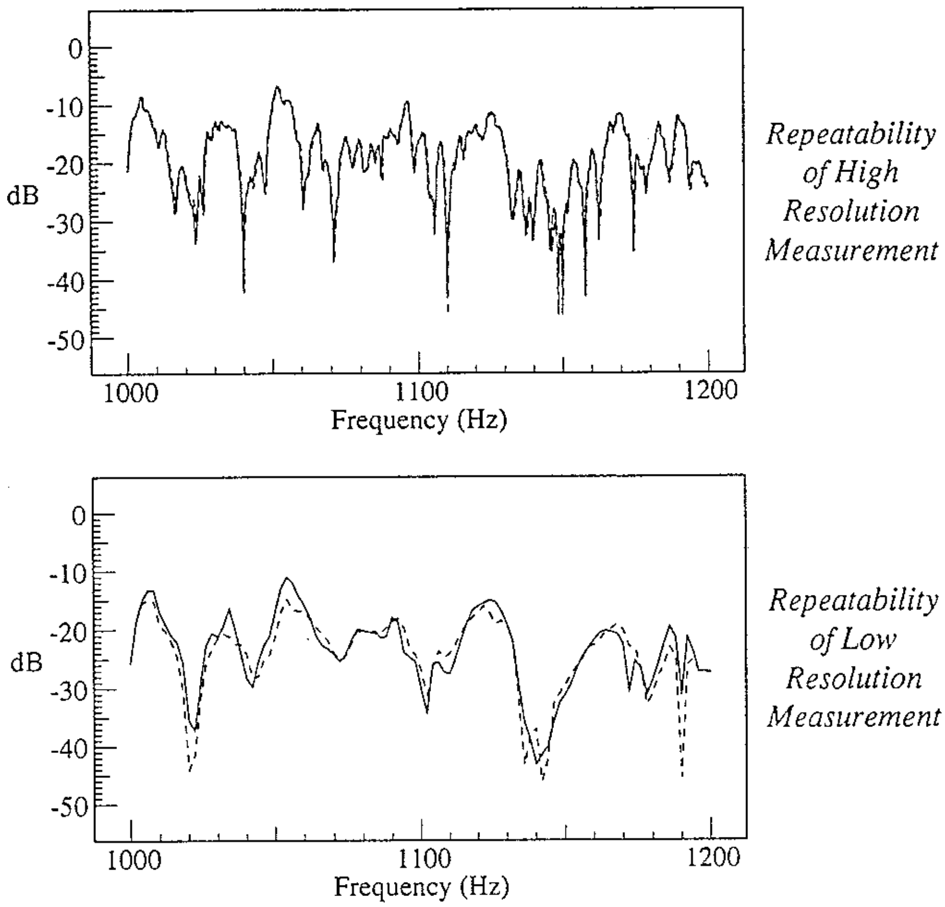

Figure 9 Influence of Frequency Resolution Bias Error on Repeatability

4.5 Measurement of Green Functions

The Green functions in equation (1) are defined as the pressure at the acoustic field point due to unit normal volume velocity of a surface element:

$$G(x \mid i, f) = \frac{p(x, f)}{S_i u_i(f)} , \qquad (16)$$

where $p(x, f)$ is the acoustic pressure at acoustic field point x and $S_i u_i(f)$ is the volume velocity of element i. By reciprocity, these Green functions are identical to the blocked pressures on the elements due to a unit volume velocity of a monopole source at position x. A source having unit volume velocity output is unlikely to be used in practice, so measurement of the Green functions at harmonic excitation frequency f involves dividing the blocked pressure measurements, $p(i \mid x, f)$ in equation (3), by a measure of the volume velocity of the source, to yield complex transfer functions $T_i(f)$:

$$T_i(f) = \frac{p(i \mid x, f)}{q(f)} . \qquad (17)$$

The volume velocity source is likely to be an electro-acoustic device (see section 6), which can be calibrated to give a known volume velocity output for a certain electrical input voltage or current, alternatively, some form of volume velocity monitor may be attached to the source.

