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Theory for the Dynamics of Acoustically-Excited Gas Bubbles in Porous Media (with Specific Application to Marine Sediment)

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FLUID DYNAMICS AND ACOUSTICS GROUP

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Authorized for issue by
Professor R J Astley, Group Chairman

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ABSTRACT

In marine sediments, bubbles of gas (often methane) exist in a variety of forms, ranging from near-spherical bubbles to coin- or slab-like gas inclusions. The phases present (gas, liquid and particulates) can co-exist in a number of forms, ranging from the presence of gas pockets contained wholly within the liquid interstices between particles, to bubbles where the solid phase forms an integral part of the effective bubble wall. All these types of bubbles can strongly influence the acoustic properties of the sediment, depending on the frequency of interest. Such an influence can be seen as an unwanted, inhomogeneous and poorly-characterised artefact in acoustic measurements. However if they are sufficiently understood, the effect of the bubbles on the sound field can be seen as providing acoustic tools for monitoring the gas bubble population, in order to provide key information for civil engineering projects, petrochemical surveying or climate modelling.

This report attempts to advance that understanding, and thereby facilitate the provision of such tools. The assumption that the bubbles interact with the sound field through volumetric pulsation and remain spherical at all times is maintained. Current bubble models for sediments assume that the bubbles exist only in the water phase or neglect the shear properties of the medium. This paper outlines a theory which is not so restricted: it does not require the above assumptions and can be applied to model any elastic or poroelastic material provided that the thermal effects are not important (i.e. provided that thermal phenomena occur over much larger spatial and temporal scales than do the acoustical processes). Moreover, the theory allows for the nonlinear pulsation of bubbles, and so can model amplitude-dependent effects which linear theories cannot predict. This feature is particularly important for gassy sediments, where the high levels of attenuation of necessitate the use of sound fields of sufficient amplitude to generate nonlinear effects. Moreover, it provides a numerical model which will be useful to those who use specifically nonlinear techniques (such as the generation of sum-and difference-frequencies) to measure gas bubbles in sediment.

LIST OF SYMBOLS

c_s	the sound speed in the host medium, for compressional waves
f	frequency
$P(t)$	time varying acoustic pressure
p_L	the liquid pressure outside the bubble wall
p_∞	the sum of all steady and unsteady pressures at $r \to \infty$
p_σ	surface tension pressure
p_v	vapour pressure
p_0	the static pressure in the liquid just outside the bubble wall
p_g	gas pressure in the bubble
p_{g0}	initial gas pressure in the bubble
Q_s	Q-factor of shear waves
R	time dependant bubble radius
R_0	equilibrium bubble radius
$\dot{R}$	bubble radius velocity
r	radial distance from the bubble centre in a spherical coordinate system

u — radial velocity of the medium surrounding a bubble

T_{rr} — the radial component the stress tensor in the host medium (gas free sediment)

$T_{rr\_elastic}$ — the elastic part of the radial component the stress tensor in the host medium (gas free sediment)

T_{rr_LA} — the elastic part of the radial component the stress tensor in the host medium (gas free sediment), when large amplitudes are considered.

T_{rr_SA} — the elastic part of the radial component the stress tensor in the host medium (gas free sediment), when small amplitudes are considered

$T_{rr\_viscous}$ — the viscous part of the radial component the stress tensor in the host medium (gas free sediment)

α_s — shear wave attenuation

κ — gas polytropic exponent

Δ — denotes incremental changes

ε_{rr} — the radial component of the strain tensor

λ_s and G_s — Lamé constants of bubble host medium (effective medium constants)

G_s^* — complex medium shear modulus (second Lamé constant)

G_1, G_2	spring constants of reological model
G'	constant of reological model
ρ_s	density of host medium
ρ_w	density of water
ρ_g	density of gas phase
σ	the surface tension
η_s	shear viscosity of the host medium
K_s	effective bulk modulus of saturated sediment
K_g	bulk modulus of sediment grains
K_w	bulk modulus of water
K_{frame}	bulk modulus of sediment drained frame
ξ	porosity
Υ	auxiliary variable in Gassmann equations

1 Introduction

Over the past few decades, there has built up a considerable body of work in the literature on the theory of acoustic propagation in marine sediment [*Biot* 1956a, 1956b; *Hampton,* 1967; *Hamilton* 1971; *Stoll*, 1972; *Hovem and Ingram* 1979; *Kibblewhite*, 1989; *Chotiros*, 1995; *Richardson and Briggs*, 1996; *Buckingham*, 1997, 1998, 2000; *Williams* 2001; *Thorsos et al.*, 2001]. However incorporation of gas bubbles into such theories is done with the inclusion of assumptions which severely limit the applicability of those models to practical gas-laden marine sediments [*Leighton*, 2007a]. Whilst there have been important advances, nevertheless the current theories require assumptions to which many gassy sediments do not conform.

