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Predictions of the modified Biot–Attenborough model for the dependence of phase velocity on porosity in cancellous bone

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Abstract

The modified Biot–Attenborough (MBA) model for acoustic wave propagation in porous media has been found useful to predict wave properties in cancellous bone. The present study is aimed at applying the MBA model to predict the dependence of phase velocity on porosity in cancellous bone. The MBA model predicts a phase velocity that decreases nonlinearly with porosity. The optimum values for input parameters of the MBA model, such as compressional speed c_m of solid bone and phase velocity parameter s_2, were determined by comparing the predictions with previously published measurements in human calcaneus and bovine cancellous bone. The value of the phase velocity parameter $s_2 = 1.23$ was obtained by curve fitting to the experimental data for 53 human calcaneus samples only, assuming a compressional speed $c_m = 2500$ m/s of solid bone. The root-mean-square error (RMSE) of the curve fit was 15.3 m/s. The optimized value of s_2 for all 75 cancellous bone samples including 22 bovine samples was 1.42 with a value of 55 m/s for the RMSE of the curve fit. The latter fit was obtained by using of a value of $c_m = 3200$ m/s. Although the MBA model relies on the empirical parameters determined from experimental data, it is expected that the model can be usefully employed as a practical tool in the field of clinical ultrasonic bone assessment.

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Keywords: Osteoporosis; Cancellous bone; Phase velocity; Porosity; Biot's theory; Modified Biot–Attenborough model

1. Introduction

Quantitative ultrasound (QUS) technologies for the diagnosis of osteoporosis have been developed rapidly for the last two decades [24,25,35]. This development is attributable to the wide availability of ultrasonic systems that provide a fracture risk assessment as an alternative to that provided by X-ray absorptiometry techniques. Most of the current clinical QUS devices measure speed of sound (SOS) and broadband ultrasonic attenuation (BUA) at peripheral skeletal sites that contain cancellous bone, such as the calcaneus and finger phalanges [35]. SOS and BUA can be combined linearly into a single index such as stiffness, which compensates for the temperature variation and offers better stability than either parameter taken alone [13,47]. However, the ultrasonic parameters are purely empirical measures and the underlying physics for their variations in cancellous bone is currently not well understood.

A range of models for wave propagation in cancellous bone have been proposed. Biot's theory [5–7] for elastic wave propagation in porous media has attracted the most attention with regard to modeling wave propagation in cancellous bone [9,12,17–21,26,27,30,31,48,49]. This application of Biot's theory has been reviewed by Haire and

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Langton [12]. Recently, Wear et al. [48] have successfully applied Biot's theory to predict the dependence of phase velocity on porosity in human calcaneus samples. One limitation of Biot's theory is that it depends on a large number of input parameters that are not necessarily easily measured. Another limitation is that Biot's theory predicts absorption due to the viscous losses at internal interfaces only. As a result, it consistently underestimates the experimental measurements of attenuation by several orders of magnitude.

As an alternative propagation model in cancellous bone, Strelitzki et al. [41] have proposed a scattering model based on velocity fluctuations in a binary mixture (marrow fat and cortical matrix) to estimate the ultrasonic attenuation in cancellous bone. Nicholson et al. [34] have also used this scattering model in cancellous bone to predict the relationship between BUA and porosity; the model has successfully predicted similar nonlinear trends to those previously observed experimentally. They have also demonstrated that the attenuation depends on scatterer size in addition to porosity. One of the potential limitations in this approach is that absorption is not included in the model.

Hughes et al. [20] first adopted the stratified model, based on a work by Schoenberg [37], to predict the angular dependence of phase velocities for the fast and the slow waves in bovine cancellous bone, modeling it as a simple layered structure of alternating parallel bone-marrow plates. Wear [44] has successfully applied the stratified model to predict negative dispersion of phase velocity in human cancellous bone. Lin et al. [28] have also used the stratified model to predict measurements of velocity and attenuation in sheep trabecular bone. An interesting feature of the stratified model is that it is essentially an anisotropic model. However, it takes no account of the effect of viscous absorption of the interstitial fluid on wave propagation.

Roh and Yoon [36] have proposed a modified Biot–Attenborough (MBA) model for acoustic wave propagation in fluid-saturated porous media such as cancellous bone and water-saturated sediments. Lee et al. [26] have successfully applied the MBA model to predict the dependences of velocity and attenuation on frequency and porosity in bovine cancellous bone. The MBA model is based on separate treatments of the viscous and the thermal effects of the fluid, since according to Attenborough, this simplifies the derivation [3,4,50]. Biot's theory has the merit of including the viscous effect of the interstitial fluid, but it does not take into account the thermal effect. Although Attenborough's theory takes into account both the viscous and the thermal effects, it does not include the fast wave of the Biot's theory because it takes the pore frame to be a rigid material. In contrast, the MBA model includes the thermal effect specified by an analytic solution and also allows for an elastic solid and fluid medium by means of a parametric fit. However, the thermal effect is relatively small for wave propagation in cancellous bone. One drawback of the MBA model is that it relies on the empirical parameters determined from experimental data. Therefore, the MBA model needs to be explored in greater depth to determine optimum values for input parameters in cancellous bone.

The present study is aimed at applying the MBA model to predict the dependence of phase velocity on porosity in cancellous bone. The optimum values for input parameters of the MBA model in cancellous bone are determined by comparing the predictions with previously published measurements in human calcaneus and bovine cancellous bone [19,26,48]. The predictions are also compared with those obtained by Biot's theory.