The transfer functions required for excitation by force inputs may be measured similarly using the free velocity measurements $u(i \mid x, f)$ in equation (4).

4.6 Measurement of Velocities and Forces

In order to take phase interference effects into account when more than one input velocity (or force) is present, the measured input quantities need to be complex. A suitable phase reference needs to be chosen (or generated) to which all of the input signals can be compared; each input velocity (or force) measurement will then have a phase relative to the other measurements.

If the inputs are generated by an electro-mechanical device, such as a loudspeaker or shaker, the electrical drive signal of the device can be used as a phase reference. The velocities (or forces) can then be measured as complex transfer functions, $U_i(f)$, between the output signal of the velocity (or force) transducers, and the electrical drive signal, thus:

$$U_i(f) = \frac{u_i(f)}{e_r(f)} \ , \tag{18}$$

where e_r is the electrical drive signal, and u_i is the transducer output signal for input i.

If the inputs are generated by external sources, such as an engine, a suitable phase reference signal is not usually available. In this case, an accelerometer attached to the vibrating structure close to the measurement points will serve as a reference. The velocities (or forces) can then be measured as complex transfer functions between the output signal of the velocity (or force) transducers, and the output signal of the fixed reference accelerometer (e_r).

Each transfer function is then complex and can be used in the vector summations of equations (3) (or (5)) in place of the measured velocity (or force).

The effect of these phase references on the absolute value of the calculated sound pressures in equations (3) or (5) can be reversed by multiplication of the sound pressure by the amplitude of the reference signal after the summation has been carried out; equations (3) would then take the form:

$$p(x, f) = |e_r(f)| \sum_i S_i \, U_i(f) \, T_i(f) \ , \tag{19}$$

where $T_i(f)$ are the reciprocally measured Green functions defined in equation (17). Note that both $U_i(f)$ and $T_i(f)$ are complex in general. Similar transfer functions can be used to redefine equation (5) for force inputs.

5 APPLICATION EXAMPLES

Some typical examples of applications for vibroacoustic reciprocity are given below to illustrate how and where the various measurement techniques described in section 4 can be applied.

Example 1. Radiation of Sound by a Thick Plate

Noise covering a frequency range from 50Hz to 2kHz is being transmitted through the walls of a boiler. The walls are flat, and are made from carbon steel with a thickness of 8mm.
Vibroacoustic Reciprocity is to be used to determine the contribution of the vibration of one of the boiler walls to the overall sound level.

Using equation (14), the wavelength of bending waves in the wall are found to be about 200mm at 2kHz, compared to about 170mm for acoustic waves in air. The maximum 'safe' sample spacing for both the vibration and the acoustic fields is therefore determined by the acoustic Nyquist criterion (equation 11) to be 85mm. The wall of the boiler is then divided into a number of area elements having maximum dimensions of 85mm and representative measurement points are chosen and marked in each element. An additional point close to the centre of the plate is chosen for a phase-reference accelerometer. While the boiler is running, an accelerometer is moved to the first element and the transfer function between the output of the accelerometer and that of the reference accelerometer is measured using a dual channel FFT analyser and stored. The accelerometer is then moved to each element in turn and the process repeated (the reference is not moved). The autospectrum of the output from the reference accelerometer is also stored.

The boiler is then switched off and an electroacoustic monopole source is placed at the position at which the noise level is required. A microphone is then placed adjacent to the first element and, with the source operating, the transfer function between the output from the microphone and the electrical (voltage or current) drive signal to the source is measured using a dual channel FFT analyser and stored. The microphone is then moved to each element in turn and the process repeated. Note that it is assumed that the plate is rigid when the boiler is not operating; the microphone measurements then represent the blocked pressures on the surface.