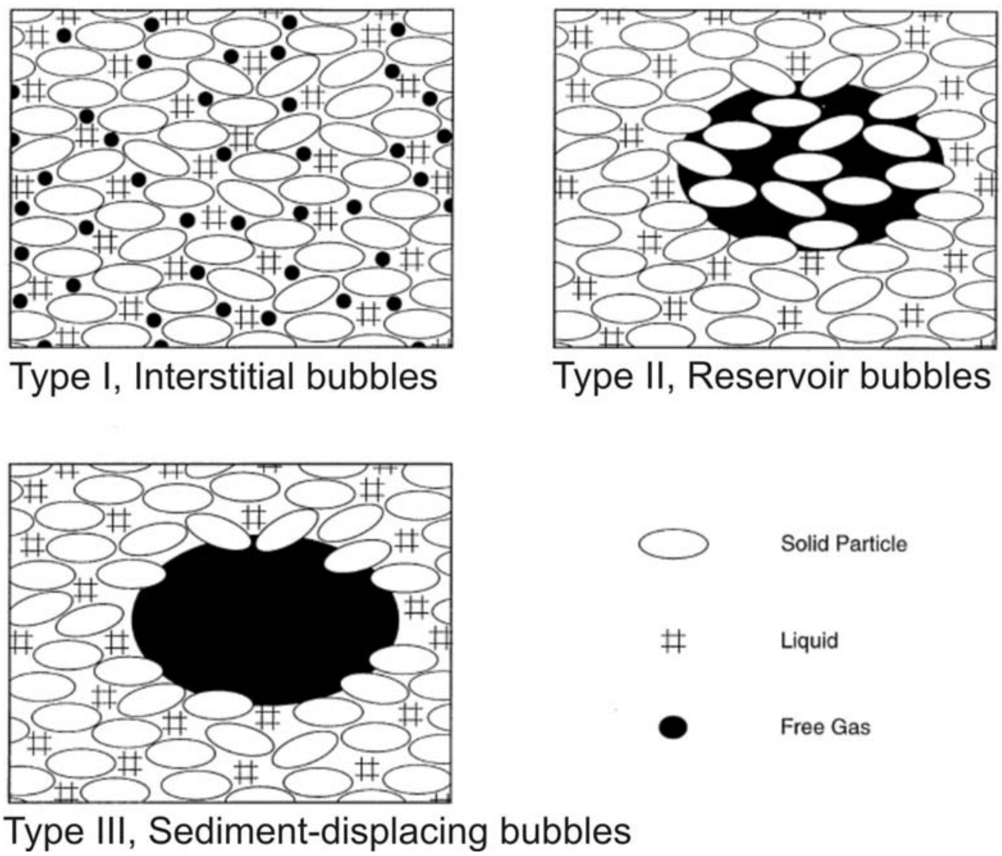

Figure 1. The characterisation by *Anderson et al.* [1998] of bubbles in gassy sediments, into three types. Type I bubbles ("interstitial bubbles"), are necessarily small, and may be free-floating or adhering to one or more solid particles, or stabilized within a crevice within a particle. In type II ("reservoir bubbles"), the gas pockets displace liquid but do not significantly affect the distribution of the solid particles. In type III ("sediment-displacing bubbles") the gas pockets displace both liquid and sediment, with a bubble wall formed from material which is substantially similar to the bulk sediment. Reproduced from *Anderson et al.* [1998].

In marine sediments, bubbles of gas (often methane) exist in many locations [*Judd and Hovland,* 1992; *Fleischer et al.,* 2001], and occur in a variety of forms, ranging from near-spherical bubbles to coin- and slab-like gas inclusions [*Abegg and Anderson,* 1997; *Anderson et al.,* 1998]. The phases present (gas, liquid and particulates) can co-exist in a number of forms, ranging from gas pockets contained wholly within the liquid interstices between particles, to bubbles where the solid phase forms an integral part of the effective bubble wall. *Anderson et al.* [1998] categorized three types of gas bubble in marine sediment, shown in Figure 1. It is of course important to appreciate that the 'bubble' is more than the pocket of gas, as in many circumstances the phenomenon in question is dominated by the coupling of that gas to the surrounding host medium (which provides the majority of the inertia for acoustic considerations [*Leighton et al.*, 2000], and which provides the reservoir of material if chemical or mass flux effects are important). Shear and thermal boundary layer occur at the bubble wall, as do depletion layers [*Birkin et al.,* 2004].

