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Theory for acoustic propagation in marine sediment containing gas bubbles which may pulsate in a non-stationary nonlinear manner

T. G. Leighton[1]

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[1] Whilst there is a considerable body of work in the literature on the theory of acoustic propagation in marine sediments, the incorporation of gas bubbles into such theories is done with the inclusion of assumptions which severely limit the applicability of those models to practical gas-laden marine sediments. This paper provides a theory which does not assume that the bubble dynamics are linear, steady-state and monochromatic. Such assumptions would be incompatible with many of the practical acoustic fields with which sediments are insonified today. These fields are necessarily high amplitude to provide adequate signal-to-noise ratios, given the high attenuation in gassy marine sediments; and often they utilise short pulses to obtain range resolution. This paper provides a theory appropriate for predicting the acoustically-driven non-stationary nonlinear dynamics of spherical gas bubbles embedded in a lossy elastic solid, and discusses how this could be incorporated into a nonlinear, time-dependent propagation model. **Citation:** Leighton, T. G. (2007), Theory for acoustic propagation in marine sediment containing gas bubbles which may pulsate in a non-stationary nonlinear manner, *Geophys. Res. Lett.*, *34*, L17607, doi:10.1029/2007GL030803.

1. Introduction

[2] Marine sediments containing gas bubbles occur at many locations [*Judd and Hovland*, 1992; *Fleischer et al.*, 2001]. They are important, first, because of the impact those bubbles have on the structural integrity and load-bearing capabilities of the sediment [*Wheeler and Gardiner*, 1989; *Sills et al.*, 1991]; second, because the presence of bubbles can be indicative of a range of biological, chemical or geophysical processes (such as the climatologically-important flux of methane from the seabed to the atmosphere [*Judd*, 2003]); and third, because of the impact which the bubbles have on any acoustic systems used to characterise the sediment [*Robb et al.*, 2006].

[3] When driven by an acoustic field, a gas bubble surrounded by a suitable host material acts as a nonlinear oscillator (which tends to linear dynamics at low pulsation amplitudes). It exhibits a pronounced breathing-mode resonance such that, when driven at frequencies much less than this resonance, its response is stiffness-controlled, and the presence of bubble reduces the sound speed (tending to quasi-static conditions at very low driving frequencies). When driven at frequencies much greater than resonance, the bubble's response is inertia-controlled, and the presence of bubbles tends to increase the sound speed, the effect decreasing with increasing frequency [*Leighton*, 1994].

[4] Whilst there is a considerable body of work in the literature on the theory of acoustic propagation in marine sediments, the incorporation of gas bubbles into such theories is done with the inclusion of assumptions which severely limit the applicability of those models to practical gas-laden marine sediments. As a result, such theories are limited in terms of which components of the above behaviour they can describe [*Leighton et al.*, 2004]. The theories most frequently used include modified versions of the Biot-Stoll Theory [*Biot*, 1956a, 1956b; *Stoll*, 1974] and an approach developed by *Anderson and Hampton* [1980a, 1980b]. The Biot model assumes that the bubble does not affect the sediment structure (i.e. it only affects the pore fluid properties). Most manifestations of the Biot model assume quasi-static bubble responses [*Domenico*, 1976, 1977; *Andreassen et al.*, 1997; *Hawkins and Bedford*, 1992; *Gregory*, 1976; *Herskowitz et al.*, 2000; *Minshull et al.*, 1994; *Smeulders and Van Dongen*, 1997]. The assumption of quasi-static gas dynamics limits the applicability of the resulting theory to cases where the frequency of insonification is very much less than the resonances of any bubbles present. It also eliminates from the model all bubble resonance effects, which often of are overwhelming practical importance when marine bubble populations are insonified. This limitation becomes more severe as gas-laden marine sediments are probed with ever-increasing frequencies.