Prior to taking any measurements, checks should be made to ensure that adequate signal-to-noise ratios exist for all measurements and the coherence should be monitored throughout the measurements. To check for time invariance, the first measurement should be repeated at the end of the measurements and compared to the original.

The contribution of the vibration of the wall to the overall noise level is then determined using equation (19); S_i is the area of element i, U_i is the transfer function between the accelerometer at element i and the reference accelerometer (equation (18)), T_i is the transfer function between the microphone at element i and the electrical drive to the monopole source (equation (17)) and e_r is the autospectrum of the reference accelerometer. The measured transfer functions have to be converted from voltage ratios to absolute units using previously determined calibrations for the microphone, accelerometer and monopole source (see section 6).

Example 2. Radiation of Sound by a Thin Plate

Aluminium plate of 0.5mm thickness is used to construct one wall of a sound proofing enclosure. The contribution of the vibration of this wall to the overall noise level at a point outside the enclosure is required when a loudspeaker is operated inside the enclosure. The highest frequency of interest is 2kHz.

Using equation (14), the wavelength of bending waves in the wall are found to be about 50mm at 2kHz, compared to about 170mm in air. Unlike in example 1, use of the acoustic Nyquist sampling criterion would result in spatial aliasing of the vibration field in this case and consequent over-determination of the radiated sound. The plate is therefore be divided into area elements having a maximum dimension of 25mm – half of a vibration wavelength at the highest frequency of interest and representative measurement points are chosen and marked in each element. Because access is available to the electrical drive signal to the loudspeaker, no phase reference accelerometer is required; the electrical drive signal to the loudspeaker provides a suitable reference. An accelerometer is moved to the first element and the transfer function between the output of the accelerometer and the electrical drive to the loudspeaker is measured using a dual channel FFT analyser. The level of the electrical drive to the loudspeaker is adjusted to avoid signal-to-noise problems (level too low) and nonlinearity (level too high) by maximising the coherence between the two signals. The accelerometer is then moved to each element in turn and the process repeated.

In order to measure the reciprocal Green functions, the blocked pressures on the surface of the plate are required. A thin aluminium plate cannot be assumed to be rigid, so a thick steel plate having the same dimensions is placed on top of the aluminium plate. An electroacoustic monopole source is then placed at the position at which the noise level is required and a microphone is placed adjacent to the first element on the steel plate. With the internal loudspeaker switched off and the monopole source operating, the transfer function between the output from the microphone and the electrical (voltage or current) drive signal to the source is measured using a dual channel FFT analyser and stored. The microphone is then moved to each element in turn and the process repeated.

Calculation of the contribution of the vibration of the plate to the overall noise level is calculated in the same way as for example 1 except for the autospectrum of a reference accelerometer; the resultant pressure level is then relative to unit electrical drive to the internal loudspeaker.

Note that the above procedure is valid regardless of the construction of the rest of the enclosure – indeed, the plate may even be a simple screen.

If an area-integrating velocity transducer is available and is used in place of the accelerometer, the restriction on the sample spacing can be relaxed to that of the acoustic Nyquist criterion (provided integration is carried out over the whole element) thereby reducing the number of vibration and pressure transfer function that are required.

6 VOLUME VELOCITY SOURCES

Many applications of vibroacoustic reciprocity require the use of a point monopole acoustic source. Ideally, such a source should be compact, it should radiate sound uniformly in all directions (omnidirectional) and should generate adequate acoustic power over the frequency range of interest. In practice, no single acoustic source can fulfil all of these requirements over a wide frequency range.

6.1 Design Considerations

In theory, a point monopole source should occupy no space. Such a source can be considered to be a pulsating sphere of zero radius. The sound power radiated by a sphere of radius a pulsating with radial velocity u is,

$$W_s = 2\rho c\, \pi a^2 \, |u|^2 \frac{(ka)^2}{1+(ka)^2} \text{ Watts} \; . \tag{20}$$

from which it is clear that, for a compact source ($ka \ll 1$), conservation of sound power requires that the surface velocity vary as the inverse of the square of the radius. In the limit of zero radius, infinite velocity is required to radiate any power; the point monopole is a useful theoretical tool, but it cannot be realised in practice.

