Natural Physical Processes associated with Sea Surface Sound

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Detection of Bubbles via Higher Order Statistics

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1. Introduction

The presence of bubbles in a medium can be detected by exploiting their characteristic scattering behaviour, specifically using the fact that they are resonant non-linear scatterers [1]. Various strategies have been proposed for achieving this goal. The simplest methods look for a resonant scatterer using a tonal probing signal. Such methods are prone to ambiguities [2]. To avoid these ambiguities methods which utilise the known non-linearities in the bubble's behaviour have also been proposed. These may probe the medium with a pair of tones (see the 'Combination frequencies' section of this volume). In this paper we discuss this problem from a system identification viewpoint and discuss a method based on a general non-linear system model. This approach uses a random excitation signal, so avoiding the need to employ a chirp or step through a range of tones at different frequencies, and yields a more complete picture of the non-linear scattering behaviour of the medium under examination.

2. Linear System Identification

The problem of measuring the scattering from a medium can be cast as a system identification problem in which both the input and output are measurable, as depicted in Fig. 1. The general problem is to infer from measurements of the x(t) and y(t) the character of the unknown system. Specifically the system will be predominantly characterised by a simple delay if the medium contains no bubbles (or other significant scatterers), whereas if a bubble is present the character of the system about the bubble's resonant frequency will be more complex.

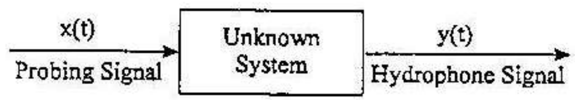

Fig. 1 System Identification

If the unknown system is assumed to be linear then the input/output relation can be expressed directly via the convolution integral or alternatively as a product in the frequency domain, *i.e.*

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \quad \text{or} \quad Y(f) = H(f)\, X(f) \qquad (1)$$

where h(t) is the system's impulse response and Y(f) denotes the Fourier transform of y(t). Such a linear system is characterised by either its impulse response, h(t), or its transfer function, H(f).

There is a plethora of techniques for measuring the transfer function associated with a linear system and these techniques can be applied to our scattering problem. For example, one can use a probing signal consisting of a single tone with variable frequency to make spot measurements of H(f). This has the drawback of requiring one to cycle through all the measurement frequencies. It is more efficient to use a single input signal which contains all the frequencies of interest. Suitable signals include impulses, swept sinusoids and random signals. We choose to consider random excitations here because of their convenience and robustness.

The cross-spectrum between the signals x(t) and y(t) is defined as

$$S_{xy}(f) = \int_{-\infty}^{\infty} r_{xy}(\tau)\, e^{-2\pi i f \tau}\, d\tau \quad \text{where} \quad r_{xy}(\tau) = E[\,x(t)\, y(t-\tau)\,]$$

in which $r_{xy}(\tau)$ is referred to as the cross-correlation function and E[] denotes the expectation operator. For a purely linear system one can show that

$$H(f) = S_{xy}(f) \,/\, S_{xx}(f) \qquad (2)$$

Hence to estimate the transfer function, H(f), one estimates the two spectra $S_{xy}(f)$ and $S_{xx}(f)$ from the data and forms their ratio. The estimator described by eq. (2) is known to be asymptotically unaffected by noise on the output measurement y(t).

A major limitation of the application of the above technique to bubble scattering data is the observation that a bubble interacts with the acoustic field in a non-linear fashion, so the underlying model, eq. (1), is flawed. It is the object of this paper to show how the techniques outlined above can be extended to non-linear models and so be applied to data from bubble scattering.

Experimental measurements were made in a 1.8m × 1.2m × 1.2m vibration isolated, reinforced plastic tank filled to a depth of 1.5m. An acoustic field was generated using an underwater loudspeaker (Gearing and Watson UW60). The received signal was measured using a B&K 8103 hydrophone. The loudspeaker was driven by broadband Gaussian noise, which was band-limited between 1.2 and 7 kHz. The driving signal and hydrophone output were acquired onto a pc at a sampling rate of 20 kHz. Measurements were made with and without a bubble, the bubble being tethered to a wire.

Figs. 2(a) and (b) show the magnitudes (in dB) of the transfer functions estimated, via eq. (2), for the cases with a bubble absent and present respectively. The peak in Fig. 2(b) at around 2.5 kHz is due to the scattering from the bubble. Fig. 2(c) shows the ratio of the two transfer functions and is one way of visualising modifications the bubble has made to the transfer function.

