COMPARISON OF THEORIES FOR ACOUSTIC WAVE PROPAGATION IN GASSY MARINE SEDIMENTS

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Abstract: *More than three decades ago, Anderson and Hampton* [1, 2] (*A&H*) *presented theories for wave propagation in gassy water, saturated sediments and gassy sediments in their two-part review, which has been cited by many researchers in the geoacoustics and underwater acoustics areas. They gave an empirical formulation based on the theory of Spitzer* [3] *for the wave propagation in gassy water by adapting that for a viscoelastic, lossy medium. Following Leighton* [4], *this paper presents a theory based on non-stationary nonlinear dynamics of spherical gas bubbles and extends that 2007 paper to include liquid compressibility and thermal damping effects. The paper then shows how that nonlinear formulation can be reduced to the linear limit, and derives the expressions for the damping coefficients, the scattering cross section, the speed of sound and the attenuation, and compares these with the A&H theory. The current formulation has certain advantages over A&H theory such as implementing an energy conservation based nonlinear model for the gas pressure inside the bubble, having no sign ambiguity for the speed of sound formula (which is important when estimating the bubble void fraction) and correcting the ambiguity on the expression for scattering cross section, as identified in the recent work of Ainslie and Leighton* [5]. *Moreover, the theory presented here forms a basis for a nonlinear, time-dependent acoustic estimation model for gas bubble distributions in viscoelastic mediums since it avoids the commonly encountered assumptions on the bubble dynamics such as linearity, steady-state behaviour and monochromaticity.*

Keywords: *gassy sediments, nonlinear propagation, acoustic scattering*

1. INTRODUCTION

The presence of gas and its effects on the physical properties of the marine sediment are of interest for several applications, including drilling operations, construction of seafloor structures, and environmental considerations such as global warming, climate change and the slope stability of the sediments. The use of acoustics in order to characterize the physical properties of ocean bottom sediments is of increasing interest. This has highlighted the need for an in-depth and accurate theoretical framework for acoustic wave propagation in gassy mediums. Currently, the theory used by the majority of investigators relies mostly on the benchmark work of Anderson and Hampton [1, 2] (abbreviated as A&H theory hereafter) which was, at the time, an extensive review of the theories for wave propagation in gassy water, saturated sediments and gassy sediments. They provide an empirical formulation based on the theory of Spitzer [3].

The A&H theory needs to be reconsidered owing to following reasons: (i) it assumes that only linear, steady state pulsations occur, which makes the method inapplicable for high amplitude pulses and second harmonic or combination frequency signals [4]; (ii) the expression for the viscoelastic losses are given *a posteriori* without a rigorous derivation, leading to some ambiguities [5-7]; (iii) at a later date, Prosperetti et al. [8] presented a formulation for the thermal behaviour of the gas pressure inside the bubble, which is more complete than the use of polytrophic relation A&H employed (especially when bubble resonance effects are present) and can be incorporated into the current problem; and (iv) the expression for the scattering cross-section, when used together with the radiation damping, involves an inconsistency in terms of frequency dependence of the expressions [5].

Leighton [4] presented a theory based the non-stationary nonlinear dynamics of gas bubbles in marine sediments, noting the requirement for follow-on work to include liquid compressibility and thermal damping effects. This paper undertakes that follow-on work, and then in the linear limit, derives expressions for the damping coefficients, the scattering cross-section, the speed of sound and the attenuation.

2. THEORY

Following Yang and Church [9], the Keller-Miksis type equation which describes the radial motion of a spherical bubble in an unbounded viscoelastic medium can be written as follows;

$$\left(1-\frac{\dot{R}}{c}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{\dot{R}}{3c}\right)\dot{R}^2=\left(1+\frac{\dot{R}}{c}\right)\frac{p_{\mathrm{L}}-p_\infty}{\rho}+\frac{R}{\rho c}\frac{d}{dt}(p_{\mathrm{L}}-p_\infty) \tag{1}$$

where

$$p_{\mathrm{L}}-p_\infty = p_{\mathrm{g}}-\frac{2\sigma}{R}-p_0+P_{\mathrm{A}}g(t)-\left[\frac{4G}{3R^3}(R^3-R_0^3)+\frac{4\mu\dot{R}}{R}\right]. \tag{2}$$

In equations (1) and (2), R is the bubble radius, the dots indicate the time derivatives, c is the speed of sound in the host medium, ρ is the density of the medium, p_L is the pressure outside the bubble wall, p_∞ is the pressure at infinity, p_g is the pressure inside the bubble, σ is the surface tension, $P_A\, g(t)$ is the time-dependent acoustic pulse with P_A being a positive real number that scales the driving pressure, p_0 is the static pressure, G is the shear modulus and μ is the shear viscosity of the surrounding medium.