The large velocities required of a compact source give rise to two forms of nonlinearity which ultimately limit either the available radiated sound power or the minimum practical source size. First, if the displacement of the wall of the sphere becomes significant compared with the radius of the sphere, the volume velocity is no longer linearly related to the surface velocity. Second, if the velocity becomes a significant fraction of the speed of sound, the propagation of sound away from the source will be nonlinear. The first source of nonlinearity can be corrected by ensuring that the ***volume velocity*** rather than the surface velocity is linearly related to the input signal (a compact piston instead of a pulsating sphere for example), but the second is common to all compact sound sources.

An approximation to a point monopole source can be realised by mounting a conventional moving-coil drive-unit in one wall of a sealed enclosure. At low frequencies, when the enclosure is small compared to a wavelength, the sound radiated from the drive-unit diffracts around the enclosure and the source can be considered to be omnidirectional. At higher frequencies the sound does not diffract around the enclosure and the source is directional. The upper frequency limit for such a source is limited by the size of the enclosure and is given by $kl \approx 1$, where l is the length of the longest side of the enclosure. The practical low-frequency limit to the power bandwidth of a conventional loudspeaker is governed by the fundamental resonance of the driver / enclosure system. The resonance frequency can be determined from,

$$f_o = \frac{1}{2\pi\sqrt{m(c_d + c_e)}} \ , \tag{21}$$

where m is the mass of the moving parts of the drive-unit, c_d is the compliance of the driver suspension system,

$$c_e = \frac{V}{\rho c^2 S_d^2} \ , \tag{22}$$

is the compliance of the air in the enclosure, S_d is the area of the drive-unit diaphragm and V is the volume of the enclosure. A low resonance frequency can therefore only be achieved with a large mass, a small diaphragm or a large enclosure volume. For a given electrical power input, the power output of a loudspeaker at frequencies above resonance is determined principally by the mass of the moving parts and the area of the diaphragm; increasing the mass or reducing the diaphragm area reduces the electroacoustic efficiency of the loudspeaker, and increasing the electrical power input to compensate leads to heat dissipation problems. A large enclosure is therefore required for the efficient radiation of low frequencies from a conventional loudspeaker, giving a correspondingly lower limit in useful high frequency performance.

Another important aspect of the performance of a source is the sensitivity of the volume velocity output to the acoustic environment within which it is operated. A large, lightweight diaphragm – desirable for the efficient radiation of low frequencies – has a low mechanical impedance and hence will be particularly sensitive to radiation conditions, whereas a small, heavy diaphragm will be less sensitive.

The exterior field generated by a pulsating sphere of finite radius is the same as that generated by a point monopole source located at its centre; the equivalent point source volume velocity is then related to the volume velocity of the pulsating sphere by,

$$\frac{q_{monopole}}{q_{sphere}} = \frac{e^{jka}}{1+jka} , \tag{23}$$

where a is the radius of the sphere. A source of this type can be realised approximately by mounting a number of moving-coil loudspeaker drive-units around a circular enclosure as shown in figure 10.