[5] Some versions of the Biot model include a simple harmonic oscillator term for the compressibility of the fluid, which incorporates the inertia, stiffness and damping terms relevant to the bubbles [*Biot*, 1962; *Stoll and Bautista*, 1998]. The acoustic theory of *Anderson and Hampton* [1980a, 1980b] similarly assumes that only linear, steady state bubble pulsations occur. As a result, neither class of theory is applicable to the propagation of fields which are sufficiently high amplitude: the ubiquitous assumption of linear bubble pulsations becomes increasingly questionable as acoustic fields of greater amplitudes are used to overcome the high attenuations, and the resulting poor-signal-to-noise ratios, that are often encountered in marine sediments. Furthermore the assumption of monochromatic steady-state bubble dynamics is inconsistent with the use of short acoustic pulses to obtain range resolution.

[6] This report outlines a theory which does not require the above assumptions. Some assumptions are still maintained, notably that the bubbles in question interact with the sound field through volumetric pulsation. Whilst this does not necessarily mean that the bubbles should be spherical at all times, it is through this assumption that the theory encompasses the volumetric pulsations. It is well-known

[1]Institute of Sound and Vibration Research, University of Southampton, Southampton, UK.

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that there are classes of bubbles in sediment which do not behave in this way (e.g. those which bear a closer resemblance of 'slabs of gas' and 'gas-filled cracks', than they do to gas-filled spheres [*Hill et al.*, 1992; *Anderson et al.*, 1998; *Reed et al.*, 2005]). The assumption is also maintained that the sediment outside of each individual bubble may be treated as incompressible. Whilst this greatly eases the analysis, the extent to which it is correct will depend on the characteristics of the sediment. The result of this assumption is that acoustic radiation damping is neglected. Furthermore the sediment outside of the bubble is assumed to be a lossy elastic solid, and no bubble-bubble interactions are assumed to occur. It should be noted that this analysis is also relevant to acoustic propagation through tissue, provided that the latter can be treated as an incompressible lossy elastic solid. *Church* [1995] pioneered the development of models of the dynamics of gas-filled biomedical contrast agent bubbles surrounded by a shell which can support stress [*Alekseev and Rybak*, 1999; *Allen and Roy*, 2000a, 2000b; *Morgan et al.*, 2000; *Zabolotskaya et al.*, 2005; *Qin et al.*, 2006]. The current paper adapts the approach of *Church* [1995] to the problem of acoustic propagation in marine sediments.

2. Dynamics of Gas Bubbles in an Incompressible Lossy Elastic Solid

[7] Consider a bubble of radius $R(t)$ which oscillates about some equilibrium radius R_0 with bubble wall velocity $\dot{R}(t)$ and radial wall displacement R_ε. In this preliminary analysis, it is assumed that the sediment outside the bubble (which will probably contain solid, liquid and gaseous phases) can be treated as an incompressible elastic solid (refinements to this assumption are discussed later). In the following derivation, the use of the dot notation in this, and the subsequent equations of motion, indicates the use of the material derivative, i.e.:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\vec{u} \cdot \vec{\nabla} \right) \tag{1}$$

where $\vec{u}$ is the particle velocity in the host medium (the material outside of the bubble wall). If the bubbles are assumed to be spherical at all times with their centres fixed in space, and the bubble population density is not too great, then the convective term can be neglected [*Foldy*, 1945]. Following *Church* [1995], if the flow is irrotational, the viscous effects appear only through the boundary conditions, all body forces are negligible, and the bubble remains spherical at all times, then the equation for the conservation of momentum for the pulsation of a single gas bubble in an incompressible lossy elastic solid reduces to:

$$\rho_s\left(\frac{\partial u_s}{\partial t} + u_s\frac{\partial u_s}{\partial r}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 T_{rr}\right) - \frac{T_{\theta\theta} + T_{\phi\phi}}{r} \tag{2}$$

where ρ_s and u_s are, respectively, the bulk density and the particle velocity in the marine sediment outside of the bubble wall (which is modelled as an elastic solid with equivalent bulk properties). Because the trace of the stress tensor is zero in elastic solids (as it also is in Newtonian liquids), its respective components (T_{rr}, $T_{\theta\theta}$ and $T_{\phi\phi}$) are related through:

$$T_{rr} = -\left(T_{\theta\theta} + T_{\phi\phi}\right). \tag{3}$$

[8] In an adaptation of the approach employed for biomedical contrast agents [*Church*, 1995], equation (2) will now be integrated through the solid (from R to $r = \infty$), using the assumption of liquid incompressibility [*Leighton*, 1994], which implies that:

$$u_s(r,t) = \frac{R^2(t)}{r^2}\dot{R}(t). \tag{4}$$

[9] Integration of (2) from R to $r = \infty$ gives:

$$\begin{aligned}
&\int_R^\infty \rho_s\frac{\partial u_s}{\partial t}\mathrm{d}r + \int_R^\infty \rho_s u_s\frac{\partial u_s}{\partial r}\mathrm{d}r = -\int_R^\infty \frac{\partial p}{\partial r}\mathrm{d}r + \int_R^\infty \frac{1}{r^2}\frac{\partial(r^2T_{rr})}{\partial r}\mathrm{d}r \\
&\quad - \int_R^\infty \frac{(T_{\theta\theta}+T_{\phi\phi})}{r}\mathrm{d}r \Rightarrow \rho_s\left(R\ddot{R} + 2\dot{R}^2\right) \\
&\quad - \frac{\rho_s\dot{R}^2}{2} = p_s(R,t) - p_\infty(t) + \Bigg(T_{rr}(r=\infty,t) \\
&\quad - T_{rr}(R,t) + \int_R^\infty \frac{2T_{rr}}{r}\mathrm{d}r\Bigg) + \int_R^\infty \frac{T_{rr}}{r}\mathrm{d}r \Rightarrow \rho_s R\ddot{R} \\
&\quad + \frac{3}{2}\rho_s\dot{R}^2 = p_s(R,t) - p_\infty(t) + T_{rr}(r=\infty,t) \\
&\quad - T_{rr}(R,t) + \int_R^\infty 3\frac{T_{rr}}{r}\mathrm{d}r
\end{aligned} \tag{5}$$

noting that for this case, T_{rr} ($r = \infty$, t) can be taken to equal zero, giving

$$\rho_s R\ddot{R} + \frac{3}{2}\rho_s\dot{R}^2 = \left(p_s(R,t) - T_{rr}(R,t)\right) - p_\infty(t) + \int_R^\infty 3\frac{T_{rr}}{r}\mathrm{d}r \tag{6}$$

[10] The bracketed term on the right of equation (6) can readily be found using the boundary condition at the bubble wall ($r = R$):

$$p_s(R,t) - T_{rr}(R,t) = p_g - \frac{2\sigma}{R} - \frac{\partial\sigma}{\partial R} \tag{7}$$

where σ is the surface tension, and $\partial\sigma/\partial R$ represents a radial force which results from the variation in the concentration of surface active molecules on the bubble wall as the bubble pulsates, although this is normally assumed to be zero [*Leighton*, 1994; *Church*, 1995]. Substitution of (7) into (5) gives:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_s}\left(p_g - \frac{2\sigma}{R} - \frac{\partial\sigma}{\partial R} - p_\infty(t) + \int_R^\infty 3\frac{T_{rr}}{r}\mathrm{d}r\right) \tag{8}$$

which can be readily evaluated to form time histories of the bubble response using the techniques familiar for gas bubbles in liquids, provided that it is possible to determine T_{rr}, the radial component of the stress tensor in the sediment.