2. Theory

2.1. Modified Biot–Attenborough (MBA) model

In this section, the equations of the MBA model are summarized without repeating the derivations. For rigorous and complete derivations, see Lee et al. [26] and Roh and Yoon [36]. In the MBA model, for simplicity, acoustic wave propagation through a circular cylindrical pore is assumed to be one-dimensional along the axis of the pore. Care is taken to treat the boundary condition at a nonrigid pore frame, which should be different from that at a rigid frame. The dimensionless parameter $\lambda(\omega)$ related to the thickness of the viscous boundary layer at the pore wall is given by

$$\lambda(\omega) = as_1(\omega/v)^{1/2}, \tag{1}$$

where a is the radius of the circular cylindrical pore, ω is the angular frequency of the wave, v is the kinematic viscosity of the pore fluid, and s_1 is the boundary condition parameter representing the rigidity of the pore frame. If the frame material is rigid, the value of s_1 is equal to unity; if it is not rigid, then s_1 is larger than unity. The effective radius r_{eff} of the cylindrical pore may be given as as_1. This can be justified because the normal component of the particle velocity at the boundary $r = a$ becomes zero for a rigid wall, but it is nonzero at $r = a$ for a nonrigid wall. This suggests that the normal component of the particle velocity for a nonrigid frame becomes zero at the effective radius $r_{\text{eff}} = as_1$.

The complex (or frequency-dependent) wavenumber $k_{\text{b}}(\omega)$ for a nonrigid porous medium with bulk cylindrical pores may be expressed by using the empirical formula

$$k_{\text{b}}(\omega) = \alpha\left[\frac{k_{\text{c}}^2 k_m^2}{(1-\beta)^{s_2}k_{\text{c}}^2 + \beta^{s_2}k_m^2}\right]^{1/2}, \tag{2}$$

where α is the tortuosity, β is the porosity, and $k_m = \omega/c_m$ is the wavenumber of the pore frame. The wavenumber k_{c} of the pore fluid depends on the complex density and the complex compressibility of the pore fluid. The phase velocity parameter s_2 represents the form of the phase velocity curve as a function of porosity. It has a value less than unity if this curve is convex. Its value is larger than unity if the phase velocity curve is concave and is equal to unity if it is linear. The phase velocity can be found as $c = \omega/\text{Re}(k_{\text{b}})$,

where $\mathrm{Re}(k_b)$ is the real component of the complex wavenumber $k_b(\omega)$ of Eq. (2).

The impedance parameter s_3 of the MBA model represents the form of the effective impedance curve as a function of porosity. A previous study introduced s_3 as a fitting parameter [36]. This was done because knowledge of the variation of the effective impedance with porosity is required in order to determine the transmission coefficient. In the present study, however, s_3 is not required because the phase velocity is here obtained from the complex wavenumber $k_b(\omega)$.

2.2. Input parameters of the MBA model

The intrinsic ultrasonic and physical parameters for cancellous bone tissue are assumed to be the same as those for solid bone (or cortical bone) comprising the skeletal frame. The parameters for fat are normally used for the pore fluid because bone marrow is mainly composed of fat with very little blood and tissue fluid. The common input parameters of the MBA model in cancellous bone are shown in Table 1. The pore radius a was assumed to be 0.5 mm, consistent with typical values for trabecular spacing of human calcaneus from the literature [11,15,42,48]. As previously applied to cancellous bone [26,27], the tortuosity α was set equal to unity based on the assumption that all of the cylindrical pores in cancellous bone have identical orientation normal to the surface and are parallel to the wave propagation direction. Moreover, consideration is restricted to motion in a single dimension. The density $\rho_m = 1800\ \mathrm{kg/m^3}$ of solid bone was taken from the work of Wear et al. [48]. The density $\rho_0 = 1000\ \mathrm{kg/m^3}$, compressional speed $c_0 = 1483$ m/s, kinematic viscosity $\nu = 1\times10^{-6}\ \mathrm{m^2/s}$, specific heat ratio $\gamma = 1.004$, and Prandtl number $N_{\mathrm{Pr}} = 7$ of the pore fluid are equal to the values for water because the predictions are compared with the experimental measurements for defatted, water-saturated bone samples. The boundary condition parameter s_1 was fixed in this study at a value of 1.5 based on previous optimal fits for propagation in nonrigid porous media [26,27,36].

Table 2 summarizes the optimum input parameters of the MBA model and the root-mean-square error (RMSE) of curve fits for 53 human samples only and for 75 combined human and bovine samples. In order to obtain a best fit to the data for the 53 human samples, a value of $c_m = 2500$ m/s for the compressional speed of solid bone was chosen to be consistent with the input parameters used by Wear et al. [48] in Biot's theory, evaluated using [39]

$$c_m = \left[\frac{E(1-\sigma)}{(1+\sigma)(1-2\sigma)\rho_m}\right]^{1/2}, \tag{3}$$

where Young's modulus $E = 8.3$ GPa, Poisson's ratio $\sigma = 0.3$, and the density $\rho_m = 1800\ \mathrm{kg/m^3}$ of solid bone were taken from the work of Wear et al. [48]. We call this set of input parameters [INPUT A]. In the present study, we will refer to four sets of input parameters [A, B, C, D] that can be described in terms of c_m and s_2 (for input to the MBA model), and Young's modulus E and the exponent n of the power law for the elastic moduli (for input to Biot's theory) where within a given set the parameters are consistent in accordance with Eq. (3). A higher value of $c_m = 3200$ m/s [INPUT B] was given by Williams [49], and this was used for the later modeling work presented here. The optimum value for the phase velocity parameter s_2 was obtained by observing the minimum RMSE of curve fits of the MBA model (varying the value of s_2 as a free parameter) to the experimental data of phase velocity as a function of porosity.