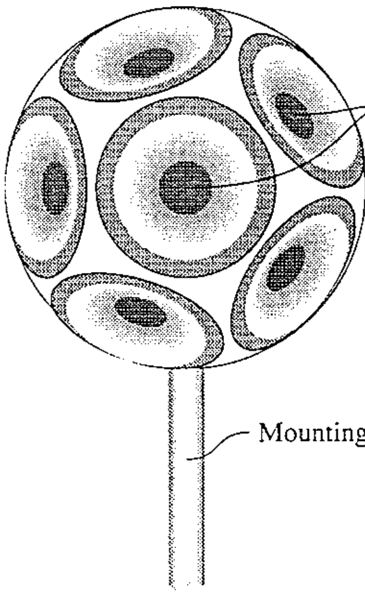

Figure 10 An Approximate Pulsating Sphere Monopole Source

At low- and mid-frequencies, the field radiated by the circular array of loudspeakers will closely approximate that radiated by a point monopole mounted at its centre. At high frequencies, when $ka_d > 1$, where a_d is the radius of a single drive-unit, the sound tends to beam from each drive-unit and the radiated field is directional. The upper frequency limit for such a source is therefore when $ka_d \approx 1$. A source comprising a large number of small drive-units will therefore operate to a higher frequency than one with a small number of large drive-units. As with a single drive-unit, the low frequency bandwidth is limited by the resonance frequency of the drive-units which is determined by the volume of the sphere, the drive-unit diaphragm area and the number of drive-units. Assuming that the surface of the sphere is entirely covered by drive-units, a large sphere will result in a lower resonance frequency than a small sphere. A wide-bandwidth approximation to a point monopole source can therefore be realised by populating the surface of a large spherical enclosure with a large number of small drive-units.

Use of a multi-drive-unit spherical source is only practical in situations where space is not limited. For other applications, where a source is to be operated inside a vehicle for example, a more compact alternative is required.

As stated above, for a single drive-unit source, a small drive-unit and enclosure is required for omnidirectional radiation of high frequencies and a large drive-unit and enclosure is required for efficient radiation of low frequencies. A reasonable compromise between these requirements can be achieved by attaching a tube to the front of the drive-unit, as shown in figure 11. The sound is radiated from the end of the tube which can be small enough to be omnidirectional at high frequencies and long enough to ensure that a reasonable sized enclosure does not effect the sound radiation – the tube may even be flexible for positioning in otherwise inaccessible places. There are two major problems with this approach. First, in order to radiate sufficient sound power from the relatively small tube-end, the particle velocity and pressure within the tube has to be high, leading to nonlinear sound propagation along the tube. Second, a standing-wave field is set up within the tube which gives rise to resonances at selected frequencies and consequently an uneven frequency response characteristic.

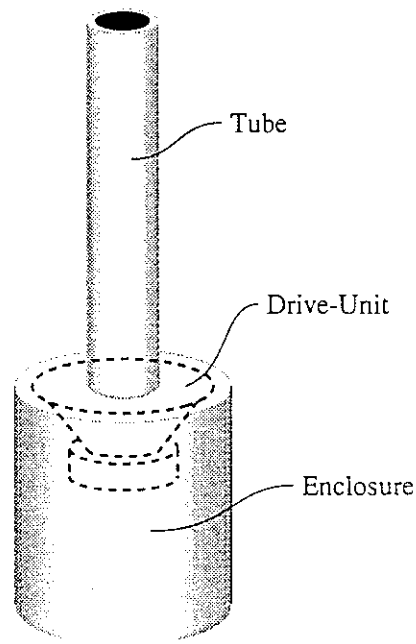

Figure 11 Omnidirectional Source using a Conventional Drive-Unit and a Tube

An alternative approach is to attach an inverse horn to the front of the drive-unit as shown in figure 12. The radiating area at the end of the horn is small enough to ensure that the radiation is near omnidirectional at high frequencies, save a narrow 'shadow' region behind the source due to the presence of the enclosure, and the nonlinearity and standing wave problems associated with the tube are reduced. The overall dimensions of the source are also reduced, and are probably the minimum practical dimensions for a source having an acceptable useful bandwidth. For example, a prototype source constructed using this technique with overall dimensions of 280mm × 150mm × 150mm, maintains acceptable omnidirectional power output over a frequency range of 60Hz to 1kHz.

