

## AN EXPERIMENTAL INVESTIGATION OF ACOUSTIC PENETRATION INTO SANDY SEDIMENTS AT SUB-CRITICAL GRAZING ANGLES

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*This paper outlines experiments into the detection of objects of low acoustic contrast buried in submerged sediment using high frequency sound (50 - 120 kHz). Particular emphasis was given to the detection of small diameter (cm scale), cylindrical targets buried to a depth of up to 1 metre below the sediment from a range of 1 metre above. The lack of an experimentally verified model to describe the transmission of acoustic waves into the sediment, where the grazing angle in the water column is sub-critical, was a complicating factor.*

*The elastic theory of wave propagation predicts that, for a measured sound speed in the region of  $1700 \text{ m s}^{-1}$  in a sandy sediment, the critical grazing angle is approximately  $28^\circ$ . At shallower angles total internal reflection is predicted. Recent experimental measurements contradict this view. One explanation stems from the Biot theory of acoustic propagation in porous sediments. It has been postulated that, although the type I (fast) P wave is reflected at the water / sediment interface, penetration of the type II (slow) P wave can still occur. An alternative explanation derives from the scattering of sound at a rough water / interface. At sub-critical grazing angles and at sufficiently high frequencies, the energy scattered into the bottom by the rough interface can be dominant.*

### 1. INTRODUCTION

A recent study has focused on the evaluation of a range of techniques to facilitate the detection of objects buried in submerged sediment. Particular emphasis was given to the detection of objects resembling lightweight telecommunications cables of low acoustic contrast, *i.e.*, polyethylene-jacketed fibre optic cables of diameter  $\sim 25 \text{ mm}$  with low metal content, thereby anticipating one of the technological challenges which may face the telecommunications industry in the near future. The overall requirement was to identify a system suitable for deployment on a seabed-crawling remotely operated vehicle and capable

of returning information to enable the location and tracking of cables buried to a depth of up to 1 m in the seabed.

It was concluded that a matched-filter approach using broadband linear-FM output pulses was near-optimal in terms of maximising the signal-to-noise at the receiver and, therefore, of maximising the likelihood of detecting the presence of a target. The essential theory and some practical considerations involving the choice of operating frequency band are discussed in the following section. An experimental investigation was performed using a water tank, part-filled with sand, and a bistatic arrangement of focused transducers inclined at an angle well above the estimated critical grazing angle for total internal reflection. Detection results for a typical cylindrical target are also presented below.

A basic assumption of this work was that the sediment could be considered to be a near-homogeneous elastic medium in the 50 - 120 kHz operating frequency range. Hence, the trajectory of the single acoustic beam formed by the focused source transducer (inclined at a large grazing angle) could be calculated by the application of Snell's law and knowledge of the speed of sound in the water column and the saturated sediment. In recent years, however, experimental evidence of anomalous high-frequency sound penetration at shallow grazing angles has been presented which serves to contradict this view. Three principal mechanisms have been proposed to explain the phenomenon: the existence of a Biot slow wave in the sediment; rough surface scattering; and scattering of the evanescent wave by volume inhomogeneities. It is clear that a better understanding of the physics of wave propagation in sediments is required before the basic assumption, cited above, can be fully justified.

## 2. THEORY

When information regarding the nature of the transmitted waveform, the target and the noise and clutter in the medium is available a filter can be designed to select and modify the shape of the target-scattered signal to maximise the likelihood of detection. So-called optimal filtering is a very useful technique for measurements containing high levels of noise. Two special cases of the optimal filter are the matched-filter and the inverse filter. These can be shown to give the maximum target signal for high levels of noise and clutter, respectively, in the scattered signal.

The matched-filter, first derived by North [1] with application to radar, constitutes the optimum linear processing of a signal. It transforms the raw data at a receiver into a form that is suitable for performing optimum detection decisions. In the frequency domain the matched-filter transfer function,  $H(\omega)$ , is defined as the complex conjugate of the spectrum of the signal to be processed:

$$H(\omega) = kS^*(\omega)\exp(-j\omega T_d) \quad (1)$$

where  $S(\omega)$  is the spectrum of the signal at the filter input and  $T_d$  is a delay constant, required to make the filter physically realisable.