Table 1
Common input parameters of the MBA model in cancellous bone

Parameter	Value
Density of solid bone (ρ_m)	$1800\ \mathrm{kg/m^3}$
Density of fluid (ρ_0)	$1000\ \mathrm{kg/m^3}$
Compressional speed of fluid (c_0)	1483 m/s
Kinematic viscosity of fluid (ν)	$1\times10^{-6}\ \mathrm{m^2/s}$
Specific heat ratio of fluid (γ)	1.004
Prandtl number of fluid (N_{Pr})	7
Pore radius (a)	0.5 mm
Tortuosity (α)	1
Boundary condition parameter (s_1)	1.5

Table 2
Optimum input parameters of the MBA model and root-mean-square error (RMSE) of curve fits for 53 human samples only [INPUT A] and for 75 combined human and bovine samples [INPUT B]

Parameter	53 human samples [INPUT A]	75 combined human and bovine samples [INPUT B]
Compressional speed of solid bone (c_m)	2500 m/s	3200 m/s
Phase velocity parameter (s_2)	1.23	1.42
RMSE of curve fits		
for 53 human samples	15.3 m/s	16.6 m/s
for 22 bovine samples	111 m/s	99 m/s
for all 75 samples	61 m/s	55 m/s

3. Results

Fig. 1 shows the experimental and theoretical phase velocities at 0.5 MHz plotted as a function of porosity for human cancellous bone. The experimental data for the 53 human calcaneus samples (with porosities from 0.86 to 0.98) were taken from Wear et al. [48]. The 23 circles in the figure denote the samples for which porosity was directly measured by using micro computed tomography (micro CT). The 30 asterisks in the figure denote the samples for which porosity was estimated from dual energy X-ray absorptiometry (DEXA) measurements. The relative orientation between the ultrasound beam and the human calcanea was the mediolateral (ML) direction, which is the same as with *in vivo* measurements performed with commercial bone sonometers [48]. The black solid curve represents the prediction of the MBA model with $c_m = 2500$ m/s [INPUT A] for the direction of propagation perpendicular

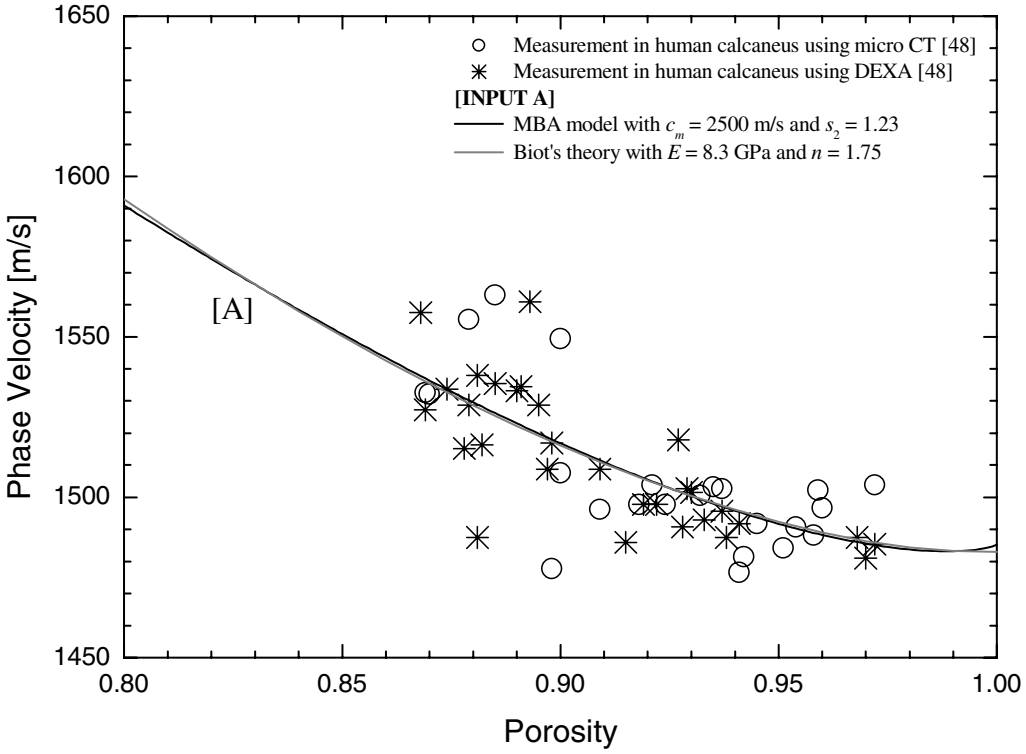

Fig. 1. Experimental and theoretical phase velocities at 0.5 MHz plotted as a function of porosity for human cancellous bone. The experimental data for the 53 human calcaneus samples were taken from Wear et al. [48].

to the trabeculae. The phase velocity parameter s_2 obtained by curve fitting to the data for all 53 samples was 1.23. The RMSE of the curve fit was 15.3 m/s (Table 2). The grey solid curve is the phase velocity predicted by Biot's theory with Young's modulus $E = 8.3$ GPa and the exponent $n = 1.75$ [INPUT A], which are the same used by Wear et al. [48]. All of the additional input parameters of Biot's theory are also taken from Wear et al. [48].