All these types of bubbles can strongly influence the acoustic properties of the sediment, depending on the frequency of interest. Such an influence can be seen as an unwanted, inhomogeneous and often unpredictable artefact in acoustic measurements [*Richardson and Briggs*, 1996; *Anderson et al.,* 1998; *Leighton and Evans*, 2007]. However if they are sufficiently understood, the effect of the bubbles on the sound field can be seen as providing acoustic tools for monitoring the gas bubble population, in order to provide key information for civil engineering projects [*Wheeler and Gardiner*, 1989; *Sills et al*., 1991], petrochemical and geophysical surveying [*Leighton*, 2007a] or the assessment of the sources and sinks of hydrocarbons and the implications for the environment [*Fleischer et al.,* 2001; *Kruglyakova et al*., 2002; *Judd*, 2003].

This report attempts to advance that understanding, and thereby facilitate the provision of such tools. Current bubble models for sediments modify in an *ad hoc* way the natural frequency and damping of the bubble [*Andersonand Hampton*, 1980] or neglect the shear properties of the medium [*Boyle and Chotiros,* 1998]. This paper outlines a theory which is not so restricted: it does not require the above assumptions and can be applied to model any elastic or poroelastic material provided that the thermal effects are not important (i.e. provided that thermal phenomena occur over much larger spatial and temporal scales than do the acoustical effects).

This derivation maintains the assumption that the bubbles interact with the sound field in the long-wavelength limit through volumetric pulsation and remain spherical at all times, and that the void fraction is sufficiently low that multiple scattering can be neglected. These are clearly important assumptions that will be violated on occasion, and will need addressing in the future. However for those sediments which conform to the above assumed conditions, the new approach overcomes other common and limiting assumptions. An earlier paper [*Leighton,* 2007a] presented the first stage in developing the new theory. It described how the approach overcame the need for common assumptions in the dynamics of gas bubbles in sediments, including:

- the assumption of quasi-static bubble dynamics, which effectively limits applicability to cases where the frequency of insonification is very much less than the resonances of any bubbles present. Furthermore, it eliminates from the theory all bubble resonance effects, which often of are overwhelming practical importance when marine bubble populations are insonified. This limitation becomes more severe as gas-laden marine sediments are probed with ever-increasing frequencies.
- the assumption of monochromatic steady-state bubble dynamics, where the bubbles pulsate in steady state. This is inconsistent with the use of short acoustic pulses to obtain range resolution.
- the assumption of monodisperse bubble populations, which is inconsistent with the wide range of bubble sizes that are found in marine sediments.
- the assumption of linear bubble pulsations, which becomes increasingly questionable as acoustic fields of increasing amplitudes are used to overcome the high attenuations. This is particularly appropriate for gassy sediments, where the high levels of attenuation of necessitate the use of sound fields of sufficient amplitude to generate nonlinear effects. The new approach allows for the nonlinear pulsation of bubbles, and so can predict amplitude-dependent effects which linear theories cannot predict [*Leighton et al.*, 2004, 2008]. Furthermore, it provides a numerical model which will be useful to those who

use specifically nonlinear techniques (such as the generation of sum- and difference-frequencies; [*Didenkulov et al.*, 2001; *Ostrovsky et al.,* 2003]) to measure gas bubbles in sediment [*Karpov et al*., 1996; *Tegolwski et al*., 2006; *Leighton et al.*, 2008].

As with the earlier paper [*Leighton*, 2007a], the current manuscript is based on consideration of the dynamics of a given bubble, and presents that next stage of development of the theory. *Leighton* [2007a] concludes that the next stage of development of the theory is to overcome the assumption that the medium outside of the bubble is incompressible. That development will now be described.

2 The dynamics of a single gas bubble in an elastic medium

2.1 The Keller-Miksis equation

The approach combines the general form of the Keller-Miksis equation with the linear Voigt model for viscoelastic solids, in the manner applied by *Yang and Church* [2005] to study the dynamics of bubbles in soft tissue. Equations from the Herring-Keller-Miksis family incorporate acoustic radiation losses through an assumed characteristic compressional wave speed in the effective host medium at the bubble wall (c_s). Although the general theory for wave propagation in porous media predicts the existence of two compressional waves [*Biot*, 1956a, 1956b] here the second (slow) compressional wave is neglected as explained in section 2.3. If, in these equations, this sound speed is set equal to infinity, they revert to the Rayleigh-Plesset equation which was the basis for the earlier paper [*Leighton*, 2007a]. The particular equation of the Herring-Keller-Miksis family used by *Yang and Church* [2005] relates the bubble radius R and wall velocity $\dot{R}$ to the inertial, forcing and dissipative terms as follows:

$$\left(1-\frac{\dot{R}}{c_{\mathrm{s}}}\right)R\ddot{R}+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3c_{\mathrm{s}}}\right)=\left(1-\frac{\dot{R}}{c_{\mathrm{s}}}\right)\frac{1}{\rho_{\mathrm{s}}}\left[p_{\mathrm{L}}(t)-p_{\infty}(t)\right]+\frac{R}{\rho_{\mathrm{s}}c_{\mathrm{s}}}\frac{\mathrm{d}\left(p_{\mathrm{L}}(t)-p_{\infty}(t)\right)}{\mathrm{d}t} \tag{1}$$