The continuity and the energy conservation equations for a perfect gas are given respectively by

$$\frac{D\rho_g}{Dt} + \rho_g \nabla \cdot \vec{v}_g = 0 \tag{3}$$

$$\rho_g C_p \frac{DT}{Dt} + \frac{T}{\rho} \frac{\partial \rho_g}{\partial t}\bigg|_p \frac{Dp_g}{Dt} = \nabla \cdot (K \nabla T). \tag{4}$$

In above equations, ρ_g is the density of the gas, $\vec{v}_g$ is the velocity field within the bubble, T is the temperature, C_p is the specific heat at constant pressure, and K is the thermal conductivity of the gas. The above formulation can be modelled to first order accuracy by using an artificial thermal viscosity term μ_{th} defined in [8].

An analytical solution to (1) may be obtained by assuming small perturbations of the bubble radius, i.e. $R=R_0(1+x(t))$ where $x<<1$:

$$\ddot{x} + 2\beta_{tot}\dot{x} + \omega_0^2 x = -\frac{P_A e^{i\omega t}}{m} \tag{5}$$

where $m = \rho R_0^2 + 4(\mu_{th} + \mu)R_0/c$ is the effective mass, β_{tot} is the total damping and ω_0 is the natural frequency. The expressions for the viscous, thermal (using thermal viscosity), acoustic, interfacial and elastic damping obtained in this way are given respectively as

$$\beta_{vis} = 2\mu \Big/ \left(\rho R_0^2 + \frac{4(\mu_{th} + \mu)R_0}{c}\right), \tag{6a}$$

$$\beta_{P\_th} = 2\mu_{th} \Big/ \left(\rho R_0^2 + \frac{4(\mu_{th} + \mu)R_0}{c}\right), \tag{6b}$$

$$\beta_{ac} = \frac{(\omega R_0/c)}{1 + (\omega R_0/c)^2} \frac{\omega}{2} (\rho R_0^2) \Big/ \left(\rho R_0^2 + \frac{4(\mu_{th} + \mu)R_0}{c}\right), \tag{6c}$$

$$\beta_{int} = -\sigma/(\rho c R_0^2 + 4(\mu_{th} + \mu)R_0), \tag{6d}$$

$$\beta_{el} = 2G/(\rho c R_0 + 4(\mu_{th} + \mu)), \tag{6e}$$

where

$$\beta_{tot} = \beta_{vis} + \beta_{P\_th} + \beta_{ac} + \beta_{int} + \beta_{el}, \tag{6f}$$

and the natural frequency satisfies

$$\omega_0^2 = \left[3\kappa p_{\mathrm{g_0}} - \frac{2\sigma}{R_0} + 4G + \frac{\omega^2 \rho R_0^2}{1 + (\omega R_0/c)^2}\right] \Big/ m. \tag{7}$$

The expressions for the non-dimensional thermal (δ_{th}), elastic (δ_{el}) and acoustic (δ_{ac}) damping coefficients in A&H theory are given in Eq. (8), (9) and (10), respectively, of [2].

Scattering cross-section

Ainslie and Leighton [5] derived the equation for scattering cross-section of gas bubbles in water as

$$\sigma_s = \frac{4\pi R_0^2}{\left(\frac{\omega_0^2}{\omega^2} - 1 - 2\frac{\beta_0}{\omega}\varepsilon\right)^2 + \left(2\frac{\beta_0}{\omega} + \frac{\omega_0^2}{\omega^2}\varepsilon\right)^2}, \tag{8}$$

where $\varepsilon \equiv \omega R_0/c$ and β_0 is the total of the damping coefficients other than acoustic damping, this is also applicable to gassy sediments provided that correct expressions for β_0 and ω_0 are used.

