***Radcliffe S*, Leighton T G and Nelson P A**, A numerical study of the use of simulated annealing for parameter identification of ocean fronts, *Proceedings of the 3rd European Conference on Underwater Acoustics, Heraklion* (ed. J. Papadakis), 1996, 385-390.

A Numerical Study of The Use of Simulated Annealing for Parameter Identification of Ocean Fronts

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This paper presents a numerical study of acoustic propagation through a generic ocean front. Low frequency sound propagation is an established method of investigating the ocean structure. In this study a parabolic equation method is used to model the propagation through a front. A Simulated Annealing search algorithm identifies the parameters that characterize an ocean front. A Linear processor is used to calculate the cost function although the use of a Minimum Variance Processor is considered for future studies.

1. THE OCEAN FRONT MODEL

1.1. Formulation of a Model for an Ocean Front

An ocean front similar to that which may be found in the Gulf Stream region may be simulated using a simple mathematical formulation [1]. This approach has been used to synthesize the thermal structure from expendable bathythermograph (XBT) data [2] demonstrating its validity.

The temperature, T, can be expressed as a function of the range, x, and depth, z, coordinates in the form:

$$T(x,z) = T_0 + \frac{1 + \tanh\left(\frac{z - z^*(x)}{H}\right)}{2}(T^* - T_0) \tag{1}$$

The depth of the midpoint of the thermocline, z^*, about which the temperature changes from that at the surface, T_0, to the deep ocean temperature, T^*, is given by:

$$z^*(x) = z_1 + \frac{1 + \tanh\left(\frac{x - x_0}{L}\right)}{2}(z_2 - z_1) \tag{2}$$

Figure 1 shows the similarities between the variation of temperature with depth, and the variation of the thermocline depth, z^*, with range from its initial depth, z_1, to its final depth, z_2. The thermocline thickness, H, is a measure of the rate of temperature change with depth, and the width of the front, L, about the front's range, x_0, is a measure of the rate at which the thermocline depth changes.

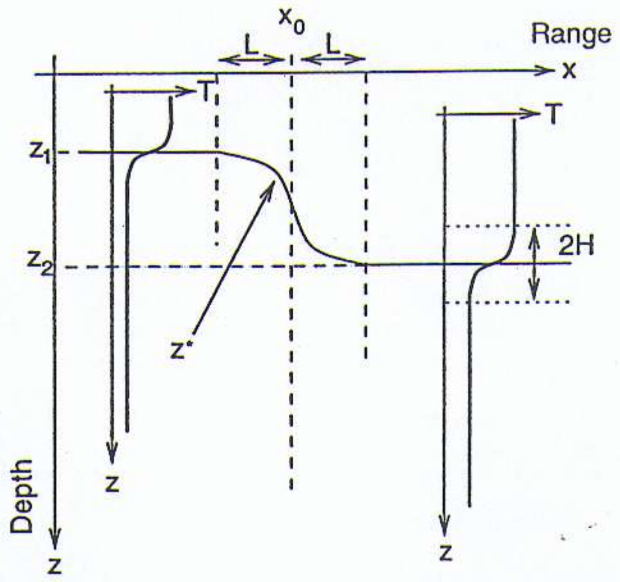

Figure 1: *The variation of the midpoint of the thermocline, z^*, with range, x, through an ocean front. Profiles showing the temperature variation, T, with depth at positions before and after the front are also shown.*

Since the temperature variation in a front can now be described, the sound speed can be easily found by using an established algorithm [3]. Figure 2 shows the sound speed profiles before and after the front produced using the previous method and which were used in this study.

1.2. Acoustic Propagation Through A Front

Figure 3 shows the effect of an ocean front centered at 40 km range with a source at 100 m depth modelled as described in Section 1.1. The amount of energy in the upper part of the ocean has increased due to the presence of a substantial surface duct (see also Figure 2). It is the distribution of this energy that is used in this simulation in order to determine the structure of the ocean.

2. GENERATING THE COST FUNCTION

2.1. Measuring the Field

In this study the receiver array was placed at a range of 100 km from the source. The array was vertical with 20 elements spaced 5 m apart beginning at 100 m depth. For the 150 Hz source placed at 100 m used in this study, this gives approximately half a wavelength spacing between measurement points. A parabolic equation model based on the Padé approximation method [4] was used for the acoustic propagation.

The 'measured' field was first generated by propagating through an ocean front defined using arbitrarily chosen parameters. This formed the reference field to which fields generated using estimates of the parameters were compared. A measure of the difference of the 'estimated' field to the 'measured' field is the cost function. Bottom interaction