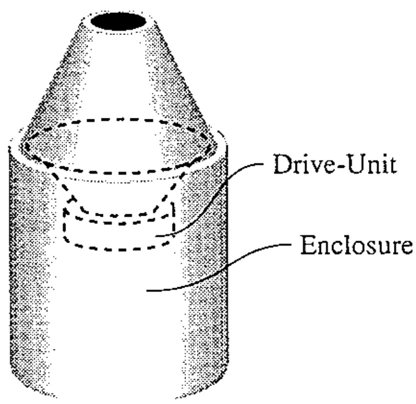

Figure 12 Omnidirectional Source using a Conventional Drive-Unit and an Inverse Horn

If the restricted low frequency power output from a compact source is unacceptable, the enclosure may, in some circumstances, be incorporated into the system under test. For example, for vehicle interior noise measurements, the enclosure could be shaped as a torso, with sound radiated from an orifice in the head; the radiation of sound need only be as omnidirectional as the human ear. The torso is then 'sat' in the passenger positions of the vehicle, and the conditions under which the measurements were taken would be those for when a passenger was present.

6.2 Source Calibration

The vibroacoustic reciprocity technique described above requires the use of a calibrated point volume velocity source. There are a number of ways in which a source can be calibrated, depending on the type of source and the available resources. One essential aspect of the calibration is to determine over what frequency range the source is omnidirectional. If the source is to be used for external excitation of a structure under free-field conditions, it need not be totally omnidirectional; it is sufficient for the sound radiation from the source to 'cover' the structure (including floor reflection if present) and the calibration method can be tailored to suit. For internal excitation or under conditions other than free-field, the source must be omnidirectional over the frequency range of interest.

6.2.1 Free-Field Calibration

If a full-space anechoic chamber is available, accurate calibration of a source can be achieved by placing a microphone at a known distance from the source. The transfer function between the calibrated output of the microphone and the electrical drive voltage (or current) to the source is then measured over the frequency range of interest. The equivalent monopole volume velocity of the source, normalised to the electrical input, is then

$$q = \frac{4\pi r}{\rho c k} T \ , \qquad (24)$$

where r is the distance from the source to the microphone, k is the acoustic wavenumber and

$$T = \frac{p}{e_s} \ ,$$

is the measured transfer function. For non-omnidirectional sources, care must be taken to 'point' the source at the microphone.

6.2.2 Half-Space Calibration

An accurate source calibration may be carried out under half-space conditions – when a rigid floor is present for example. Under these conditions, the source is mounted a known distance above the floor and the microphone is placed on the floor beneath the source. The equivalent monopole volume velocity is then half of that calculated in equation (24).

6.2.3 Near-Field Calibration

If free-field conditions are not available, some sources – particularly ones having a single output orifice such as those in figures 11 and 12 – can be calibrated using a near-field method. A miniature microphone is mounted as close as possible to the orifice and the transfer function between the calibrated microphone output and the electrical drive to the source is measured. The volume velocity, normalised to the electrical input, is then the division of the transfer function by the radiation impedance of the orifice multiplied by the area of the orifice. For small orifices at the end of a tube or at least positioned away from the body of the source the radiation impedance can be approximated by that of a piston in the end of a tube and the calibration is then

$$q = \frac{T(\pi a^2)}{Z_r} \ , \ \text{where } Z_r \approx \left(\frac{(ka)^2}{4} + j0.6ka\right) , \qquad (25)$$

and a is the radius of the orifice .

Alternatively, a microphone placed a small distance away from the source orifice can be thought of as being on a sphere having the source orifice as its centre. The radiation impedance in equation (25) is then replaced by that of a sphere,

$$Z_r = \frac{(kr)^2 + jkr}{1 + (kr)^2} ,$$

where r is the distance from the source to the microphone.

Neither of these calibration methods will yield as accurate a result as the free-field calibration methods above and they do not strictly yield the equivalent monopole volume velocity as required for vibroacoustic reciprocity.

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