Speed of sound and attenuation

The complex speed of sound in a bubbly mixture, c_{m}, is given by the expression [10, 11]

$$\frac{c^2}{c_{\mathrm{m}}^2} = 1 + 4\pi c^2 \int_0^\infty \frac{R_0 n(R_0)}{\omega_0^2 - \omega^2 + 2i\beta_{\mathrm{tot}}\omega} \mathrm{d}R_0 \tag{9}$$

where $n(R_0)\mathrm{d}R_0$ is the number of bubbles per unit volume with radii between R_0 and $R_0 + \mathrm{d}R_0$. Note that β_{tot} includes the elastic damping and the interfacial damping in addition to the other damping mechanisms for bubbles in water. Setting $c/c_{\mathrm{m}} = u - iv$ yields expressions for phase velocity V [10]

$$V = c / u, \tag{10}$$

and attenuation A in dB/cm

$$A = 8.6859\ (\omega v/c). \tag{11}$$

3. RESULTS

Application of the model to sediments

The proposed model can be applied to sediments with several advantages over A&H model such as having no sign ambiguities in the speed of sound formula and defining the higher order scattering coefficients. In this section, the results obtained by applying the model to the marine sediments are presented and compared to those obtained by A&H model. The formulation of the A&H model is not explicitly stated in this paper. Two different sediments types, ocean bottom silt and harbour mud, which were investigated by A&H will be examined here as well, to facilitate a direct comparison of two models. The mixture properties of ocean silt and harbour mud, as given in A&H, are repeated here in Table 1. Air bubbles (with polytrophic exponent $\kappa = 1.4$) are assumed to be embedded in sediments. Density of seawater is taken as 1030 kg/m^3 based on a salinity of 0.3%.

	Harbour Mud	Ocean Silt
Porosity	0.75	0.68
Shear Modulus (G)	1 GPa	250 GPa
Bubble void fraction (Γ)	0.075	0.068
Density (ρ)	1400 kg/m^3	1550 kg/m^3
Speed of sound (c)	1488 m/s	1552 m/s

Table 1: Model input parameters for harbour mud and ocean silt

Linear damping coefficients

In this section, damping constants of the current formulation are plotted for air bubbles in ocean sediments and compared to the predictions from the A&H theory by assuming $\delta \equiv 2\beta/\omega$, where δ is the non-dimensional and β is the dimensional damping coefficient.

In Fig. 1a and 1b, the linear damping coefficients for acoustic propagation in harbour mud are plotted as a function of frequency using Eq. (6) and A&H theory, respectively, and in silt, they are plotted in Fig. 1c and 1d, respectively. First of all, the two formulations show identical results for the acoustic damping. As Ainslie and Leighton [5, 7] recently noted, there existed a contradiction for many years for the expression of acoustic damping, A&H being among the few who have reported this issue and used the correct expression. For the thermal damping, the two formulations predict quite different results, especially in terms of the trend they show with increasing applied driving frequency. This is mainly due to the thermal models used. This paper uses a nonlinear, energy conservation formulation given in Eqns. (6) and (7) for the gas inside bubble whereas A&H modifies Devin [12]'s approach which solves energy conduction equation assuming variable gas stiffness which can take values between the isothermal and adiabatic, and intermediate stages.

It is interesting to note that the two formulations predict slightly different value for the applied frequency at which the minimum damping is observed (also where the maximum bubble resonance is expected). For instance, the resonance frequency of a 1 mm bubble in mud is ~8.5 kHz according to current theory and it is ~8.9 kHz according to A&H. Considering the fact that the use of thermal viscosity is not involved in the expression for the resonance frequency (thus neglecting such thermal effects on resonant frequency) the current theory predicts a cross-over of elastic and acoustic damping at around that frequency. The minimum damping in A&H occurs at a slightly higher frequency. This discrepancy can be explained as follows: A&H use complex dynamic shear modulus $G^* = G + iG'$ where the imaginary part G' is taken *a priori* as $G/5$. The expression for the resonance frequency involves the real part of G^* whereas the viscoelastic losses are evaluated by introducing a term analogous to viscosity,

$\mu_f \equiv G'/\omega$. The latter fact also explains the reason why the viscoelastic damping in Figs. 1b and 1d decreases with increasing driving frequency. The resonance frequency for a 1 mm bubble in silt is predicted ~127 kHz from both theories, which is slightly underestimated by A&H theory according to the minimum damping observed in Fig. 1d.