In this application the target lies in a densely cluttered environment. The seabed may be considered to be composed of randomly distributed, independent, broadband scatterers spaced

much more closely than the resolution capability of the matched-filter. Therefore, the clutter signal is expected to resemble a stationary Gaussian random process as a result of the overlap of the many individual signals that are present. The filter that optimises the signal-to-noise ratio for the general non-flat spectrum case is given by,

$$H(\omega)_{\text{opt}} = \frac{S^*(\omega)}{\frac{N_0}{2} + K_c |S(\omega)|^2} \quad (2)$$

where  $K_c |S(\omega)|^2$  is the clutter power spectrum and  $N_0$  is the one-sided noise power density at the filter input. (The delay constant,  $T_d$ , is generally ignored.) If clutter dominates equation 2 becomes the inverse filter and if noise dominates it becomes the matched filter.

Having determined the optimal filter the question arises as to what output waveform gives the highest resolution and accuracy. An important finding by Woodward was that resolution is a function of the signal bandwidth and not of the transmitted pulse width [2]. A pulse can be as wide as necessary to achieve a high total transmitted power and still meet the resolution requirement. In principle, therefore, the matched-filter concept can be applied to any waveform, though certain waveforms give rise to less ambiguous measurements. In the case where it is known that the waveform will be unaffected by a Doppler displacement the linear-FM pulse approximates the optimum bandlimited waveform, *i.e.*, the waveform which results in the least ambiguity for resolving amongst several signals [3].

It should be noted that extra consideration must be given to the attenuating effect of the medium, the scattering characteristic of the target and the background noise spectrum when selecting the operating frequency band. In many cases the backscattered signal level can be significantly higher at lower frequencies. There is also further scope for improving the signal-to-noise ratio at the filter output if distinctive scattering properties of the target can be identified and used to improve the accuracy of the matched-filter function.

The principle of optimal filtering was tested using a variety of cylindrical objects, each buried separately in the laboratory tank. The bistatic source / receiver arrangement was used to insonify and receive the acoustic energy scattered from around discrete sampling points within the sediment. The underwater extent of the tank was 180 cm × 125 cm × 116 cm deep, the bottom 50 cm of which was sand. Each target was buried up to 30 cm ± 3 cm deep in the sediment. The frequency band of the output waveform extended from 50 kHz to 120 kHz which made best use of the dynamic range of the transducer elements. As noted, a bandwidth centred on a lower frequency, *e.g.*, around 20 kHz, may have given an improvement in the backscattered signal-to-noise ratio at the receiver. However in this case, where the background noise level was low, a wider frequency band could be afforded. The duration of each output pulse was 1 ms.

For each buried target the time-windowed peak of the squared output of the matched-filter at every sampling point was combined to form a 3-dimensional map of the sediment volume. A vertical slice through the map obtained for a buried polyethylene cylinder, 20 mm in diameter, is shown as a contour plot in figure 1. The cylinder was oriented perpendicular to the sample plane. (It should be noted that the positive *z*-direction is taken from the bottom of the tank upwards.) A maximum is seen to occur at  $z = 26 \text{ cm} \pm 1 \text{ cm}$ . The discrepancy

between the calculated depth and the actual burial depth ( $z = 20 \text{ cm} \pm 3 \text{ cm}$ ) arises from inaccuracies in the measurement of the speed of sound in the sediment and in the geometry of the transducers. The filter output is also seen to maintain a high level over much of the sample plane in the x-direction. This is simply an artefact of the relatively wide beam profile of the acoustic source / receiver elements.

