Nonlinear Transient Bubble Oscillations

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Abstract. The investigation considers issues associated with transient regime of nonlinear bubble oscillations: the ring up times near the acoustic threshold of parametrically-driven distortion modes [2], the subharmonic resonance of breathing mode for the pump pulse duration shorter then the time of the transient processes [3], the peculiarities of transient bistable bubble oscillations.

INTRODUCTION

The highly nonlinear dynamics of a gas bubble in a liquid renders difficult the investigation of the transient oscillation associated with the bubble forcing by a powerful sound field [1]. Is new physics (beyond that implied by steady state) responsible for peculiarities of the transitional response in bubble oscillations? The results presented here illustrate the existence of new effects near the bifurcation of bubble dynamic states. In this domain, one of the eigenvalues of the linear stability problem λ_1 is small (it equals zero at the threshold, which would be likened to zero damping). The growth of an instability is very slow, causing the transient processes associated with establishing steady state to be of very long duration. This article reviews transient processes near the acoustic threshold of parametrically-driven bubble shape oscillations [2]; the subharmonic resonance of bubble radial pulsation for the pump pulse duration shorter than the duration of the transient processes [3]; the peculiarities of transient bistable bubble oscillations.

PARAMETRICALLY-DRIVEN BUBBLE SHAPE OSCILLATIONS

The nonlinear response of a gas bubble to a low frequency w_p pumping wave results in parametrically-generated shape oscillations above a well-defined threshold. This may be detected through insonification of the bubble by a high-frequency w_i imaging wave, leading to the scattering of signals at $w_i \pm w_p$, $w_i \pm w_p/2$ (corresponding to

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pulsation and subharmonic wall oscillation, respectively) [4]. Ramble *et al.* [5] discovered that there exists a significant difference in the transient times taken to establish steady-state subharmonic and fundamental combination frequency signals (the so-called ‘ring-up’ times).

The complete theory of the transient processes near the threshold of excitation of distortion modes has been derived. The approach is as follows. Expansion of the bubble wall displacement and the potential in terms of the spherical harmonics leads to the evolution equations for the complex amplitudes. A phase space analysis of the system is made. Having identified the regions of instability of surface waves, the transient regime has been examined in that region to determine how rapidly the instability will develop. The solution of the system equations for amplitudes is based on the use of the center-manifold reduction. The explanation proposed here for the difference in the ring-up times of subharmonic and fundamental combination frequency signals (see Fig. 1), relies on the fact that these measurements were made slightly above the threshold of excitation of the distortion mode [5], where the growth of an instability is very slow, which leads to a very long duration transient interval while establishing steady state surface oscillations.

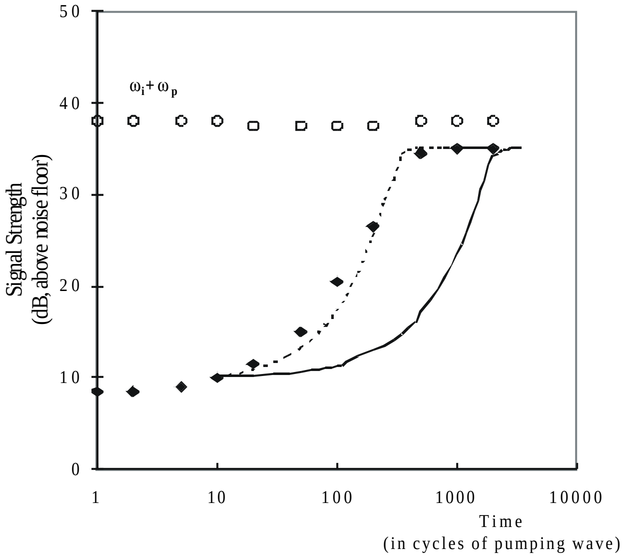

FIGURE 1. The comparison of experimental data on the transient regime of excitation of the fundamental ○ and the subharmonic ● combinative components with the derived theoretical results plotted as solid line. The dashed line is obtained by increasing the value of Lyapunov exponent that gives better agreement with the data.

SUBHARMONIC RADIATION BELOW THRESHOLD

The detection of subharmonic component is a general characteristic of nonlinear phenomena in liquids containing phase inclusions. At the same time, as first observed by Neppiras [6] a small subharmonic component is also observed below threshold, so the growth of this spectral component (see Fig. 2) is not, strictly speaking, of a threshold character.