Viscous and interfacial damping values of current theory are not shown in the figures because they have much lower values compared to other sources of damping. However, the curves for β_{tot} include also those values in Fig. 1a and 1c.

Among all damping mechanisms, the elastic damping is apparently the most important. The two formulations use slightly different constitutive models which is the reason for the observed differences in elastic damping. However this observation is currently under further investigation.

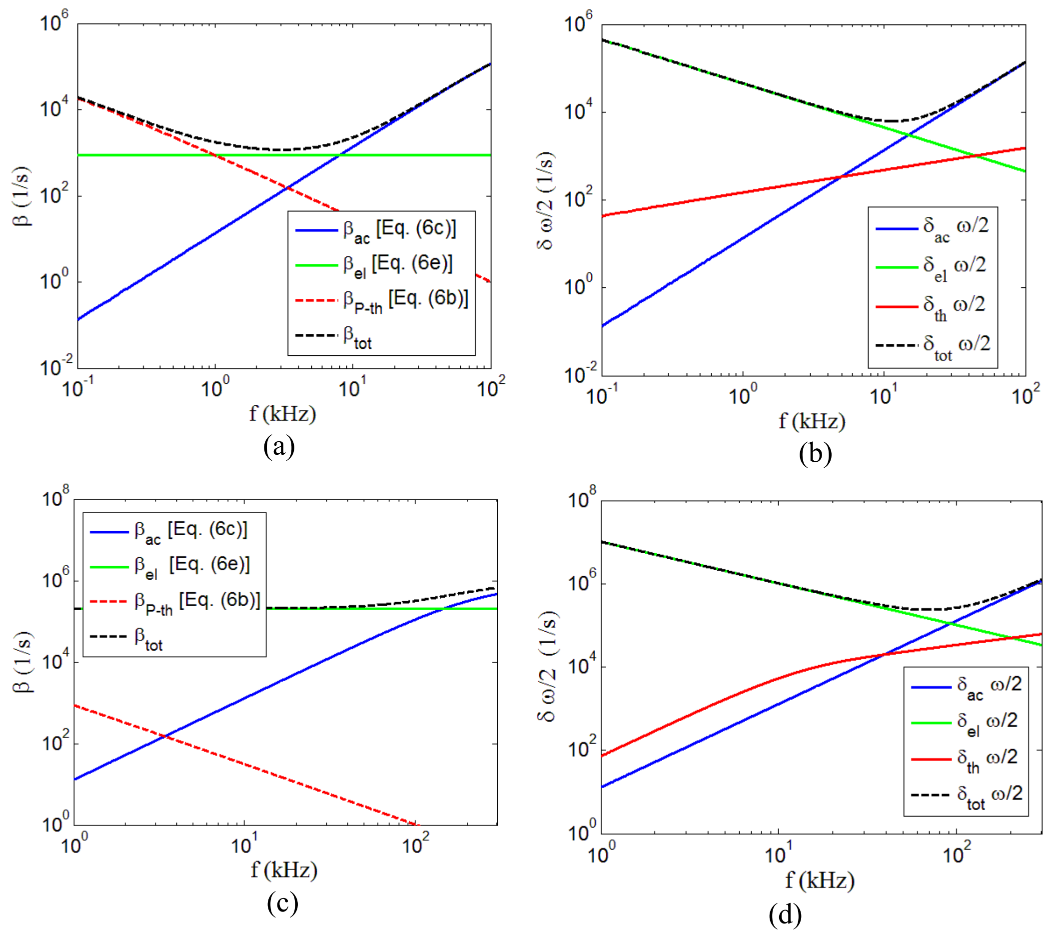


Fig.1: Damping coefficients vs driving frequency for a 1 mm equilibrium radius spherical bubble in mud using (a) Eq. (6) and (b) the A&H theory with dimensional coefficients (δ ω/2), and in ocean silt using (c) Eq. (6) and (d) the A&H theory with dimensional coefficients (δ ω/2). Viscous and interfacial damping coefficients are not plotted for clarity though β_{tot} includes them as in Eq. (6f).

Scattering cross section

Calculated values of scattering cross-section of bubbles in harbour mud and ocean silt, normalized with respect to their geometrical cross sections, are plotted in Fig. 2a and 2b, respectively. The current theory predicts a cross section almost two orders of magnitude higher for a bubble with radius approximately 1 mm in mud, whereas it predicts a smaller value for near resonant bubbles in ocean silt.