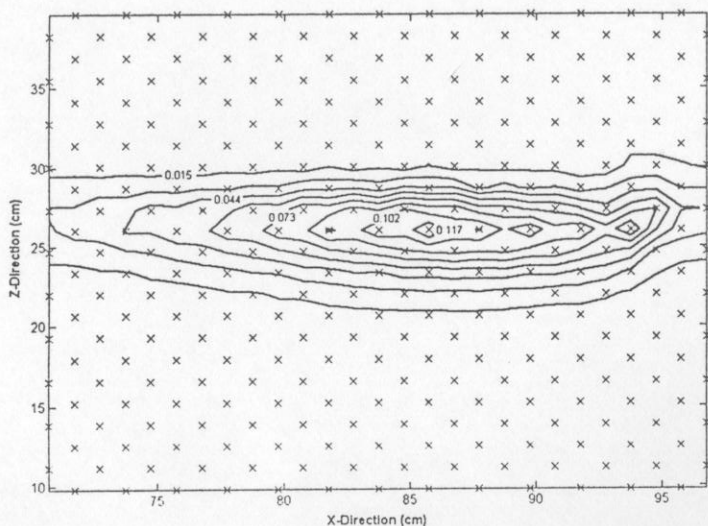


Figure 1. The matched-filter peak output ( $V^2$ ) interpolated over the range of sample points,  $x$ , in a vertical plane through the sediment. The cylindrical target was oriented perpendicular to the sample plane. (Base of tank,  $z = 0 \text{ cm}$ . Top of sediment,  $z = 50 \text{ cm}$ . Walls of tank:  $x = 0 \text{ cm}$  and  $x = 180 \text{ cm}$ .)

The design of a transducer system that would be suitable for use in the field requires a knowledge of the mechanisms by which acoustic waves propagate into and within sediments. As noted in the introduction, however, the physics of acoustic wave propagation in sediments is not fully understood and requires further investigation. An appropriate starting point is to review extant theoretical models and to compare their predictions with experiment. One model in particular is considered in the remainder of this section: the Biot theory of acoustic wave propagation in porous media.

A porous medium can be thought of as mineral grains in a skeletal frame which is saturated with a fluid. Biot proposed a set of coupled differential equations to describe wave propagation in such fluid-filled porous media. A common form is that used by Stoll and Kan [4] in which they are presented as a pair of coupled displacement equations:

$$\mu \nabla^2 \mathbf{u} + (H - \mu) \nabla (\nabla \cdot \mathbf{u}) - C \nabla (\nabla \cdot \mathbf{w}) = \rho \ddot{\mathbf{u}} - \rho_f \ddot{\mathbf{w}} \quad (3)$$

$$C \nabla (\nabla \cdot \mathbf{u}) - M \nabla (\nabla \cdot \mathbf{w}) = \rho_f \ddot{\mathbf{u}} - (c \rho_f / \beta) \ddot{\mathbf{w}} - (F \eta / \kappa) \dot{\mathbf{w}} \quad (4)$$

where  $\rho_f$  and  $\rho$  are the mass densities of the fluid and the saturated sediment,  $\eta$  is the viscosity of the fluid,  $\kappa$  is the permeability of the porous solid and  $\mu$  is the shear modulus of the solid. The term  $c$  accounts for the virtual mass of the liquid associated with its motion relative to the solid and the term  $F$  accounts for the frequency dependence of the drag force between the solid and the fluid. Porosity is denoted by  $\beta$  and the displacement vector of the solid frame is denoted by  $\mathbf{u}$ . The displacement vector of the fluid relative to the solid is  $\mathbf{W}$  and  $\mathbf{w} = -\beta\mathbf{W}$ . The remaining coefficients  $M$ ,  $C$  and  $H$  are defined in terms of the bulk moduli of the fluid, the solid material and the solid frame.

The Biot theory predicts the existence of three propagating waves: two compressional waves, termed the type I (fast) and type II (slow)  $P$  waves, and a shear wave. The fast wave, which is often referred to as the "normal" compressional wave, results from the in-phase motion of the fluid and the solid frame. The slow wave results from the out-of-phase motion and the shear wave results from the interaction between the two.