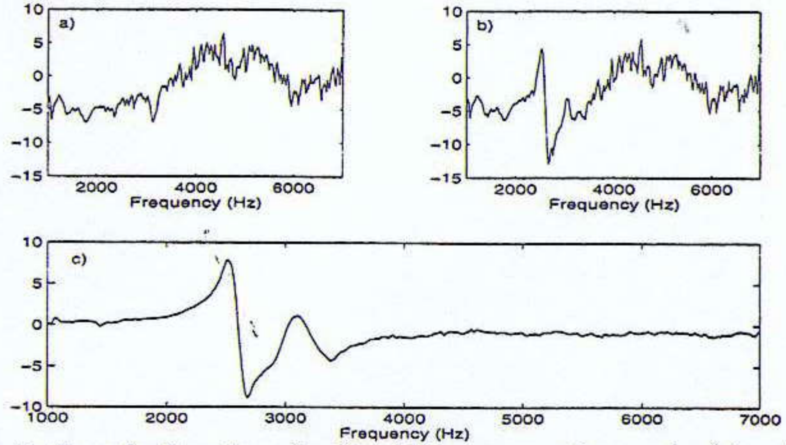

Fig. 2 Transfer Functions (in dB with common reference for (a) and (b)): (a) Without Bubble, (b) With Bubble, (c) Ratio of (b) to (a)

Care must be exercised when interpreting the transfer functions in Fig. 2 since the bubble's scattering behaviour is known to be non-linear and the theory of transfer functions is only relevant to linear systems. However at low excitation levels a linear approximation may be reasonable.

An alternative method for comparing the linear scattering results with and without bubbles is to compare the residual spectra. The method is to use the data set in which a bubble is absent to estimate the impulse response, $h_a(t)$, of the tank-loudspeaker-hydrophone system. This impulse response can then be used to form a residual signal, z(t), as

$$z(t) = y(t) - \int h_a(\tau)x(t-\tau)d\tau$$

This signal represents the scattered signal components not accounted for by the tank-loudspeaker-hydrophone system. Fig. 3 shows two examples of such residual spectra. Fig. 3(a) shows the case where there is no bubble present. Fig. 3(b) shows the residual spectrum obtained when a bubble is introduced. Its effect is to contribute the majority of energy around the peak at 2.54 kHz (±0.02kHz), which corresponds to an estimate of the resonant frequency of the bubble, and in turn can be related to the bubble's mean radius.

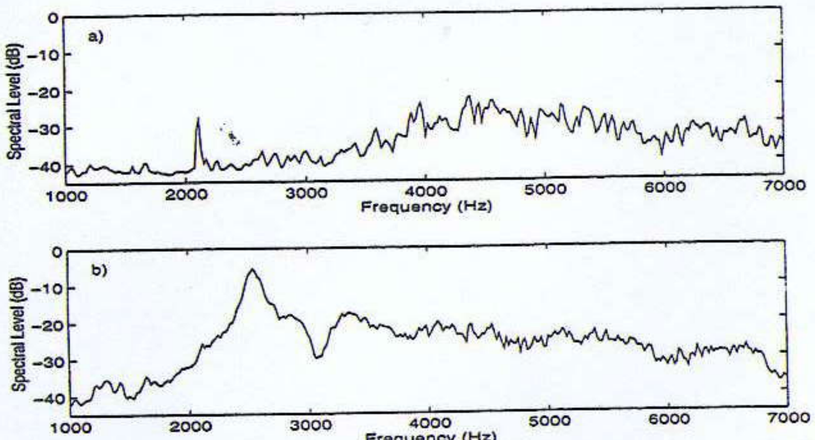

Fig. 3 Spectra of Residual Signals (in dB with common reference): (a) Without Bubble, (b) With Bubble

By comparing Figs. 2 and 3 we see that the method based upon residual spectra gives a less ambiguous result, generating only a single peak.

3. Non-Linear System Identification

The linear method described above can be used to identify whether a resonant scatterer is present. However this procedure fails to include the known non-linear behaviour associated with bubbles. In this section we detail a method for partially characterising the non-linear scattering of a bubble.