[11] Taking both the elastic and lossy characteristics of the solid together [*Church*, 1995], the radial component of the stress tensor is:

$$T_{rr} = -\frac{4R^2}{r^3}\left(G_s R_\varepsilon + \eta_s \dot{R}\right) \tag{9}$$

where G_s is the modulus of rigidity (the shear modulus, or second Lamé constant) and η_s is the shear viscosity of the material outside of the bubble wall. Given the complexity of the problem, it may be necessary to progress towards the correct values for these for a nonlinear non-stationary gassy marine sediment using an iterative scheme. The integral for the solid in equation (8) can be evaluated:

$$\int_R^\infty \frac{3T_{rr}}{r}\mathrm{d}r = -\frac{4}{R}\left(G_s R_\varepsilon + \eta_s \dot{R}\right) \tag{10}$$

[12] Equation (8) can now be expressed with the integrals evaluated using (10):

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_s}\left(p_g - \frac{2\sigma}{R} - \frac{\partial\sigma}{\partial R} - p_\infty(t) - \frac{4}{R}\left(G_s R_\varepsilon + \eta_s \dot{R}\right)\right). \tag{11}$$

3. Discussion and Conclusions

[13] Equation (11) forms the basis of predicting the dynamics of a single bubble in a lossy elastic solid. The assumption that the host medium is incompressible means that bubble-mediated acoustic radiation losses are neglected. Incorporation of these would require the introduction of appropriate terms [*Vorkurka*, 1986; *Morgan et al.*, 2000; *Zabolotskaya et al.*, 2005]. If the gas pressure p_g is calculated using a polytropic index κ which is constant throughout the oscillatory cycle (i.e. assuming $p_g\,(4\pi R^3/3)^\kappa =$ constant), then there will be no bubble-mediated thermal dissipation, since such a technique only adjusts the stiffness of the bubble gas for reversible heat flux across the bubble wall [*Leighton et al.*, 2004]. Rather than including thermal dissipation by artificially increasing the viscosity [*Chapman and Plesset*, 1971] (which at best assumes linear conditions), thermal dissipation would more properly be included by combining the continuity and energy relations for a perfect gas with spatially uniform pressure to provide an exact expression for the velocity field in terms of the temperature gradient [*Prosperetti and Hao*, 1999]. This would preserve the nonlinear character of the bubble dynamics so that it could be incorporated into a nonlinear acoustic propagation scheme. In such a scheme, if the bubble dynamics are considered in the space plane formed by plotting the driving pressure against the bubble volume response, the attenuation is given by the area mapped out, and the sound speed by the gradient of the spine of the locus of points mapped out by the bubble oscillation [*Leighton et al.*, 2004; *Leighton*, 2004, 2007].

[14] The next stages of this work will be to use equation (11) to predict the attenuation and sound speed in gassy marine sediments, using the route outlined in the above paragraph to incorporate thermal and radiation damping. This will require values for G_s and η_s to use in equation (11) and, as outlined after (9), these may not be simple to come by. Given that the propagation method considers, through equation (11), the nonlinear dynamics of each bubble in turn [*Leighton et al.*, 2004], the issue is to what extent the effect of the 'neighbouring bubbles' (i.e. those in the sediment beyond the wall of the bubble in question) contribute to the values of G_s and η_s. The zeroth order solution would be to set G_s equal to zero, and set η_s equal to the value for bubble-free water. In such a case, the equations would reduce to modelling nonlinear bubble pulsations in water, and the method for predicting the attenuation and sound speed would be no different to that already accomplished by *Leighton et al.* [2004]. It would of course be an improvement to use G_s and η_s values that pertain to a bubble-free saturated sediment. However the true first-order solution would be to assign to G_s and η_s values which include the effect of the neighbouring bubbles derived from the dynamics of bubble pulsating in the quasi-static fashion (i.e. driven at frequencies much less than resonance), as described in the Introduction. A more sophisticated approach would be to include the resonator properties of these neighbouring bubbles to derive frequency-dependent values for G_s and η_s (either from the Biot or Anderson-Hampton approach, again as described in the Introduction). In such a circumstances, whilst the propagation would be based on a consideration of the nonlinearly dynamics of each bubble in turn [*Leighton et al.*, 2004], the material outside of the bubble wall would be characterized by values of G_s and η_s that are derived from linear bubble dynamics. This is the reason why the iterative approach discussed after equation (9) might be necessary to ensure that the nonlinear pulsations of the bubbles involved in the calculation of sound speed and attenuation, are themselves modified by a gassy sediment in which the bubbles can pulsate nonlinearly.

