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STUDY OF BUBBLE FORMATION DYNAMICS BASED ON ASSOCIATED ACOUSTIC RADIATION

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The release of gas into water can be found in many industrial applications. This process results in the formation of bubbles, and both associated bubble size distribution and bubble volume flow rate have an important impact on the designed outcomes. Research has been carried out both experimentally and theoretically, in order to explore the underlining physics of a gas-water system of dispersed bubbles. Many researchers have used acoustic techniques to study a quasi-steady bubbly plume. However, it is relative rare to study the dynamics of turbulent release of gas into water using an acoustic techniques, in which both experimental and theoretic analyses become fairly challenging. For those conditions, the widely used optical technique is no longer reliable if an experimental study is performed. However, the use of acoustic technique has been found to be a promising approach because in injection conditions, the formation of each individual bubble emits sound. In this paper, the bubble formation dynamics will be studied using acoustic techniques. The acoustic emission of the gas discharge into water will be measured accurately and the bubble formation dynamics will be modelled based on their acoustic emission. The characteristics of the bubbly plume are explored. The bubble are generated using three nozzles having internal diameters of 4 mm, 6 mm and 9 mm and the gas discharge rate is varied between 2.5 L/min to 40 L/min. It has been found that the gas discharge rate has an strong impact on the sound of bubble formation. The choice of nozzle imparts some influence on the bubble formation.

Keywords: bubble acoustics, bubbly plume, bubble size distribution, bubble generation, underwater nozzle

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1. Introduction

The release of gas into water can be found in many industrial applications. This process results in the formation of bubbles. Both bubble size distribution and bubble generation rate have an important impact on the designed outcomes. Examples occur in bubble-mediated medical diagnostics, the significance of bubbles to the stealth of naval platforms, and the contribution of bubbles to ocean background noise. Such noise (whether produced under breaking waves [1] or as gas leaks from the seabed into the ocean from natural or anthropogenic gas reserves [2]) can be inverted to estimate the bubble size distribution and the amount of gas being entrained in the liquid. This paper will restrict itself to detecting bubble injection through passive acoustic techniques. Research, usually at low injection rates where the sound from each individual bubble can be identified, has explored the underlying physics both experimentally and theoretically ([3] -[8]). For a dispersed bubbly flow, the widely used techniques are optical-based such as PIV and high-speed photograph. A recent example of the use of an acoustic technique to measure volume flow rate can be found in Leblond et al.[10].

Those applications are mainly for low gas flow rates. However, very often there is a requirement to characterise a turbulent bubbly plume for both size distribution and bubble generation rate. When the gas flow rate is high, the bubbly plume becomes turbulent. According to recent research by ([11] - [13]), at a high gas discharge rate, the gas-water flow will exhibit a transition from bubbling to jetting. The bubble concentration will increase significantly. Under those situations, the conventional optical techniques are no longer applicable. An acoustic based technique may be the only option and can be very powerful because every bubble being formed emits sound.

Noise generated by bubble formation is an interesting and complex phenomena owing to the rich physics. A comprehensive coverage of the fundamentals of bubble acoustics can be found in [14]. Most studies have mainly focused on a single bubble and its natural frequency, or bubble formation dynamics at a low discharge rate. The research on the modelling number of bubble generation based on their acoustic emission are limited, and it becomes even rarer when a turbulent gas discharge is involved. This is because of the challenges associated with the model validation and complication associated with sound transmission in the bubbly medium.

Modelling and measurement of sound generated by a highly turbulent discharge water-gas flow (bubbling plume) can be very challenging. If solving the forward problem to predict the sound from turbulent injection is challenging, the task of inverting such sound using an appropriate model to estimate bubble generation, size distribution and gas flux is even more so. There is no simple theory that can be used to predict the associated acoustic field because of the complicated dynamics in the bubble formation process [3],[12] and [15]. For example, the sound emission due to bubble fragmentation caused by turbulence may be different to that of a bubble excited by detachment from a nozzle. Multiple contacts at the nozzle [16] and fragmentation/coalescence after release [7] can create ambiguities. At high gas discharge rates, the formation of bubbles can be strongly influenced by the nozzle size and the induced liquid flow. Many experimental and theoretical studies have been reported to explore in this area (e.g. [3], [8] and [17] - [19]).