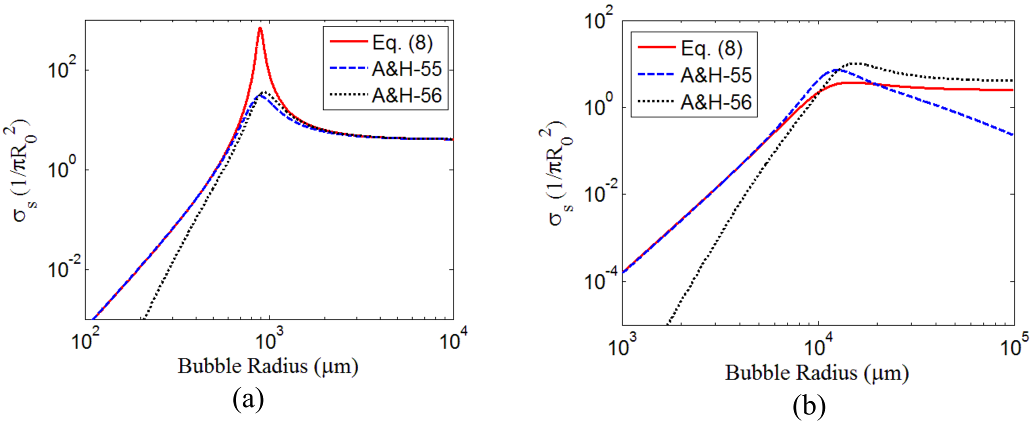


Figure 2: Scattering cross-sections for bubbles in (a) harbour mud and (b) in ocean silt, driven by a 10 kHz pulse, calculated using Eq. (8) and A&H theory.

Speed of sound

Computed values of speed of sound are plotted in Fig. 3 for bubbles in harbour mud for two different values of bubble void fraction specifically 0.075% and 7.5%. It is observed that for wave propagation in harbour mud the current formulation estimates reduced damping in the frequency range where bubble resonance effects are more significant. This is consistent with the results in Fig. 1a and 1b where the total damping, in the range 1-100 kHz, for a 1 mm bubble is predicted to be less than that in A&H.

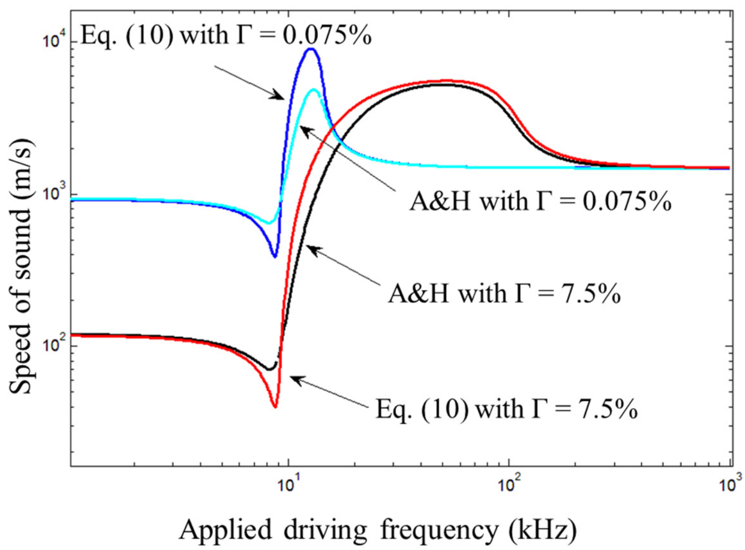


Figure 3: Speed of sound through a mono-disperse bubble population in harbour mud plotted using Eq. (10) and the A&H theory for bubble void fraction values of 0.075% and 7.5%.

In Fig. 4, the wave propagation through a mono-disperse bubble cloud in ocean silt is plotted for the gas void fraction values 0.068% and 6.8%. One may notice that the below resonance and the above resonance speed of sound values are similar to each other and that the transition near the resonance regime is smooth.