Fig. 2 shows the experimental and theoretical phase velocities at 0.5 MHz plotted as a function of porosity for both human (at 0.5 MHz) and bovine (at 1 MHz) cancellous bones. The experimental data for the 53 human calcaneus samples (with porosities from 0.86 to 0.98) are as for Fig. 1. The 10 open triangles in the figure denote the bovine femur samples (with porosities from 0.69 to 0.93) taken from Hosokawa and Otani [19], and the 12 solid triangles the bovine tibia samples (with porosities from 0.67 to 0.92) from Lee et al. [26]. The comparison of measurements at two different frequencies is not problematic given the relatively nondispersive nature of bovine cancellous bone from 0.5 to 1 MHz [19,26]. The validity of combining data for two species will be discussed later. The bovine cancellous bone samples were also all oriented in the ML direction in relation to the bone. The black solid curve is the prediction of the MBA model with $c_m = 2500$ m/s and $s_2 = 1.23$ [INPUT A] as shown in Fig. 1. Values for the RMSE of the curve fit were 111 m/s for the 22 bovine samples and 61 m/s for all 75 samples (Table 2). The MBA model of [INPUT A] seems to underestimate the velocity at low porosities; this may be due to the low value for the compressional speed c_m used for the best fit to the human data of Fig. 1. The black dashed curve is the theoretical fit of the MBA model to the data for the 75 combined human and bovine samples [INPUT B]. The prediction was obtained by using a value of $c_m = 3200$ m/s and then varying the phase velocity parameter s_2 to obtain an optimum fit to the data. The optimized parameter s_2 for all 75 samples was 1.42. Values for the RMSE of the curve fit were 16.6 m/s for the 53 human samples, 99 m/s for the 22 bovine samples and 55 m/s for all 75 samples (Table 2). For comparison, the phase velocities of Biot's theory corresponding to those of the MBA model are also plotted in Fig. 2. The grey solid curve is the prediction of Biot's theory with $E = 8.3$ GPa and $n = 1.75$ [INPUT A] as shown in Fig. 1. The grey dashed curve is the prediction of Biot's theory with $E = 13.7$ GPa and $n = 2.10$ [INPUT B]. Other input parameters are the same as used by Wear et al. [48].

4. Discussion

In the present study, the MBA model has been applied in an attempt to predict the dependence of phase velocity on porosity in human cancellous bone, by using previously published measurements in human calcaneus [48]. As Fig. 1 shows, the MBA model with $c_m = 2500$ m/s and $s_2 = 1.23$ [INPUT A] performs well in predicting the dependence of phase velocity on porosity for the 53 human calcaneus samples. Wear et al. [48] have also shown that Biot's theory performs well for predicting the dependence of phase velo-

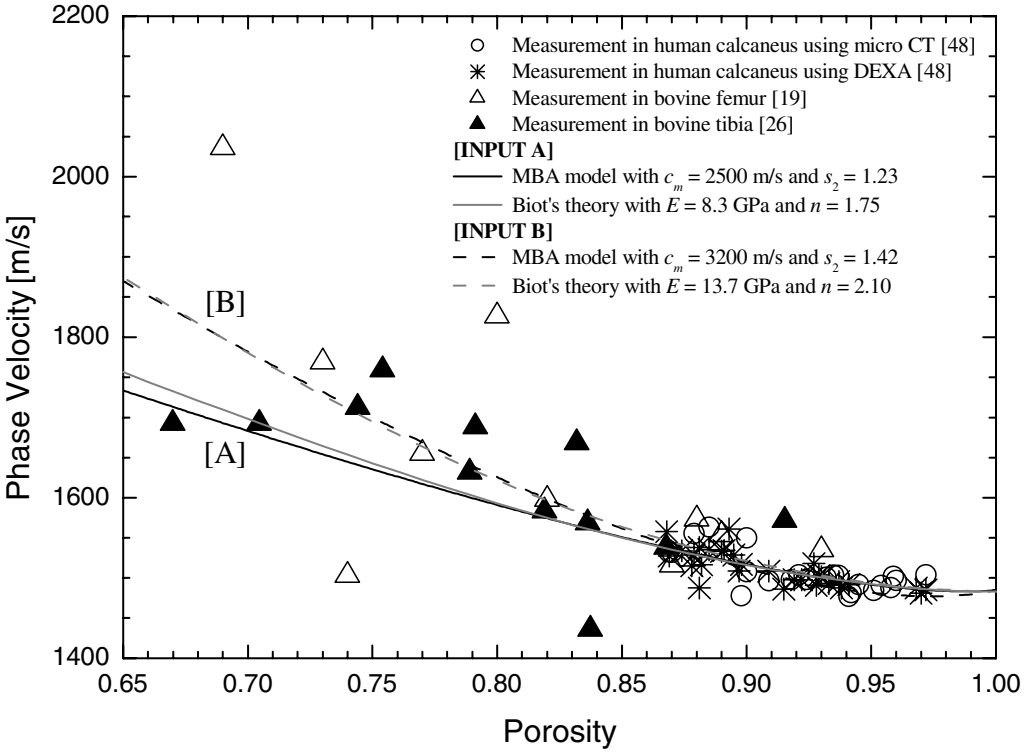

Fig. 2. Experimental and theoretical phase velocities at 0.5 MHz plotted as a function of porosity for both human (at 0.5 MHz) and bovine (at 1 MHz) cancellous bones. The experimental data for the 53 human calcaneus samples are as for Fig. 1, and that for the 22 bovine samples were taken from Hosokawa and Otani [19] and Lee et al. [26].

city on porosity in the human calcaneus samples. Good agreement can be found between the predictions of Biot's theory and the MBA model as shown in Fig. 1. This may be attributable to the use of consistent values for the input parameters required by these two models. One feature of this range of models is the numerous parameters used as input or as fitting parameters. For example, Biot's theory requires 14 parameters including one fitting parameter, of which around four are not easy to estimate [49,48]. The MBA model also requires 14 parameters including three empirical parameters determined from experimental data [26,27]. Therefore, the two models can be considered to be equally efficient in terms of the number of input parameters. However, the MBA model uses commonly known input parameters, at the cost of introducing simplifications such as an enforced tortuosity of unity.