Leighton and White [17] developed a model to estimate bubble number generation of a bubbly plume from a submerged nozzle based on acoustic emission. They highlighted the need to know the amplitude of excitation of each bubble, and the fact that current experimental data did not allow this. Until such data became available, they took a fixed value for the percentage by which the bubble radius is first perturbed when it is mechanically excited. They warn 'Use of a constant factor would facilitate the inversion, but may not reflect the real relationship' [17]. Berges et al. [2] and Chen et al. [21] confirmed that indeed an excitation based on a constant percentage radius perturbation is unable to adequately describe the bubble formation dynamics and to predict the associated acoustic emission. The model of Chen et al. [21] based on [17] has taken the effect of gas flow rate on bubble formation dynamics into consideration. In this paper, the model of Chen et al. [21] will be used to investigate the characteristics of a turbulent bubbly plume.

2. Theory

Of the many applications of bubble acoustics, this paper focuses on the estimation of the number and size formed by their passive emissions. This was first done in the natural world by Leighton and Walton [22] to count bubbles formed by brooks and waterfalls, and because bubble entrainment rates were low, each bubble formation event was clearly distinguished as an exponentially decaying sinusoid, the frequency of which gave the bubble size. As such there was no need to know the amplitude of excitation of each bubble. At higher entrainment rates, the overlap between bubble signatures makes it difficult to distinguish them from one another, and although signal processing routines can help [23], as the entrainment rate increases further it is necessary to abandon the method of counting individual signatures, and instead measure the overall power of the sound emission, and count bubbles by dividing it by the assumed power contributed by each bubble into each frequency band. In this way Leighton and White [17] estimated the bubble size distribution being formed.

Modelling the relationship between acoustic emission and bubble generation rate for high flow rates is more challenging because of the range of bubble sizes and interaction of bubbles in the bubbling plume. In addition, estimating the size distribution of bubbles in this flow requires an accurate measurement of the sound pressure. Leighton and White [17] provided a model to calculate the acoustic pressure radiated by a single bubble pulsating at natural frequency ω_0 at time t and a distance r (far field)

$$P(t) \approx Re\left(\rho \frac{(\omega R_0)^2}{r} R_{0\epsilon i} e^{j\omega_0 (t-t_i)} e^{\frac{-\omega_0 \delta_{tot} (t-t_i)}{2}} H(t-t_i)\right). \tag{1}$$

In Eq.1 R_0 is the initial bubble size, $R_{\epsilon 0i}$ is the initial displacement of the bubble wall, ρ is the liquid density, H is the Heaviside step function, and t_i is time at which the bubble emission first occurs. The decay of the pressure induced by the oscillating bubble is determined by the dimensionless damping constant δ_{tot} defined by [14] and [24] as

$$\delta_{tot} = \delta_{th} + \delta_{vis} + \delta_{rad}. \tag{2}$$

The thermal damping constant δ_{th}, the viscous damping constant δ_{vis} and the radiation damping constant δ_{rad} can be found in [14].

$$\mid P(\omega, R_0) \mid^2 = \left(\omega_0^2 R_0^3 \frac{\rho_0}{\pi r} \frac{R_{0\epsilon i}}{R_0}\right)^2 \times \frac{(4\omega_0 \delta_{tot} + 16\omega^2)^2}{\left[(\omega_0 \delta_{tot})^2 + 4(\omega_0 - \omega)^2\right]\left[(\omega_0 \delta_{tot})^2 + 4(\omega_0 + \omega)^2\right]}. \tag{3}$$

If the initial bubble excitation ratio, $\frac{R_{0\epsilon i}}{R_0}$, is known, the sound pressure spectrum of the acoustic emission of bubble generation can be calculated.

Considering a bubbling jet and assuming that the oscillation of each bubble is not correlated to the motion of surrounding bubbles, the monopole emissions of individual bubbles are then uncorrelated. If the bubble generation rate, (where $D(R_0)dR_0$ is the number of bubbles per second produced that have radii between R_0 and $R_0 + dR_0$), of the bubbling plume is specified (for example using the results from a computational fluid dynamics (CFD) simulation), the power spectral density $S(\omega)$ of far-field sound can be calculated by using

$$S(\omega) = \int_0^\infty D(R_0) \mid P(\omega, R_0) \mid^2 dR_0. \tag{4}$$

The relation between R_0 and the resonance is given in the adiabatic limit [14] by

$$\omega_0 = \frac{1}{R_0}\left(\frac{3\gamma p}{\rho}\right)^{0.5}. \tag{5}$$

in which p_0 is the static pressure and γ is the ratio of specific heats. Assume that the bubble generation rate is divided into n bins and number of bubble generated in each bin is given as

$$\psi(n) = \int_{R_{l,n}}^{R_{u,n}} D(R_{0,n}) dR_{0,n}. \tag{6}$$

If the acoustic emission at far-field is given, the number of bubbles generated from each bin at R_0 will be

$$S(\omega) \approx \sum_{n=1}^{N_b} \varphi(n) \mid P(\omega, R_{0,n}) \mid^2. \tag{7}$$