It should be noted that the following formulation is also valid:

$$\left(1-\frac{\dot{R}}{c_{\mathrm{s}}}\right)R\ddot{R}+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3c_{\mathrm{s}}}\right)=\left(1-\frac{\dot{R}}{c_{\mathrm{s}}}\right)\frac{1}{\rho_{\mathrm{s}}}\left[p_{\mathrm{L}}(t)-p_{\infty}(t)\right]+\frac{R}{\rho_{\mathrm{s}}c_{\mathrm{s}}}\frac{d\left(p_{\mathrm{L}}(t)\right)}{dt} \tag{2}$$

[*Prosperetti and Lezzi*, 1986]. The pressure p_{L} at the bubble wall can be evaluated in a number of ways, one of which (as described in the earlier paper [*Leighton,* 2007a] is to equate it to the sum of the gas pressure (p_{g}), surface tension pressure ($p_{\sigma}=2\sigma/R$, where σ is the surface tension) and stress tensor of the near field (T_{rr}):

$$p_{\mathrm{L}}=p_{\mathrm{g}}-p_{\sigma}+T_{\mathrm{rr}}(R,t) \tag{3}$$

The pressure far from the bubble (at range $r\rightarrow\infty$ from the bubble centre) will be represented by p_{∞}. As described in earlier papers [*Church,* 1995; *Yang and Church*, 2005; *Leighton* 2007a], it can be found from the static pressure p_0, a function $P(t)$ which is the time-varying external acoustic pressure at $r\rightarrow\infty$, and the stress tensor of the near and far field, given by $\int_R^{\infty}(T_{\mathrm{rr}}/r)dr$:

$$p_{\infty}(t)=p_0+P(t)+T_{\mathrm{rr}}(R,t)+3\int_R^{\infty}(T_{\mathrm{rr}}/r)dr \tag{4}$$

Similarly, the time-varying gas pressure in the bubble can be found through assumption of a polytropic gas law:

$$p_{\mathrm{g}}=p_{\mathrm{g}0}\left(\frac{R_0}{R}\right)^{3\kappa}, \tag{5}$$

Assuming that there are no residual stresses in the surrounding medium, when the bubble is at its equilibrium radius R_0, the gas pressure is $p_{\mathrm{g}0}$:

$$p_{g0} = p_0 + \frac{2\sigma}{R_0}. \tag{6}$$

2.2 Evaluating viscoelastic stress components

In this section, the viscoelastic stress components that may be used in equation (1) are evaluated. Because of the assumed spherical symmetry, the radial stress component depends only on the radial deformation of the medium at distance r from the bubble centre. Assuming Hookean medium behaviour [*Reismann and Pawlik*, 1980], the elastic part of the tensor component T_{rr} is expressed in terms of the radial strain ε_{rr} as [*Reismann and Pawlik*, 1980]:

$$T_{rr\_elastic} = (\lambda_s + 2G_s)\frac{\partial \varepsilon_{rr}}{\partial r} + 2\lambda_s \frac{\varepsilon_{rr}}{r}. \tag{7}$$

where λ_s is the first Lamé constant and G_s is the modulus of rigidity (the dynamic shear modulus, or second Lamé constant) [*Church* 1995; *Yang and Church*, 2005; *Leighton* 2007a]. In incompressible conditions, spherical divergence or convergence of particle velocity gives the radial strain ε_{rr} through conservation of mass [*Leighton* 2007a] as follows:

$$u = \left(R/r\right)^2 \dot{R} \tag{8}$$

where u is the radial velocity of the medium surrounding a bubble.

Assuming small deformations, the viscous dissipation is proportional to the viscosity of the medium [*Church*, 1995]. As a result, the viscous part of the radial stress tensor can be expressed as function of the shear viscosity η_s of the host medium outside of the bubble wall:

$$T_{rr\_viscous} = 2\eta_s \frac{\partial \varepsilon_{rr}}{\partial t}. \tag{9}$$

The radial strain rate is approximated from equation (8) according to:

$$\frac{\partial \varepsilon_{\mathrm{rr}}}{\partial t} \cong \frac{\partial u}{\partial r} \Rightarrow$$
$$\frac{\partial \varepsilon_{\mathrm{rr}}}{\partial t} = -2\frac{R^2}{r^3}\dot{R} \tag{10}$$

Radial strain for small amplitudes

For small amplitudes, the radial strain component is approximated as follows:

$$\varepsilon_{\mathrm{rr}} \cong \frac{\Delta r}{r} = \left(\frac{R}{r}\right)^2 \frac{\Delta R}{r} \Rightarrow$$
$$\frac{\Delta r}{r} = \left(\frac{R}{r}\right)^2 \frac{\left(R - R_0\right)}{r} \tag{11}$$