There are a variety of models available for parameterising non-linear systems. Here we shall concentrate on one of the most popular, namely the Volterra series [3]. The Volterra series extends the convolution integral in eq. (1) to higher orders, so the input/output equations become

$$\begin{aligned} y(t) = &\int h_1(\tau)x(t-\tau)d\tau + \iint h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 \\ &+ \iiint h_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \cdots \end{aligned} \quad (3)$$

in which case the non-linear system is characterised by the infinite set of kernel functions $h_k(\tau_1,\ldots,\tau_k)$. Even eq. (3) can only model some classes of non-linear systems. In practice the Volterra series has to be truncated at a low order and in this paper we shall restrict our attention to the simplest non-linear model, *i.e.* that in which only the linear and quadratic terms appear. Also our model must be discrete in form, so the actual model we exploit is more correctly written as

$$y(n) = \sum_p h_1(p)x(n-p) + \sum_p \sum_q h_2(p,q)x(n-p)x(n-q) \quad (4)$$

The problem is now how to estimate the kernel functions, $h_1(p)$ and $h_2(p,q)$ from the measured data x(n) and y(n). In fact we choose to estimate the Fourier transforms of these functions, $H_1(f)$ and $H_2(f_1,f_2)$, which are referred to as the first and second order transfer functions, respectively.

The problem of estimating the higher order transfer functions can be solved by extensions to eq. (2). If one assumes that the input, x(t), is Gaussian then it can be shown [3,4] that for the model (4)

$$H_1(f) = \frac{S_{xy}(f)}{S_{xx}(f)} \quad \text{and} \quad H_2(f_1,f_2) = \frac{B_{xxy}(f_1,f_2)}{2S_{xx}(f_1)S_{xx}(f_2)} \quad (5)$$

Note that the estimate of the first order transfer function is exactly that used for linear system identification. This a consequence of our choice of a Gaussian input which serves to decouple the problems of estimating the first and second order transfer functions. The quantity $B_{xxy}(f_1,f_2)$ is termed the cross-bispectrum and is defined as the double Fourier transform of the second order cross-correlation function, $r_{xxy}(\tau_1,\tau_2)$ defined as

$$r_{xxy}(\tau_1,\tau_2) = E[x(t)x(t-\tau_1)y(t-\tau_2)]$$

Eq. (5) represents the standard technique for estimating the second order transfer function. It has been shown [5] that the variability of this estimator can be improved by modifying eq. (5), so that one forms the cross-bispectrum between x(t) and a new signal z(t). This new signal is the residual formed by removing the effect of the linear transfer function, *i.e.*

$$z(t) = y(t) - h_1(t) * x(t) \tag{6}$$

where $h_1(t)$ is the first order kernel (linear impulse response) and * denotes convolution. Hence the procedure used to estimate the second order transfer function is: i) use eq. 2) to estimate the linear transfer function; ii) inverse Fourier transform this transfer function to yield an estimate of $h_1(t)$; iii) form z(t) using eq. (6); iv) estimate the cross-bispectrum between x(t) and z(t); v) use eq. (5) to estimate the second order transfer function.

There is an obvious parallel between the method used to compute the residual spectrum, shown in Fig. 3, and the method used to estimate the second order transfer function. However, it should be emphasised that in the case of the second order transfer function one does not need to exploit a measurement made in the absence of a bubble.

Fig. 4 shows the result of estimating the second order transfer function, based on eq. (5), using the bubble scattering data. This transfer function shows a distinct peak around the bi-frequency (2.6, 2.6) kHz, indicating that there is significant non-linear behaviour in that region. This, in turn, may be used to infer the presence of a bubble in the medium and further to estimate its radius based on a resonant frequency of 2.6 kHz. This characteristic resonant behaviour allows one to distinguish between non-linearities due to a bubble and, say, non-linearities due to a transducer. In the latter case the second order transfer function would not exhibit the narrow band behaviour shown in Fig. 4.

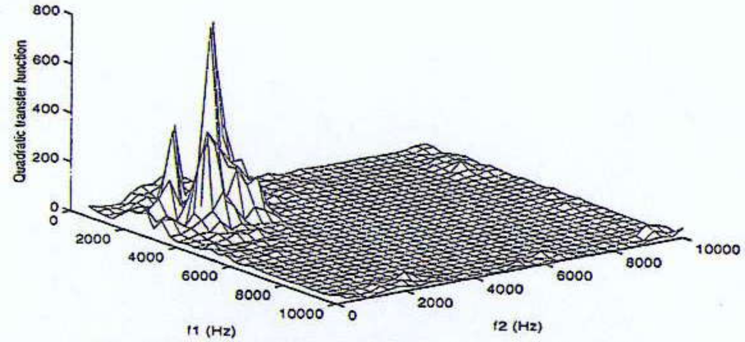

Fig. 4 Estimated Second Order Transfer Function

4. Conclusions

This paper has demonstrated how one can use a Volterra series as a model of the non-linear scattering behaviour of a bubble. This parameterisation of the bubble behaviour allows one to be confident whether or not a given scatterer is a bubble. Further, this approach also naturally yields an estimate of a bubble's resonant frequency.

Acknowledgements

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