Solving for the bubble generation rate is the inverse problem of equation 7 for a given sound pressure level. If the bubbles are divided into groups N_b, the bubble number spectrum $S(\omega)$ is a single column vector of N_k elements. Eq.7 forms a $(N_k \times N_b)$ spectral matrix, which will be square if the number of frequencies N_k is equal to the number of bubble groups N_b. In such a case, Eq.7 can be solved by inverting the spectrum matrix to obtain the bubble number spectrum as a function of the initial bubble size R_0 for a given power spectral density of the sound field. The total flow rate of the system, Q, can be then obtained by integrating the bubble number spectrum.

According to Deane and Stoke [7], the initial excitation mechanism at very low gas injection rates is determined mainly by balancing between buoyancy force and surface tension, although other authors [25] disagree with their scaling laws and consequently with the importance of suface tension. At a high gas discharge rate the amplitude of the emission is bound to change in a nonlinear fashion with flow speed because the form of the flow qualitatively changes. Leighton et al. [26] identified that multiple excitations can occur because of coalescence and fragmentation at the nozzle. Furthermore, Berges et al. [2] noted that the excitation $R_{0\epsilon i}$ can vary with nozzle type for the same gas flow; and Leighton et al. [29] demonstated that relative flow between needle and surroundings (caused by vibrating the needle) dramatically affected the bubble formation. Chen et al. [21] proposed that, there are extra forces acting on forming bubble because the separation between bubbles is small, for example the wake-induced lift force generated by the upstream bubbles. Such force will encourage bubble detachment from a nozzle or the fragmentation. This should lead to a conclusion that the ratio $\frac{R_{0\epsilon i}}{R_0}$ should depend on the surrounding liquid flow, Q. Instead of being a constant, it should be $\frac{R_{0\epsilon i}}{R_0} = f(Q)$. Chen et al. [21] has suggested that this parameter is indeed dependent on the gas discharge rate and proposed an empirical expression. However, when the gas discharge flow rate is high, a plume consisting of many bubbles will be generated, and could have affect the measured sound field. In this paper, a refined expression for including sound screening effect is proposed as

$$\frac{R_{0\epsilon i}}{R_0} = 0.0446e^{-0.4841Q} + 0.0183e^{-0.0129Q}. \tag{8}$$

The exponent Q is flow rate in L/min. This empirical formula 8 is derived from the best fitting of the values used in the model to provide a good agreement with the measured gas flow rate. It is based on the results for a single round orifice of a diameter of 9 mm. The nozzle oriented vertically and located on the bottom of a water tank of dimensions of $10m \times 10m \times 6m$. This expression can not be considered as universal but are applicable for the flow ranges and nozzles investigated in the paper. The deviation from this expression by different nozzles has been observed. However, the principle reflected in this expression is valid. That is the initial bubble excitation ratio is dependent of flow.

3. Acoustic measurement

To calculate the bubble generation rate, the acoustic emission of bubbles must be measured. In this study, which are measured in the acoustically calibrated water tank with dimensions of $10m \times 10m \times 6m$. Although this tank is large, the emissions from the plume were continuous, and so it was not possible to time gate out reflections, and it should be noted that reflections can drive a bubble and affect both its natural frequency [27] and damping [28], and therefore reduce the accuracy of an inversion. Acoustic measurements were obtained by four carefully located RESON® Type 4013 hydrophones to ensure the measurements being in the diffuse field. Data was recorded and analysed using a 16-channel analyser DEWETRON. Regulated and filtered air was released into the tank through a single round nozzle located vertically on the bottom of the tank to generate bubbles. The air flow rate was measured with a flowmeter located on top of the tank and held constant for each measurement. Sound generated by air discharging one at a time from round nozzles of three different diameters were measured. The details of the acoustic characteristics of the water tank can be found in [21]. The measured sound pressure levels in third octave bands were corrected to 1 m away from the bubble plume. The measurements were performed at the conditions given in table 1 and used as the input of the inverse Eq. 7 to calculate bubble generation rates at different flow rates and from three

Table 1: Summary of measurement conditions (took on top of the tank)

L/min, Diameter(inter)	case 1	case 2	case 3	case 4	case 5	case 6
4 mm	2.5	5	7.5	15	30	40
6 mm	2.5	5	7.5	15	30	40
9 mm	2.5	5	7.5	15	30	40

round nozzle sizes. The diameters of three nozzles listed in 1 are internal diameters.