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References

Alekseev, V. N., and S. A. Rybak (1999), Gas bubble oscillations in elastic media, *Acoust. Phys.*, *45*(5), 535–540.

Allen, J. S., and R. A. Roy (2000a), Dynamics of gas bubbles in viscoelastic fluids. II: Linear viscoelasticity, *J. Acoust. Soc. Am.*, *107*(6), 3167–3178.

Allen, J. S., and R. A. Roy (2000b), Dynamics of gas bubbles in viscoelastic fluids. II: Nonlinear viscoelasticity, *J. Acoust. Soc. Am.*, *108*(4), 1640–1650.

Anderson, A. L., and L. D. Hampton (1980a), Acoustics of gas-bearing sediment. I: Background, *J. Acoust. Soc. Am.*, *67*(6), 1865–1889.

Anderson, A. L., and L. D. Hampton (1980b), Acoustics of gas-bearing sediment. II: Measurements and models, *J. Acoust. Soc. Am.*, *67*(6), 1890–1905.

Anderson, A. L., F. Abegg, J. A. Hawkins, M. E. Duncan, and A. P. Lyons (1998), Bubble populations and acoustic interaction with the gassy floor of Eckernforde Bay, *Cont. Shelf Res.*, *18*(14–15), 1807–1838.

Andreassen, K., P. E. Hart, and M. MacKay (1997), Amplitude versus offset modelling of the bottom simulating reflection associated with submarine gas hydrates, *Mar. Geol.*, *137*(1–2), 25–40.