It is well known that the relationship between attenuation and bone density is essentially nonlinear over a wide range of densities [1,8,14,16,23,38]. In contrast, significant linear correlations between velocity and density have been reported for both human and bovine cancellous bones [14,32,38]. Recently, Wear [46] has reported that the phase velocity is linearly proportional to porosity over a porosity range from 0.88 to 0.98 in trabecular-bone-mimicking phantoms consisting of parallel nylon wires. Meanwhile, Strelitzki et al. [40] have suggested that a quadratic model provides a more appropriate fit for a different bone phantom, which consisted of small cubic gelatin granules randomly distributed in epoxy with porosities from 0.66 to 0.83. It should be noted that the two phantom designs are not necessarily expected to exhibit the same features on phase velocity versus porosity, because they used different materials and different shapes of inclusions and the ranges of porosities spanned in each phantom were substantially different. In human cancellous bone, however, there is some indication of nonlinearity at very low densities [14]. As seen in Fig. 1, the nonlinearity of phase velocity at very high porosities can be observed from the predictions of both the MBA model and Biot's theory. Lin et al. [28] have also demonstrated that the theoretical relationship between velocity and porosity is not strictly linear, by using the stratified model.

We have also attempted to compare the MBA model with measurements of phase velocity made in bovine cancellous bone as well as in human calcaneus. As shown in Fig. 2, a degree of scatter in bovine bone is larger than that in human calcaneus because of the heterogeneity of the trabecular structure of the bovine samples with high density. A difference is observed between the prediction of the MBA model with $c_m = 2500$ m/s and $s_2 = 1.23$ [INPUT A] and the bovine data. This discrepancy at relatively low porosities may be due to uncertainty associated with input parameters relating to the elastic property of bone. Indeed, the value of $c_m = 2500$ m/s for the compressional speed of solid bone is somewhat lower than the experimental data from the literature [2,22,45]. Therefore, the MBA model with $c_m = 2500$ m/s and $s_2 = 1.23$ [INPUT A] may only have validity in a limited range of porosity for human

calcaneus. In contrast, the MBA model with $c_m = 3200$ m/s and $s_2 = 1.42$ [INPUT B] performs reasonably well for predicting the porosity dependence of phase velocity over a wide range of porosities. However, this refinement does not significantly reduce the RMSE because of the scatter in the bovine data. Fig. 2 includes the predictions of Biot's theory corresponding to those of the MBA model. The values for Young's modulus E of Biot's theory were chosen to be consistent with the compressional speed c_m of the MBA model, by using Eq. (3) with Poisson's ratio $\sigma = 0.3$ and the density $\rho_m = 1800$ kg/m^3 of solid bone.

Combining human and bovine data for the prediction of models based on a single set of input parameters may be problematic because structural parameters such as phase velocity parameter s_2, exponent n, pore size, and tortuosity might not be the same for both human and bovine bones. Furthermore, material parameters such as density, sound speed, and Young's modulus for constituents of bone might not be the same owing to biological differences between the two species. Indeed, Lee et al. [26] have obtained a value of $s_2 = 1.5$ with a compressional speed $c_m = 3500$ m/s of solid bone in bovine cancellous bone. In the present study, however, the phase velocity parameter $s_2 = 1.23$ was obtained using the human calcaneus samples, with a compressional speed $c_m = 2500$ m/s. Williams [49] has found the exponent $n = 1.23$ in bovine cancellous bone with an oriented columnar structure, and $n = 2.35$ with a random structure. Hosokawa and Otani [18,19] have obtained $n = 1.46$ in the direction parallel to the trabeculae of bovine cancellous bone, and $n = 2.14$ in the perpendicular direction. In contrast, Wear et al. [48] have made use of $n = 1.75$ in human calcaneus. As stated by Wear et al. [48], this may result from a more complex trabecular structure of human calcaneus compared to bovine femur and tibia. However, the variability of the exponent n may also be attributable to the use of different values for Young's modulus E of solid bone. Indeed, the value of $E = 8.3$ GPa was chosen by Wear et al. [48], but $E = 20$ GPa was used by Hosokawa and Otani [19] and Williams [49].

Bone is considered a highly heterogeneous and anisotropic medium [19,20,29,33,43]. Acoustic anisotropy implies that the structure affects acoustic properties independent of density since the volumetric density of a given sample is independent of direction. Ultrasonic velocity in cortical bone is greatest in the axial direction, least in the radial direction, and intermediate in the tangential direction. Therefore, it is important to examine the sensitivity of the theoretical fit to the assumed value for the compressional speed c_m of solid bone. Fig. 3 shows the minimum RMSE fits of the MBA model to the 75 data for three different assumed values of c_m spanning the range of measurements from the literature [2,22,45]. Typical measurements of velocity in human cortical bone range from 3200 to 4200 m/s. The black solid curve in the figure corresponds to $c_m = 3200$ m/s [INPUT B], the black dashed curve to $c_m = 3700$ m/s [INPUT C], and the black dotted curve to $c_m = 4200$ m/s [INPUT D], with each phase velocity parameter s_2 optimized by curve fitting. It is shown that the fit of the MBA model is sensitive to the value used for the compressional speed c_m of solid bone. Fig. 3 also illustrates the phase velocities of Biot's theory corresponding to those of the MBA model. The grey solid curve corresponds to Young's modulus $E = 13.7$ GPa [INPUT B], the grey dashed curve to $E = 18.3$ GPa [INPUT C], and the grey dotted curve to $E = 23.6$ GPa [INPUT D], with each exponent n optimized by curve fitting. The values for E consistent with c_m were evaluated using Eq. (3) with Poisson's ratio $\sigma = 0.3$ and the density $\rho_m = 1800$ kg/m^3 of solid bone.