Using parameters from the literature, other researchers have calculated that the slow wave speed should increase with frequency, reaching a maximum of roughly  $400 \text{ m s}^{-1}$  at a frequency of around 1 kHz. It has always been assumed that the slow wave makes no significant contribution to the propagating sound field since the calculations have also shown it to be highly attenuated. However, slow waves measured by Chotiros for a range of sandy sediments were found to have speeds in the region of  $1200 \text{ m s}^{-1}$  and experienced significantly lower attenuation than predicted [5]. It is certain that these were not attributable to shear waves which are known not to exceed  $200 \text{ m s}^{-1}$  and they were unlikely to be the result of interface waves since they were observed to propagate below the interface zone.

Chotiros showed that by adjusting the bulk moduli of the solid material and the solid frame the predicted slow wave attenuation would be lower and the slow wave speed would be significantly higher than previous model predictions. The new speed was in agreement with the measured values but attenuation was still notably different. It has been argued that the choice of parameters was incompatible with a fundamental requirement of the Biot theory; that is the assumption of an equivalent homogeneous solid. However, it has been shown that it is possible to choose a compatible set of parameters for the Biot theory that meets this requirement and fits the velocity observations [6].

### 3. EXPERIMENT

A series of hydrophones were buried in the laboratory tank at a range of depths. The sediment was insonified with broadband output pulses, centred around 85 kHz, using a focused transducer. This was inclined at different grazing angles, both above and below the estimated  $28^\circ$  critical angle for total internal reflection of the normal compressional wave. An example of the beam pattern produced by the "normal" wave within the sediment is shown in figure 2. In this case the grazing angle was measured to be  $57^\circ \pm 1^\circ$  and the angle of transmission into the sediment was measured to be  $37^\circ \pm 1^\circ$ . (In a separate experiment the speeds of sound in the water and the sediment were measured to be  $1478 \text{ m s}^{-1} \pm 7 \text{ m s}^{-1}$  and  $1692 \text{ m s}^{-1} \pm 8 \text{ m s}^{-1}$  respectively which, by the application of Snell's law, is in agreement with the measured angle of refraction.) A similar result was obtained with the transducer

inclined at  $26^\circ \pm 1^\circ$ , *i.e.*, just below the critical grazing angle. The speed of sound of the propagating wave was not observed to vary significantly over the operating frequency range and was calculated to be  $1193 \text{ m s}^{-1} \pm 70 \text{ m s}^{-1}$ . This result is in excellent agreement with the values of slow wave speed measured by other authors discussed in section 2.

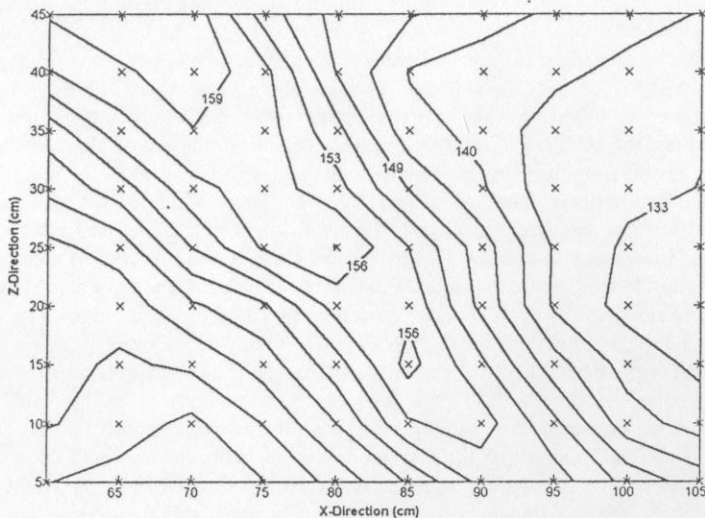


Figure 2. The sound field (dB re:  $1 \mu\text{Pa}$  @ 1 m) produced within the sediment by a focused transducer inclined at a large grazing angle. Each sample point, x, represents the location of a receiver hydrophone.

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