Hence the expression of instantaneous strain of equation (11) can be written as:

$$\varepsilon_{\mathrm{rr}} \cong \frac{R^2}{r^3}\left(R - R_0\right). \tag{12}$$

Substituting the expression of equation (12) into (7) leads to cancellation of the terms containing the first Lamé parameter in equation (7) and the elastic component $T_{\mathrm{rr\_elastic}}$ becomes:

$$T_{\mathrm{rr\_elastic}} = 2G_{\mathrm{s}} \frac{\partial \varepsilon_{\mathrm{rr}}}{\partial r}. \tag{13}$$

Taking both the elastic and lossy characteristics of the medium together (equations (13) and (9) respectively) [*Church,* 1995], the radial component of the stress tensor is:

$$T_{\mathrm{rr\_SA}} = -4\frac{R^2}{r^3}\left(G_{\mathrm{s}}\left(R - R_0\right) + \eta_{\mathrm{s}}\dot{R}\right). \tag{14}$$

and the integral for the medium in equation (4) can be evaluated with equation (14) [*Leighton*, 2007a]:

$$3\int_R^{\infty} \frac{T_{\mathrm{rr}}}{r} dr = -4G_{\mathrm{s}} \frac{R - R_0}{R} - 4\eta_{\mathrm{s}} \frac{\dot{R}}{R}. \tag{15}$$

Radial strain for large amplitudes.

For large amplitude pulsations equation (12) is not valid. In this case the radial strain is better approximated by equation (16) according to *Yang and Church* [2005]:

$$\varepsilon_{\mathrm{rr}} = -\frac{2}{3r^3}\left(R^3 - R_0^3\right). \tag{16}$$

The viscous part of the radial strain is by definition the strain rate $\partial\varepsilon_{\mathrm{rr}} / \partial t$. Differentiation of equation 16 with respect to time results in the expression of equation (9). Hence taking both the lossy and elastic characteristics of the medium together the radial component of the stress tensor becomes:

$$T_{\mathrm{rr\_LA}} = -\frac{4}{r^3}\left(-\frac{G_{\mathrm{s}}\left(R^3 - R_0^3\right)}{3} + \eta_{\mathrm{s}} R^2 \dot{R}\right). \tag{17}$$

The stress tensor integral of equation (4) results from integration of (17) at R according to :

$$3\int_R^{\infty} \frac{\mathrm{T}_{rr}}{r} dr = -\frac{4}{3} G_{\mathrm{s}} \frac{R^3 - R_0^3}{R^3} - 4\eta_{\mathrm{s}} \frac{\dot{R}}{R}. \tag{18}$$

In both the small (equation (15)) and large amplitude (equation (18)) case, the radial stress can be expressed also in terms of radial strain as:

$$3\int_R^{\infty} \frac{T_{\mathrm{rr}}}{r} dr = -4\left(G_{\mathrm{s}}\varepsilon_{\mathrm{rr}} + \eta_{\mathrm{s}} \frac{\partial \varepsilon_{\mathrm{rr}}}{\partial t}\right). \tag{19}$$

The bubble model for large amplitudes in porous materials

Since here we are interested at large amplitudes, the tensor in the form of equation (18) will be used rather than from equation (15). The model we present in this report results from substitution of equations (3), (4), (5) and (18) in (2):

$$\left(1-\frac{\dot{R}}{c_\mathrm{s}}\right)R\ddot{R}+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3c_\mathrm{s}}\right)= \left(1-\frac{\dot{R}}{c_\mathrm{s}}\right)\frac{1}{\rho_\mathrm{s}}\left[p_\mathrm{g}-\frac{2\sigma}{R}-p_0-P(t)\right]+ \frac{R}{\rho_\mathrm{s}c_\mathrm{s}}\left[\left(-3\kappa p_\mathrm{g}+\frac{2\sigma}{R}\right)\frac{\dot{R}}{R}-4G_\mathrm{s}\frac{R_0^3\dot{R}}{R^4}-4\eta_\mathrm{s}\left(\frac{\ddot{R}}{R}-\frac{\dot{R}^2}{R^2}\right)\right] \tag{20}$$

2.3 Definition of model parameters

Equation (20) will be used for predicting the nonlinear time-dependent dynamics of a single bubble in a gassy marine sediment. This approach is identical to that provided for tissue by *Yang and Church* [2005]. However the bubble dynamics as described by equation (20) is restricted to media that behave as Voigt solids. In addition, the meanings of the model parameters $\rho_\mathrm{s}, G_\mathrm{s}$ and η_s are different to the interpretations for tissue given by *Yang and Church* [2005], and the sound speed entering equation (1) needs further consideration. This section is devoted to the adaptation of these parameters for sediments.