The bubble number distributions estimated via the acoustic inversion at different flow rates for three nozzles were made from the measured sound fields, and integrated across bubble sizes to estimate the gas flux rate from the nozzle. This was compared with the injected gas flux.

4. Results and discussion

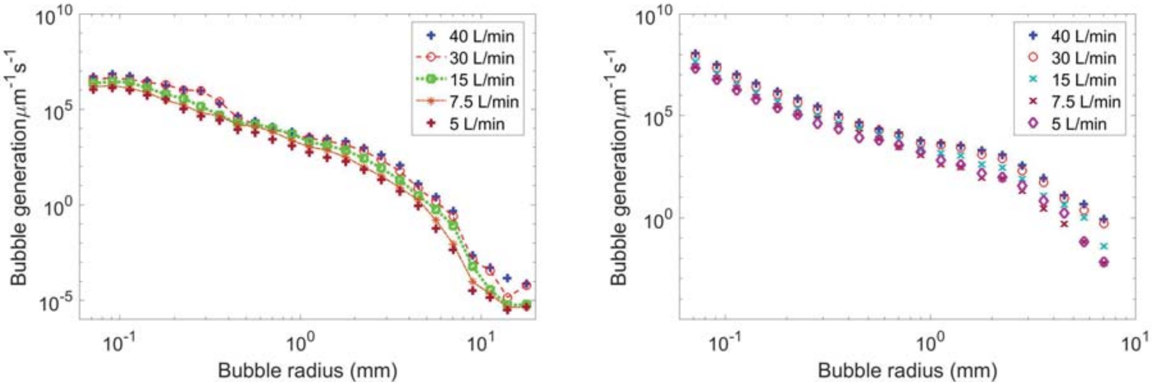
Space constraints mean that only a subset of the results for the 9 mm and 4 mm nozzles are shown in Figure 1. It must be seen from all those figures that an increase in gas flow rate results in an apparent increase in number of bubbles. Nevertheless, the flow rate shows an impact on the apparent rate of generation of large bubbles.

The acoustically estimated bubble volume densities for nozzles of $9mm$ is shown in Figure 2 as an example. The number in the bracket of the legend is the acoustically inferred total gas flow rate. Here the bubble volume distribution is obtained by normalising the bubble volumetric flux per μm by the total flux. There appears to be a tendency to exhibit two peaks in the apparent size distribution of bubbles injected: one occurs at bubbles of a radius around 0.2 mm, and another one is observed at bubbles of a radius of 2 mm (Note that the logarithmic scale suppresses the visual impact of the two-peak structure). This trend has been also observed for the 6 mm nozzle, which has not been included in this paper.

The effect of nozzle size on the characteristics of the bubbly plume is shown in Figures 3. Figure 3a is for bubble distribution. Here the bubble generation rate per μm increment in bubble radius is normalised by the total number of bubbles generated per second for all bubble sizes, which was obtained by integrating the estimated bubble generation over all bins using the inverse of Eq. 7, to have a better presentation of the characteristics of the bubbly plume. With this normalisation, the bubble number distributions collapse to a single line in Figure 3a for the 9 mm and the 6 mm nozzles. The results of the bubble number distributions between different flow rates for the same nozzle also show a similar trend. Those results may imply that the bubbly plumes are in a similar flow region. Whether this trend will persist at a flow rate higher than those investigated, and when the transition to different flow region occurs, will be subjected to further investigation. However, for the 4 mm nozzle, the bubble number distribution deviates from the other two. The 4 mm nozzle produces a higher proportion of small bubbles. Those results indicate that the nozzle size indeed has an impact on the bubble generation. The comparison of the bubble volume distribution between three nozzles at the gas flow rate equal to 30 L/min is given in Figure 3b. For the 6 mm and 9 mm nozzles, two peaks are observed. For the 9 mm nozzle, bubbles of a radius close to 0.2 mm have the highest volume density, and the bubbles of a radius close 2 mm have the highest volume density for 4 mm and 6 mm nozzles. It is also noted that the first peak produced by the 6 mm nozzle is at the bubble radius smaller than 0.2 mm. The volume concentration of the bubbly plume produced by the 4 mm nozzle has one peak. Three nozzles produce a similar peak at a bubble radius close to 2 mm.