Since the initial suggestion that the inversion of ultrasonic measurements might (through use of a suitable model) be used to infer material parameters of cancellous bone, and thereby potentially indicate bone health [24], a range of propagation models have been proposed. There have been numerous studies comparing the predictions of one or more of models with measurements of propagation through cancellous bone and through phantoms [9,12,17–21,26–28,30,31,34,41,44,48,49]. A key issue is in identifying whether any observed variations in the predictions of different models are the result of differences in the physics of the models, or due to inconsistency in the input values used for each model. In the present study, the use of consistent values for the input parameters of the MBA model and Biot's theory leads to good agreement between them, as shown in Figs. 1–3. The two models equally tell us that the range of porosities encountered in normal and osteoporotic cancellous bone are particularly unsuited for analysis using inversion of the sound speed. As can be seen in Fig. 1, at porosities higher than 0.9, the models tell us the sound speed will be particularly insensitive to the porosity. Considering the scatter of the data, if there were no other information available (such as on the anisotropy of the measurement), then measurement of sound speed as a function of porosity would present significant challenges in differentiating between normal and osteoporotic bone. The more difficult task, of monitoring changes in porosity (either as a disease progresses, or in response to drug treatment), would seem unapproachable by this technique.

Cancellous bone is a highly porous, anisotropic medium composed of a cellular network of calcified strands or plates called trabeculae, filled with fatty bone marrow. However, neither the MBA model nor Biot's theory, as previously applied to cancellous bone, allows for the angular dependence of acoustic properties with direction. Biot's theory can account for acoustic anisotropy in cancellous bone, by introducing the adjustable exponent n of the power law for the elastic moduli [10]. Testing along the direction of trabecular alignment results in a value of n close to 1. When the structure is more random, *i.e.*, the trabeculae are not aligned in any direction, or when the material is tested in a direction other than that of the major alignment of the trabeculae, n falls between 2 and 3. The value of n can be determined by curve fitting to the experimental data of phase velocity as a function of porosity [49]. In predicting the phase velocity in cancellous bone,

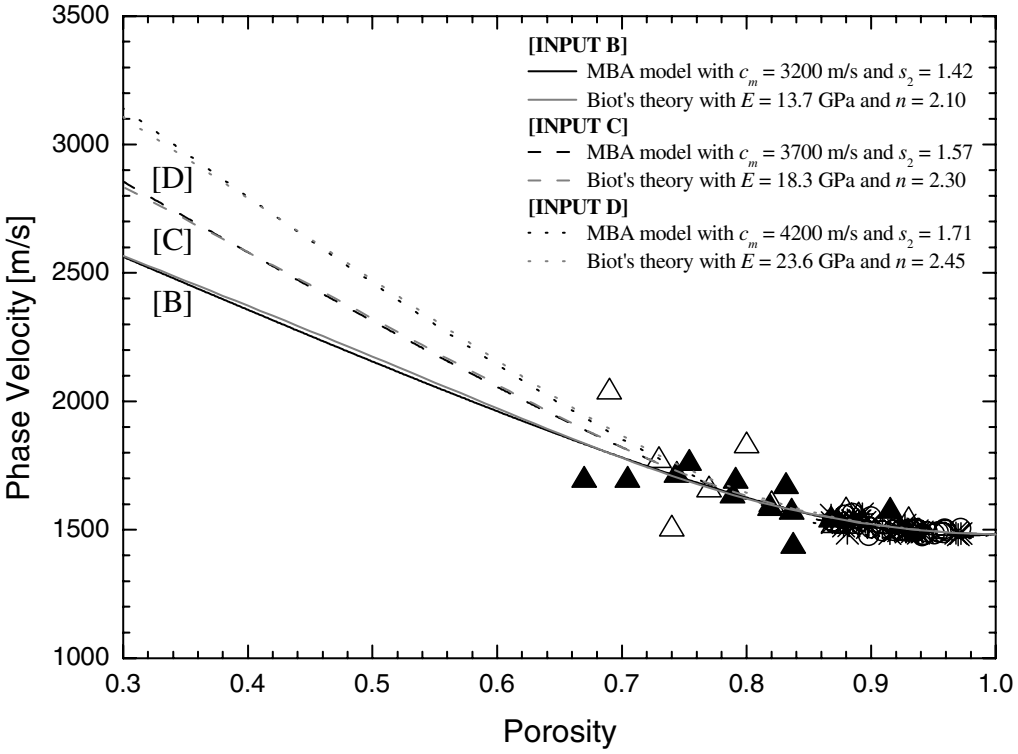

Fig. 3. Minimum RMSE fits of the MBA model to the 75 cancellous bone samples for three different assumed values of the compressional speed c_m of solid bone with each phase velocity parameter s_2 optimized by curve fitting. For comparison, the phase velocities of Biot's theory corresponding to those of the MBA model are also plotted.

it may be regarded that the phase velocity parameter s_2 of the MBA model plays a role equivalent to that of the exponent n of Biot's theory. The value of s_2 also depends on the direction of loading (or the direction of propagation) varying from 0 to 2 [26,27]. It has a value close to 0 when the material is loaded along the direction of trabecular alignment and has between 1 and 2 in the transverse direction. As with n, s_2 can also be optimized by curve fitting to the experimental data of phase velocity as a function of porosity. Therefore, the parameter s_2 may have potential for predicting an anisotropic response of cancellous bone. Further development is currently underway with the introduction of an empirical angle-dependent phase velocity parameter into the MBA model for predicting acoustic anisotropy in cancellous bone.