Marine sediments can be regarded as a two-phase porous continuum which is composed of a grain skeleton and pore fluid [*Biot*, 1956a, 1956b]. An alternative approach is to regard the sediment as a continuous medium (frame plus fluid) with effective properties; an example of this approach is given by *Williams* [2001]. In this effective medium, the volume of a particle must be much smaller than the entire domain but larger than the pore and the grain size to permit meaningful statistical averages of the properties in question [*Bear*, 1972]. Therefore this treatment is applicable to the Type III bubbles of Figure I, which Anderson *et al*. [1998] (with some qualifications) indicate are probably the most common form of marine sediment bubble observed.

The following discussion seeks to assign effective medium values for key parameters which the previous sections showed to be relevant to predicting the bubble dynamics. It is important to note that these parameters are here defined as those attributed to a bubble-free medium surrounding a Type III bubble. The presence of bubbles in the medium will cause some of the parameter values to be potentially frequency- and amplitude-dependent [*Leighton*, 2007a].

Under the effective medium assumption, the mixture low applies and the density entering the equation (20), is the effective density, which is defined as:

$$\rho_{\mathrm{s}} = \xi \rho_{\mathrm{w}} + (1-\xi)\rho_{\mathrm{g}} . \tag{21}$$

where ρ_{w} is the water density, ρ_{g} is the sediment grain density, and $\xi = V_{\mathrm{w}} / V_{\mathrm{s}}$ is the porosity (given that a volume V_{w} of pore water is contained within a volume V_{s} of the bubble free sediment). The sound speed c_{s} entering equation (20) can be interpreted as the compressional wave speed in the bubble free medium:

$$c_{\mathrm{s}} = \sqrt{\frac{K_{\mathrm{s}} + \frac{4}{3} G_{\mathrm{s}}}{\rho_{\mathrm{s}}}} , \tag{22}$$

where G_{s} and K_{s} are, respectively, the shear and bulk moduli of that effective medium. The bulk modulus K_{s} can be expressed as function of the grain bulk modulus K_{g}, the fluid bulk modulus and the solid frame (drained) bulk modulus K_{frame} [*Gassmann*, 1951]:

$$K_{\mathrm{s}} = K_{\mathrm{g}} \frac{K_{\mathrm{frame}} + \Upsilon}{K_{\mathrm{g}} + \Upsilon} . \tag{23}$$

where:

$$\Upsilon = \frac{K_w\left(K_g + K_{frame}\right)}{\xi\left(K_g + K_w\right)}. \quad (24)$$

Equations (21) and (22) assume no relative motion between the fluids and the skeletal frame, thus they are a low frequency approximation for the effective density and sound speed of the medium surrounding the bubble. According to Biot theory [*Biot*, 1956a, 1956b], in the low frequency regime the skeletal frame (which consists of the solid grains and the intersitial fluid) move together. In contrast, in the high frequency regime they move out of phase, so that at high frequencies the inertia depends on the coupling between solid and fluid. Equation (22) is the Biot fast wave low- frequency asymptotic hence this approximation is valid for that frequency regime where the Biot theory predicts P-waves with constant value. If marine sediments of high permeability are considered (i.e. very fine slits and clays), the relative fluid motion is limited and therefore the Biot low frequency regime spans over a large frequency range. Under the assumption of the limited relative motion, the slow wave can be neglected and the fast wave can be approximated from equation (22) in the low frequency range. An example is shown with the simulations of Figure 2 where the fast P-wave velocity is plotted according to Biot theory (after the modifications of Stoll [1]). The values used for both models are shown in table 1.

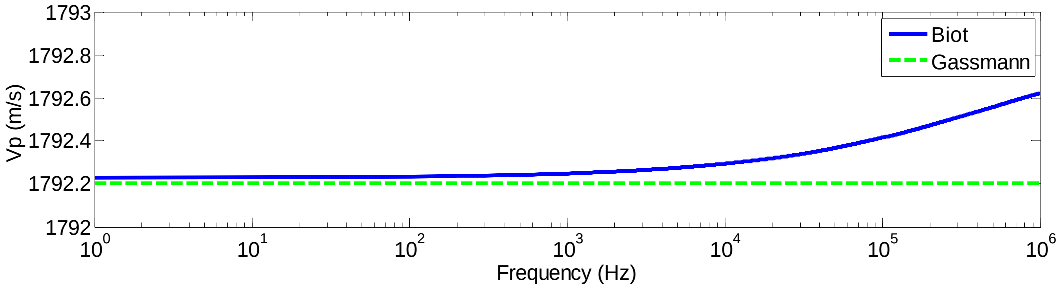

Figure 2 P wave sound speed according to Biot theory (solid line) and Gassmann approximation (dashed line). The values used for the simulation are typical for sediments with high silt content , parameter values shown in table 1.

[1] Biot code written by A.I.Best

The results of figure 1 show that Gassmann's equation can be a good approximation over a wide frequency range in the low frequency regime. For this example, frequencies above 1 kHz render the global flow phenomena important and therefore Gassmann's approximation fails. An additional requirement is the porosity remains constant.