Biot, M. A. (1956a), The theory of propagation of elastic waves in fluid-saturated porous solids: 1. Low-frequency range, *J. Acoust. Soc. Am.*, *28*(2), 168–178.
Biot, M. A. (1956b), The theory of propagation of elastic waves in a fluid-saturated porous solid: 2. Higher frequency range, *J. Acoust. Soc. Am.*, *28*(2), 179–191.
Biot, M. A. (1962), Generalized theory of acoustic propagation in porous dissipative media, *J. Acoust. Soc. Am.*, *34*(9), 1254–1264.
Chapman, R. B., and M. S. Plesset (1971), Thermal effects in the free oscillation of gas bubbles, *Trans. ASME J. Basic Eng.*, *93*, 373–376.
Church, C. C. (1995), The effects of an elastic solid surface layer on the radial pulsations of gas bubbles, *J. Acoust. Soc. Am.*, *97*(3), 1510–1521.
Domenico, S. N. (1976), Effect of brine-gas mixture on velocity in an unconsolidated sand reservoir, *Geophysics*, *41*(5), 882–894.
Domenico, S. N. (1977), Elastic properties of unconsolidated porous sand reservoirs, *Geophysics*, *42*(7), 1339–1368.
Fleischer, P., T. H. Orsi, M. D. Richardson, and A. L. Anderson (2001), Distribution of free gas in marine sediments: A global overview, *Geo Mar. Lett.*, *21*, 103–122.
Foldy, L. L. (1945), The multiple scattering of waves, *Phys. Rev.*, *67*, 107–119.
Gregory, A. R. (1976), Fluid saturation effects on dynamic elastic properties of sedimentary rocks, *Geophysics*, *41*(5), 895–921.
Hawkins, J. A., and A. Bedford (1992), Variational theory of bubbly media with a distribution of bubble sizes. 2: Porous solids, *Int. J. Eng. Sci.*, *30*(9), 1177–1186.
Herskowitz, M., S. Levisky, and I. Shreiber (2000), Attenuation of ultrasound in porous media with dispersed microbubbles, *Ultrasonics*, *38*, 767–769.
Hill, J. M., J. P. Halka, R. Conkwright, K. Koczot, and S. Coleman (1992), Distribution and effects of shallow gas on bulk estuarine sediment properties, *Cont. Shelf Res.*, *12*(10), 1219–1229.
Judd, A. G. (2003), The global importance and context of methane escape from the seabed, *Geo Mar. Lett.*, *23*, 147–154.
Judd, A. G., and M. Hovland (1992), The evidence of shallow gas in marine sediments, *Cont. Shelf Res.*, *12*(10), 1081–1095.
Leighton, T. G. (1994), *The Acoustic Bubble*, Academic, London.
Leighton, T. G. (2004), From seas to surgeries, from babbling brooks to baby scans: The acoustics of gas bubbles in liquids, *Int. J. Mod. Phys. B*, *18*(25), 3267–3314.
Leighton, T. G. (2007), What is ultrasound?, *Prog. Biophys. Mol. Biol.*, *93*(1–3), 3–83.
Leighton, T. G., S. D. Meers, and P. R. White (2004), Propagation through nonlinear time-dependent bubble clouds, and the estimation of bubble populations from measured acoustic characteristics, *Proc. R. Soc. London, Ser. A*, *460*(2049), 2521–2550.
Minshull, T. A., S. C. Singh, and G. K. Westbrook (1994), Seismic velocity structure at a gas hydrate reflector, offshore western Colombia, from full waveform inversion, *J. Geophys. Res.*, *99*(B3), 4715–4734.
Morgan, K. E., J. S. Allen, P. A. Dayton, J. E. Chomas, A. L. Klibanov, and K. W. Ferrara (2000), Experimental and theoretical evaluation of microbubble behavior: Effect of transmitted phase and bubble size, *IEEE Trans. Ultrason. Ferroelectr. Frequation Control*, *47*, 1494–1509.
Prosperetti, A., and Y. Hao (1999), Modelling of spherical gas bubble oscillations and sonoluminescence, *Philos. Trans. R. Soc. London, Ser. A*, *357*, 203–223.
Qin, B., J. Chen, and J. Cheng (2006), Influence of the surrounding pressure on acoustic properties of slightly compressible porous media permeated with air-filled bubbles, *Acoust. Phys.*, *52*(4), 418–424.
Reed, A. H., B. P. Boudreau, C. Algar, and Y. Furukawa (2005), Morphology of gas bubbles in mud: A microcomputed tomographic evaluation, paper presented at the 1st International Conference of Underwater Acoustic Measurements: Technologies and Results, Found. for Res. and Technol. HELLAS, Heraklion, Greece, 28 June–1 July.
Robb, G. B. N., T. G. Leighton, J. K. Dix, A. I. Best, V. F. Humphrey, and P. R. White (2006), Measuring bubble populations in gassy marine sediments: A review, *Proc. Inst. Acoust.*, *28*(1), 60–68.
Sills, G. C., S. J. Wheeler, S. D. Thomas, and T. N. Gardiner (1991), Behaviour of offshore soils containing gas bubbles, *Geotechnique*, *41*(2), 227–241.
Smeulders, D. M. J., and M. E. H. Van Dongen (1997), Wave propagation in porous media containing a dilute gas-liquid mixture: Theory and experiments, *J. Fluid Mech.*, *343*, 351–373.
Stoll, R. D. (1974), *Sediment Acoustics*, *Lecture Notes Earth Sci.*, vol. 26, Springer, New York.
Stoll, R. D., and E. O. Bautista (1998), Using the Biot theory to establish a baseline geoacoustic model for seafloor sediments, *Cont. Shelf Res.*, *18*(14–15), 1839–1857.
Vorkurka, K. (1986), Comparison of Rayleigh's, Herring's, and Gilmore's models of gas bubbles, *Acustica*, *59*, 214–219.
Wheeler, S. J., and T. N. Gardiner (1989), Elastic moduli of soils containing large gas bubbles, *Geotechnique*, *39*(2), 333–342.
Zabolotskaya, E. A., Y. A. Ilinskii, G. D. Meegan, and M. F. Hamilton (2005), Modifications of the equation for gas bubble dynamics in a soft elastic medium, *J. Acoust. Soc. Am.*, *118*(4), 2173–2181.

T. G. Leighton, Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton SO17 1BJ, UK. (tgl@soton.ac.uk)