5. Conclusions

The MBA model has been applied to predict the dependence of phase velocity on porosity in cancellous bone. The optimum values for input parameters of the MBA model in cancellous bone were determined by comparing the predictions with previously published measurements in human calcaneus and bovine cancellous bone. This modeling effort is relevant to the use of QUS in the diagnosis of osteoporosis because the relationship between phase velocity and porosity is closely related to diagnostic measurements in current use for the assessment of osteoporosis. Although the MBA model relies on the empirical parameters determined from experimental data, it is expected that the model can be usefully employed as a practical tool in the field of clinical ultrasonic bone assessment.

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References

[1] J.M. Alves, W. Xu, D. Lin, R.S. Siffert, J.T. Ryaby, J.J. Kaufman, Ultrasonic assessment of human and bovine trabecular bone: a comparison study, IEEE Trans. Biomed. Eng. 43 (1996) 249–258.
[2] P.P. Antich, J.A. Anderson, R.B. Ashman, J.E. Dowdey, J. Gonzales, R.C. Murry, J.E. Zerwekh, C.Y. Pak, Measurement of mechanical properties of bone material in vitro by ultrasound reflection: methodology and comparison with ultrasound transmission, J. Bone Miner. Res. 6 (1991) 417–426.
[3] K. Attenborough, Acoustical characteristics of porous materials, Phys. Rep. 82 (1982) 179–227.
[4] K. Attenborough, Acoustic characteristics of rigid fibrous absorbents and granular materials, J. Acoust. Soc. Am. 73 (1983) 785–799.
[5] M.A. Biot, Theory of propagation of elastic waves in a fluid-saturated solid. I. Low-frequency range, J. Acoust. Soc. Am. 115 (1956) 168–178.
[6] M.A. Biot, Theory of propagation of elastic waves in a fluid-saturated solid. II. Higher frequency range, J. Acoust. Soc. Am. 28 (1956) 179–191.
[7] M.A. Biot, Generalized theory of acoustic propagation in porous dissipative media, J. Acoust. Soc. Am. 34 (1962) 1254–1264.