The explanation proposed involves the analysis of transient processes near a subharmonic resonance [3]. When an external field is switched on (as a rule, experiments are performed with modulated pulsed signals containing from ten to hundred pump cycles), besides forced oscillations, pulsation having a natural frequency Ω_0 (the so-called 'Minnaert' frequency) are excited such that $\Omega_0 \approx w_p / 2$. Near threshold, the damping of natural oscillations is very small because of parametric energy transfer in this component (it vanishes at the threshold). For this reason, the duration of the transient processes can be greater than that of the pump pulse, and the corresponding component in the radiation spectrum can be interpreted as the appearance of a subharmonic component below threshold.

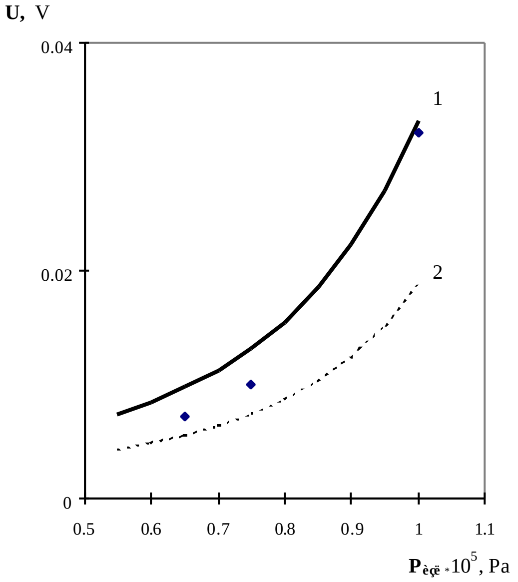

FIGURE 2. Subharmonic radiation below threshold.

Fig. 2 shows the signal level U of acoustical radiation at 0.6 MHz measured by a selective detector as a function of driving pressure amplitude of ultrasonic pulses with a 1.2 MHz carrier frequency w_p and 10μs duration. The data points are redrawn from [7] and fit to these data based on analytical solution of Rayleigh-Plesseet equation. Curve 1 corresponds to exact subharmonic resonance $w_p/2=W_0$, and curve 2 corresponds to detuning in a half width of subharmonic resonance.

TRANSIENT BUBBLE BISTABLE OSCILLATIONS

The spectrum of the radiation caused by acoustic cavitation has the form of a series of harmonics and subharmonics each of which has a narrow bandwidth, superimposed upon a noise continuum of generally lower amplitude. To describe the shape of the individual bands, including random components in addition to the driving harmonic from the real spectrum of the acoustic pressure that acts on the single bubble in a cavitation zone has been proposed in [8]. The effect of fluctuations associated with the random field component has been found to be most pronounced in the vicinity of the bifurcational values of the control parameters (field amplitude and detuning $h = (\Omega_0^2 / w_p^2 - 1)$).

In analyzing bifurcations of nonlinear bubble oscillations, the dependence of maximum radius $R_{\max}$ attained by a single bubble during its expansion half-cycle on control parameters is traditionally used. Fig. 3 shows the distribution density of maximum bubble radii $f(x_{\max}, h)$ as a function of detuning h for the domain of fundamental resonance ($x_{\max} \equiv (R_{\max} - R_0)/R_0$). For reference, the thin solid lines show Gaussian distributions having the same mean and dispersion as the series of $x_{\max}$ that has been used for computation of the distribution density. In the plane of x, h markers show the values of $x_{\max}$ for the steady state regime in the absence of random force for the same values of detuning that permits us to carry out the comparison with known data. The distribution density has a complex structure, and markedly differs from the Gaussian form. The spreading of $f\left(x_{\max}, h\right)$ indicates the weak stable character of bubble oscillations and suggests an increase in duration of the transient regime that is confirmed by the run of trajectories on the phase plane.

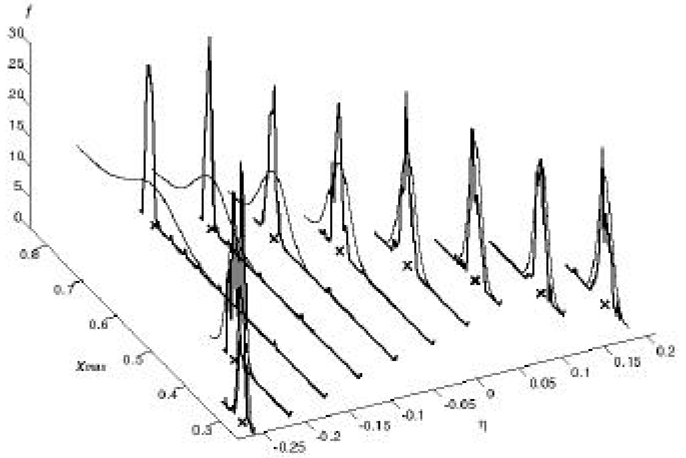

FIGURE 3. The distribution density of maximum bubble radii

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