5. Conclusion

The bubble formation dynamics from a turbulent gas discharge through submerged nozzles and the characteristics of the formed bubbly plumes are investigated using acoustic inversion. The gas



(a) 9 mm nozzle

(b) 4 mm nozzle

Figure 1: Estimated bubble generation for different flow rates for 9 mm and 4 mm nozzles

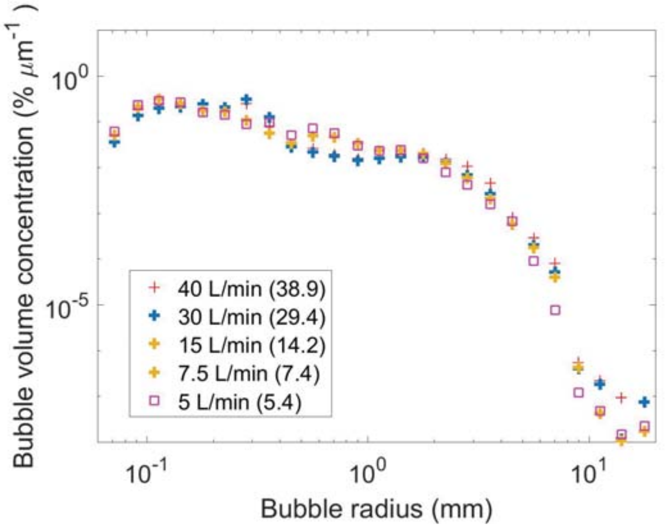
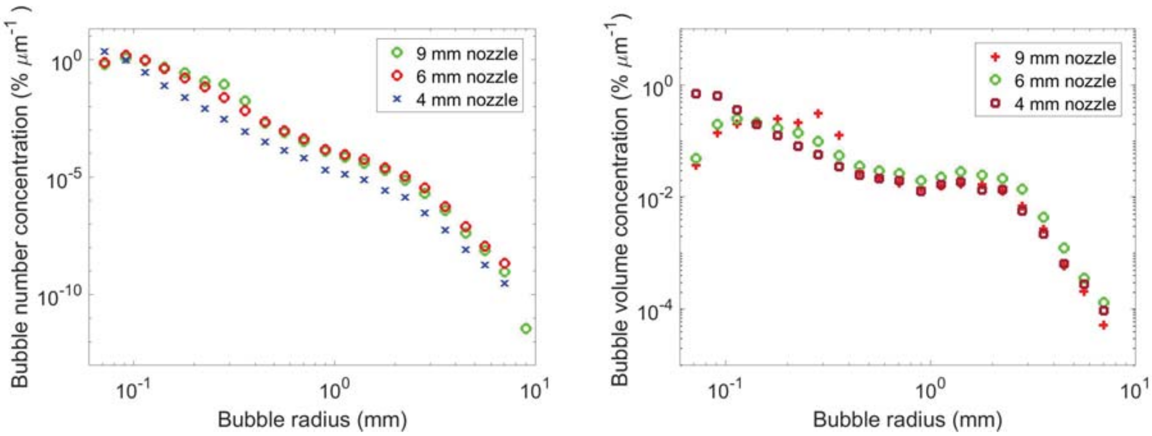


Figure 2: comparison of bubble volume distributions at different flow rates for 9 mm (number in bracket is acoustically inferred flow rate)

flow rates varies between 2.5 L/min to 40 L/min, and the bubbly plumes formed by three nozzles of 9 mm, 6 mm and 4 mm in diameter are studied.

It has found that the gas flow rate and nozzle size have an impact on the bubble formation. An increase in gas flow rate leads to an increase in the number density particularly for bubbles larger than 5 mm while the smaller bubbles maintain nearly constant number density; the bubble number distribution decreases with an increase in bubble radius. However, for bubble volume distribution, multi-peaks are observed, and those peaks strongly depend on flow rate and nozzle size. For 9 mm two peaks are observed at bubble radius close to 0.2 mm and 2 mm; for 6 mm nozzle, one peak is observed at bubbles smaller than 0.2 mm; only one peak is obtained for the 4 mm nozzle. For the flow rates investigated, the normalised bubble number distribution shows a consistent trend for two nozzles used. The bubble volume distribution reveals more details of the bubbly plume. The current study has demonstrated that the acoustic-based approach is able to provide a good insight of the characteristics of a turbulent bubbly plume. However the results of the inversion depend on the assumptions of the model, which is that each bubble contributes once to the overall sound field, as predicted by Eq. 1. Multiple excitations and departures from the free field (as show in [26] and [28] respectively) will introduce errors, as will emissions that do not conform to the assumptions inherent in Eq. 1 and Eq.7.



(a) Bubble number distribution

(b) Bubble volume distribution

Figure 3: comparison of bubble distributions between 9 mm, 6 mm and 4 mm nozzles at flow rate equal to 30 L/min

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