[8] J.A. Evans, M.B. Tavakoli, Ultrasound attenuation and velocity in bone, Phys. Med. Biol. 35 (1990) 1387–1396.
[9] Z.E.A. Fellah, J.Y. Chapelon, S. Berger, W. Lauriks, C. Depollier, Ultrasonic wave propagation in human cancellous bone: application of Biot theory, J. Acoust. Soc. Am. 116 (2004) 61–73.
[10] L.J. Gibson, The mechanical behavior of cancellous bone, J. Biomech. 18 (1985) 317–328.
[11] C.C. Gluer, C.Y. Wu, M. Jeregas, S.A. Goldstein, H.K. Genant, Three quantitative ultrasound parameters reflect bone structure, Calcif Tissue Int. 55 (1994) 46–52.
[12] T.J. Haire, C.M. Langton, Biot theory: a review of its application to ultrasound propagation through cancellous bone, Bone 24 (1999) 291–295.
[13] S. Han, J. Medige, I. Ziv, Combined models of ultrasound velocity and attenuation for predicting trabecular bone strength and mineral density, Clin. Biomech. 11 (1996) 348–353.
[14] S. Han, J. Rho, J. Medige, I. Ziv, Ultrasound velocity and broadband attenuation over a wide range of bone mineral density, Osteoporosis Int. 6 (1996) 291–296.
[15] D. Hans, M.E. Arlot, A.M. Schott, J.P. Roux, P.O. Kotzki, P.J. Meunier, Do ultrasound measurements on the os calcis reflect more the bone microarchitecture than the bone mass? A two-dimensional histomorphometric study, Bone 16 (1995) 295–300.
[16] R. Hodgkinson, C.F. Njeh, M.A. Whitehead, C.M. Langton, The non-linear relationship between BUA and porosity in cancellous bone, Phys. Med. Biol. 41 (1996) 2411–2420.
[17] A. Hosokawa, Simulation of ultrasound propagation through bovine cancellous bone using elastic and Biot's finite-difference time-domain methods, J. Acoust. Soc. Am. 118 (2005) 1782–1789.
[18] A. Hosokawa, T. Otani, Ultrasonic wave propagation in bovine cancellous bone, J. Acoust. Soc. Am. 101 (1997) 558–562.
[19] A. Hosokawa, T. Otani, Acoustic anisotropy in bovine cancellous bone, J. Acoust. Soc. Am. 103 (1998) 2718–2722.
[20] E.R. Hughes, T.G. Leighton, G.W. Petley, P.R. White, Ultrasonic propagation in cancellous bone: a new stratified model, Ultrasound Med. Biol. 25 (1999) 811–821.
[21] E.R. Hughes, T.G. Leighton, G.W. Petley, P.R. White, R.C. Chivers, Estimation of critical and viscous frequencies for Biot theory in cancellous bone, Ultrasonics 41 (2003) 365–368.
[22] R. Lakes, H.S. Yoon, J.L. Katz, Ultrasonic wave propagation and attenuation in wet bone, J. Biomed. Eng. 8 (1986) 143–148.
[23] C.M. Langton, C.F. Njeh, R. Hodgkinson, J.D. Currey, Prediction of mechanical properties of human calcaneus by broadband ultrasonic attenuation, Bone 18 (1996) 495–503.
[24] C.M. Langton, S.B. Palmer, R.W. Porter, The measurement of broadband ultrasonic attenuation in cancellous bone, Eng. Med. 13 (1984) 89–91.
[25] P. Laugier, An overview of bone sonometry, Int. Congress Ser. 1274 (2004) 23–32.
[26] K.I. Lee, H.S. Roh, S.W. Yoon, Acoustic wave propagation in bovine cancellous bone: application of the modified Biot–Attenborough model, J. Acoust. Soc. Am. 114 (2003) 2284–2293.
[27] K.I. Lee, S.W. Yoon, Comparison of acoustic characteristics predicted by Biot's theory and the modified Biot–Attenborough model in cancellous bone, J. Biomech. 39 (2006) 364–368.
[28] W. Lin, Y.X. Qin, C. Rubin, Ultrasonic wave propagation in trabecular bone predicted by the stratified model, Ann. Biomed. Eng. 29 (2001) 781–790.
[29] G. Luo, J.J. Kaufman, A. Chiabrera, B. Bianco, J.H. Kinney, D. Haupt, J.T. Ryaby, R.S. Siffert, Computational methods for ultrasonic bone assessment, Ultrasound Med. Biol. 25 (1999) 823–830.
[30] M.L. McKelvie, S.B. Palmer, The interaction of ultrasound with cancellous bone, Phys. Med. Biol. 36 (1991) 1331–1340.
[31] M.M. Mohamed, L.T. Shaat, A.N. Mahmoud, Propagation of ultrasonic waves through demineralized cancellous bone, IEEE Trans. Ultrason. Ferroelectr. Freq. Control 50 (2003) 279–288.
[32] P.H.F. Nicholson, M.J. Haddaway, M.W.J. Davie, The dependence of ultrasonic properties on orientation in human vertebral bone, Phys. Med. Biol. 39 (1994) 1013–1024.
[33] P.H.F. Nicholson, R. Muller, G. Lowet, X.G. Cheng, T. Hildebrand, P. Ruegsegger, G. Van Der Perre, J. Dequeker, S. Boonen, Do quantitative ultrasound measurements reflect structure independently of density in human vertebral cancellous bone? Bone 23 (1998) 425–431.
[34] P.H.F. Nicholson, R. Strelitzki, R.O. Cleveland, M.L. Bouxsein, Scattering of ultrasound in cancellous bone: predictions from a theoretical model, J. Biomech. 33 (2000) 503–506.
[35] C.F. Njeh, D. Hans, T. Fuerst, C.C. Gluer, H.K. Genant, Quantitative Ultrasound: Assessment of Osteoporosis and Bone Status, Martin Dunitz, London, 1999.
[36] H.S. Roh, S.W. Yoon, Acoustic diagnosis for porous medium with circular cylindrical pores, J. Acoust. Soc. Am. 115 (2004) 1114–1124.
[37] M. Schoenberg, Wave propagation in alternating solids and fluid layers, Wave Motion 6 (1984) 303–320.
[38] L. Serpe, J.Y. Rho, The nonlinear transition period of broadband ultrasound attenuation as bone density varies, J. Biomech. 29 (1996) 963–966.
[39] R.E. Sheriff, L.P. Geldart, Exploration Seismology, Cambridge University Press, New York, 1995.
[40] R. Strelitzki, J.A. Evans, A.J. Clarke, The influence of porosity and pore size on the ultrasonic properties of bone investigated using a phantom material, Osteoporosis Int. 7 (1997) 370–375.
[41] R. Strelitzki, P.H.F. Nicholson, V. Paech, A model for ultrasonic scattering in cancellous bone based on velocity fluctuations in a binary mixture, Physiol. Meas. 19 (1998) 189–196.
[42] D. Ulrich, B. van Rietbergen, A. Laib, P. Ruegsegger, The ability of three-dimensional structural indices to reflect mechanical aspects of trabecular bone, Bone 25 (1999) 55–60.
[43] K.A. Wear, Anisotropy of ultrasonic backscatter and attenuation from human calcaneus: implications for relative roles of absorption and scattering in determining attenuation, J. Acoust. Soc. Am. 107 (2000) 3474–3479.
[44] K.A. Wear, A stratified model to predict dispersion in trabecular bone, IEEE Trans. Ultrason. Ferroelectr. Freq. Control 48 (2001) 1079–1083.
[45] K.A. Wear, The effect of trabecular material properties on the frequency dependence of backscatter from cancellous bone (L), J. Acoust. Soc. Am. 114 (2003) 62–65.
[46] K.A. Wear, The dependencies of phase velocity and dispersion on trabecular thickness and spacing in trabecular bone-mimicking phantoms, J. Acoust. Soc. Am. 118 (2005) 1186–1192.
[47] K.A. Wear, D.W. Armstrong, Relationships among calcaneal backscatter, attenuation, sound speed, hip bone mineral density, and age in normal adult women, J. Acoust. Soc. Am. 110 (2001) 573–578.
[48] K.A. Wear, A. Laib, A.P. Stuber, J.C. Reynolds, Comparison of measurements of phase velocity in human calcaneus to Biot theory, J. Acoust. Soc. Am. 117 (2005) 3319–3324.
[49] J.L. Williams, Ultrasonic wave propagation in cancellous and cortical bone: prediction of some experimental results by Biot's theory, J. Acoust. Soc. Am. 91 (1992) 1106–1112.
[50] C. Zwikker, C.W. Kosten, Sound Absorbing Materials, Elsevier, Amsterdam, 1949.