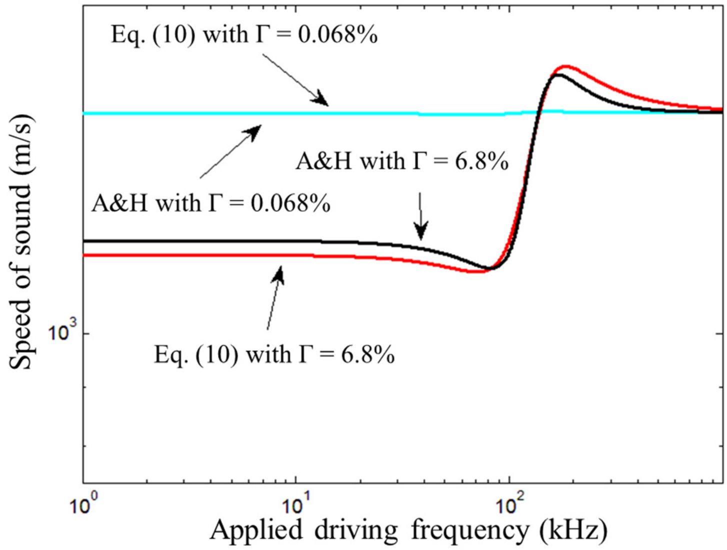
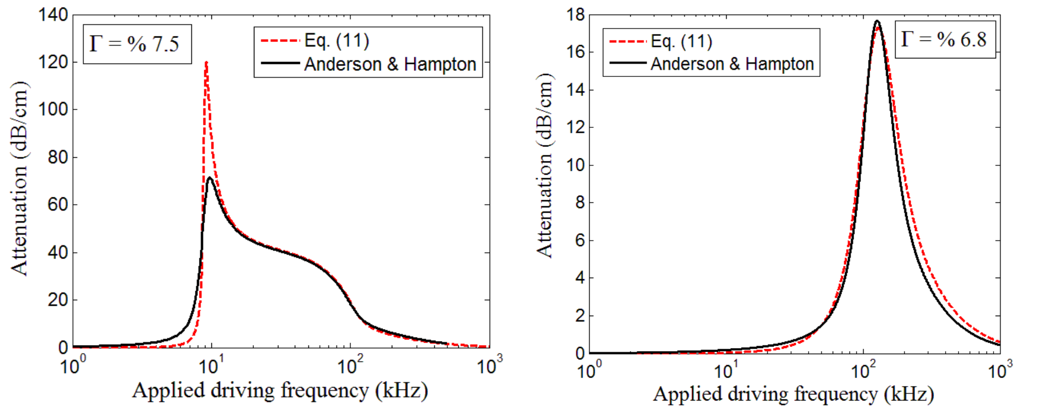


Figure 4: Speed of sound through a mono-disperse bubble population in ocean silt plotted using Eq. (10) and the A&H theory for bubble void fraction values of 0.068% and 6.8%.The results from both formulations for Γ = 0.068% are very similar to each other such that the two curves overlie one another.

Attenuation

Computed values of attenuation in gassy sediments are plotted in Fig. 5a and Fig. 5b for bubbles in harbour mud and ocean silt, respectively. The values are computed using the same values as in the previous plots for speed of sound, i.e. using Γ=7.5% for harbour mud and Γ=6.8% for ocean silt. The solid black lines in Fig. 5 give the result obtained by A&H (their Fig. 16 of Part II). It is observed that the current formulation predicts lower attenuation in harbour mud for pulse frequency of less than 10 kHz. For ocean silt the predictions from the two formulations are quite similar, this is a fact consistent with the results obtained for the speed of sound.



(a) (b)

Figure 5: Attenuation of acoustic wave through a mono-disperse bubble population (a) in harbour mud for Γ=7.5% and (b) in ocean silt for Γ=6.8%, plotted using Eq. (11) and the A&H theory.

4. CONCLUSIONS

The new formulation proposed in this paper has removed the ambiguities with the scattering cross-section and speed of sound expressions from the A&H formulations, and is not restricted to linear monochromatic bubble pulsations. This is important because many methods for detecting bubbles rely on bubble behaviours which are not restricted to these limitations [12, 13]. However when reduced to the linear regime to allow comparison with the predictions of the A&H formulation, the new method shows considerable and important differences in damping, bubble resonance, sound speed and attenuation. These differences need to be checked to ascertain if the new predictions are correct, because errors in the well-used A&H formulation would be inherent (though it is too soon to say whether they are also important) in many studies that use of the A&H formulation.

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