Table 1 Model parameters for the models of figure1.

Fluid density ρ_s	1000 kg/m^3	Frame bulk modulus	2 GPa
Fluid bulk modulus	2.2 GPa	Frame shear Modulus	0.7 MPa
Fluid shear viscosity	0.001 Pas	Mean grain size (microns)	5
Grain density	2650 Kg/m^3	Porosity	0.6
Grain Bulk Modulus	36.5 GPa		

The definition of shear modulus is less complicated since fluids cannot carry shear forces. Hence the effective shear modulus is assumed to be equal to the skeletal frame shear modulus [*Gassmann*, 1951], which is the same for drained and saturated conditions [*Gassmann*, 1951]. Additionally only one shear wave has ever been observed and Biot theory predicts the existence of one shear wave. The effective shear modulus, entering the bubble model, is therefore an inherent property of the matrix material, and it can be either inferred from geotechnical properties of the sediment or measured in situ by shear vane testing.

It is more difficult to establish the physical correspondence between the effective medium viscosity η_s and the sediment viscosity (a rheological property of the sediment). The effective viscosity in the equation (20) represents energy dissipation during shear deformation of the medium. Hence the parameter η_s can be inferred from shear wave attenuation provided that the medium is behaving in shear according to the Voigt viscoelastic model in the frequency rage of interest. He following paragraph discusses the modelling of shear losses in sediments.

The question is if this model (Figure 3(a)) can describe the velocity and attenuation of acoustic waves propagating in bubble free sediments. At this point the kind of sediment under consideration must be distinguished because observations cannot be generalised. In this report we are mainly interested in muddy sediments. That is sediments with high silt or clay content which are characterised by high permeability.

Referring to the Voigt model of figure 3(a), considerable body of measurements in this field has shown that the Voigt model is *not* adequate (see page 4046 of *Hamilton et al.* [1970] for an extensive discussion on this topic) for silts and clays because a linear dependence of the shear wave attenuation with frequency has been observed (see for example [*Wood and Weston*, 1964]. According to these experimental observations the model of (Figure 2(b)) is more adequate to describe the shear wave velocity and attenuation because a rather constant –Q behaviour has been observed in muddy sediments. The model of Figure 2(b) allows for this by fitting the imaginary part of the shear modulus.

A wave propagating in a Voigt viscoelastic medium (i.e. according to the model of Figure 3(a)) undergoes shear wave attenuation (α_s, in Np m^{-1}) which is proportional to the square of the frequency f (α_s [Np/m] $\propto f^2$) [*Kolsky,* 1953]. This is equivalent to stating that the logarithmic decrement is proportional to the frequency ($1/Q_s \propto f$), where the Q_s-factor relates the shear wave speed to the attenuation ($Q_s = \pi f /(\alpha_s \, c_s)$ [2]). There are a few works that conform to this model: The measurements of *Stoll* [1985] show regimes where there is linear dependence of the logarithmic decrement with the frequency (see Figure 3 of *Stoll* [1985]). Also the work of *Leurera and Brown* [2008] shows that linear dependence of the logarithmic decrement as function of frequency can be observed at high frequencies. According to these measurements, the shear wave showed a logarithmic decrement proportional to *f* in sediment with high clay content. An explanation to these measurements is that the sediment has a high intrinsic attenuation because of the 'house-of-cards' frame structure in combination with clay content and high porosity [*Stoll and Bautista,* 1998].

[2] Muddy sediments exhibit in general no dispersion

The results of *Stoll* [1985] and *Leurera and Brown* [2008] are two of the few examples that show that the effective viscosity of the bubble model *cannot* be fitted form the modified Voigt model (figure 3 (b))'. The constant-Q behaviour, which has been mostly observed, can be modelled with the viscoelastic model shown in figure 3(b) which will be further considered. However it must be noted that this model has acausal behaviour if not purely sinusoidal excitations are considered. Hence, the Kelvin-Voigt model (shown in Figure 3 (c)) [*Kolsky*,1953] is a more rigorous viscoelastic model to describe the behaviour of the background material. Tuning of the model parameters (i.e. the dashpot G' and spring constants G_1, G_2) can lead to a good approximation of the experimental results. The main drawback is that the constitutive reological of figure 3 (c) requires apparent material properties. That is the spring constants G_1, G_2 and dashpot constant G' in Figure 3(c) must be coupled to the geotechnical sediment properties. That is, the model constants should be expressed as material properties: shear modulus, bulk modulus etc (see *Schanz, and Cheng* [2001] and the extensive discussion in *Abousleiman et al.* [1996]).
For this reason this report proceeds with the use of the model (figure 3(b)) which is in accordance with the experimental observations that the attenuation increases linearly with the frequency α_s [Np/m] $\propto f$. The model parameters are described below:

The visco elastic operator $G_s + \eta_s \frac{\partial}{\partial t}$ of equation (21) in the time domain is assumed to have a correspondent form in the frequency domain:

$$G_s^* = G_s + jG_s', \tag{25}$$

and the corresponding Q-factor for this viscoelastic material model is [*Whorolow*, 1992]:

$$Q = \frac{G_s}{G_s'}. \tag{26}$$

According to this model, the dynamic shear modulus is assumed to be a complex number with constant real and imaginary parts. The real part is equal to the dynamic shear modulus. The correspondence between the imaginary part and the medium viscosity is found by applying a Laplace transform (More details on this approach can be found in *Whorolow* [1992]:

$$\eta_s = \frac{G_s^{'}}{2\pi f} . \tag{27}$$

According to this model, *at one particular frequency* the steady state behaviour of the linear medium corresponds to the behaviour of a single Voigt element shown in Figure 2(a) [*Whorolow*, 1992]. For a frequency range there is no correspondence between the two models.

Considering single frequency insonification, for the bubble model, equation (20), the effective viscosity η_s is found from equation (27). The value of $G_s^{'}$ equals to the best-fit value the material, sediment in this case. This ‘value-fit' value will result from an iterative process from attenuation measurements. Predictions and applications of this model is a topic of current research.

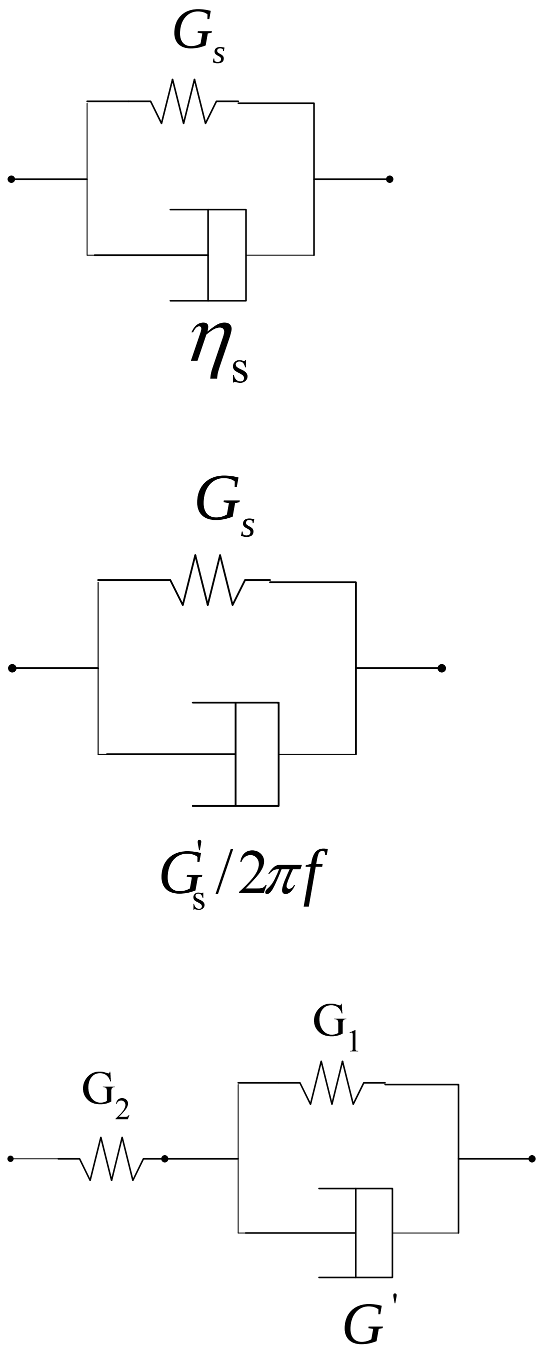

Figure 3. Reological models: (a) Voigt model with constant parameters; (b) Modified Voigt model with the dashpot parameter inversely proportional to frequency (equation (27)); and (c) Voigt model with auxiliary spring (Kelvin-Voigt model).

Conclusions

This paper has outlined a progression from the theory for a single bubble in a gassy marine sediment outlined by *Leighton* [2007a]. As with *Leighton* [2007a], the bubble dynamics need not be assumed to be quasi-static, monochromatic, steady-state or linear. However whereas *Leighton* [2007] assumed that the medium outside of the bubble wall was incompressible, the current paper includes finite compressibility (and therefore radiation losses) by adapting the Keller-Miksis equation, using an approach of *Yang and Church* [2005]. As outlined by *Leighton* [2007a], in the nonlinear regime this approach is more appropriate than other options, such as artificially enhancing the shear viscosity to account for radiation and thermal losses. The analysis is applicable to marine sediments where global flow phenomena are limited. Further work is expected to refine the estimates of the key parameter values.

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