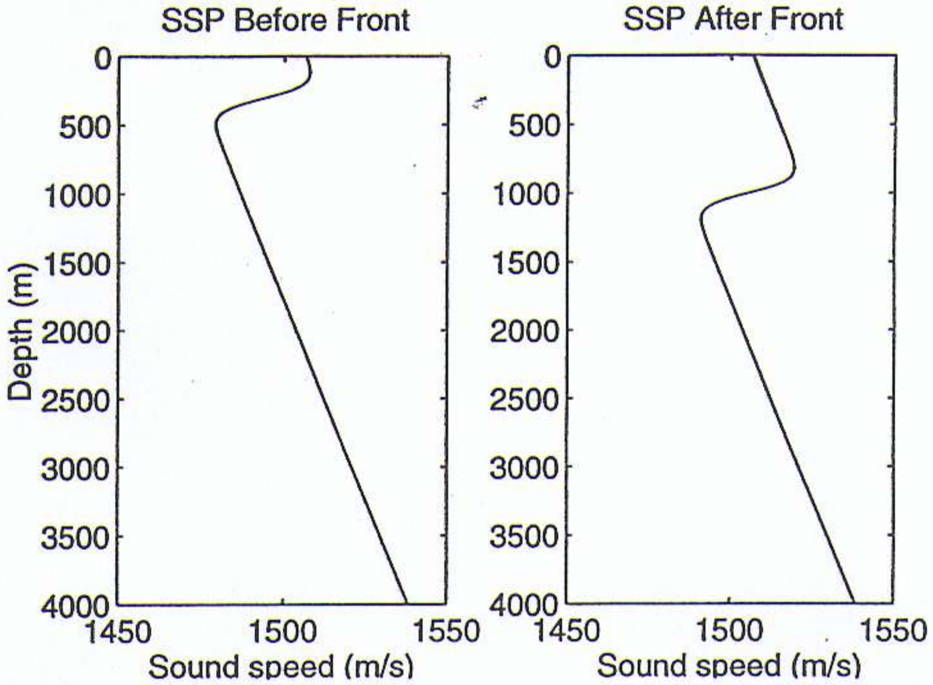

Figure 2: *Sound speed profiles before and after a front calculated using the method described in Section 1.1. In this example the thermocline thickness, H, has been taken to be 100 m with the surface temperature, T_0, at 15 Celsius and the deep ocean temperature, T^*, at 5 Celsius.*

effects were suppressed by having a flat, perfectly absorbing ocean bottom.

2.2. Calculating the Cost Function

The most straightforward method of calculating the cost function[1] is to use a 'Linear' or 'Bartlett' approach [5]. If p is the vector representing the 'measured' pressure field at a specified range and $\hat{\mathbf{p}}$ is the 'estimated' pressure field, then the cost, J_{lin}, is given by Equation 3. It is simply the sum of the squared errors between the 'measured' complex pressures and the 'estimated' complex pressures.

$$J_{lin} = (\mathbf{p} - \hat{\mathbf{p}})^H(\mathbf{p} - \hat{\mathbf{p}}) \tag{3}$$

The ocean front parameters can be varied systematically in order to deduce their effect on acoustic propagation. If an optimization method such as simulated annealing[2] is used then the characteristics of the front may be inferred from the measured field.

In this study the measured field was obtained by simulating acoustic propagation through a front generated using the method described in Section 1.1. If the front's parameters are correctly identified by a search algorithm then the cost function will have a value of zero.

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[1] Also known as the error or objective function.

[2] See also Section 3

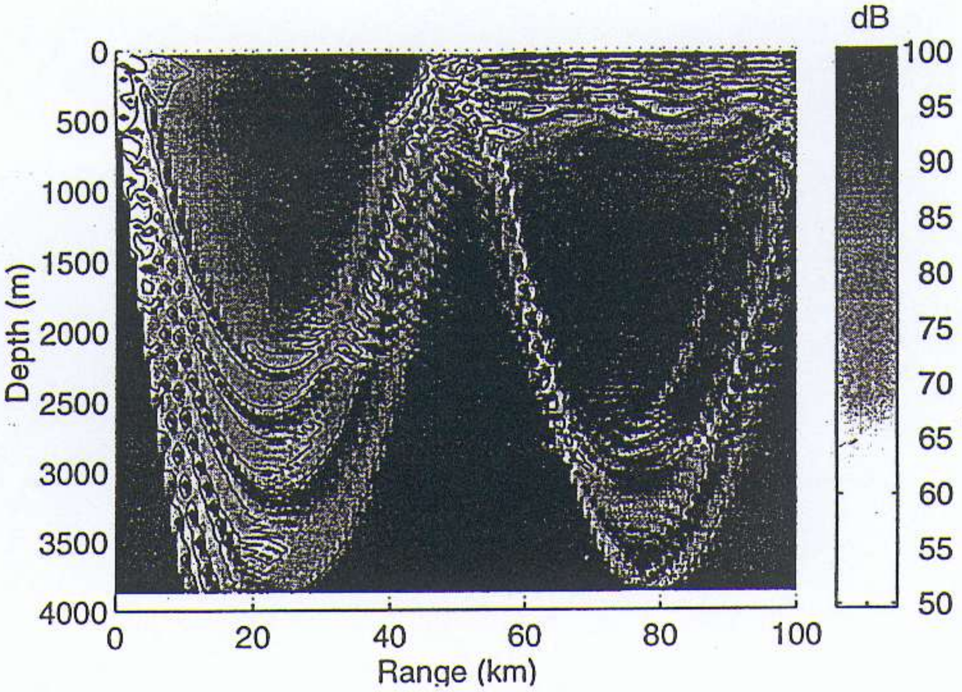

Figure 3: *A combined intensity and contour plot that shows transmission loss in decibels for propagation through a front centered at 40 km from the source. The front extends to 15 km either side of its center.*

3. SIMULATED ANNEALING RESULTS

3.1. The Form of the Cost Function

In the initial stages of this project two parameters are being investigated: the range of the center of the front from the source, x_0, and the width of the front, L. By using these two parameters the cost function can be visualized as a surface. When a linear processor is used there is a minimum in the cost for the set of parameters which correspond to the 'true' values which completely describe the front.

In order to reduce time and effort, an optimization routine is used to find the set of parameters that best describe the front. This removes the need to form a complete map of the cost function surface which, while instructive, is computationally intensive. If the cost function surface formed using the linear processor is considered, then the surface can be seen to consist of many hills (maxima) and valleys (minima). While there are many local minima there is only one global minimum and this is what is sought.

3.2. Results of the Simulation

Figure 4 shows the result of a simulated annealing search using a linear processor to calculate the cost function. The results have been overlaid onto a plot of the cost function over the entire surface. By using simulated annealing [6] the likelihood of finding a local minimum and not the global minimum is reduced. In Figure 4 it can be seen how the simulated annealing method has allowed uphill moves but has finished close to the global

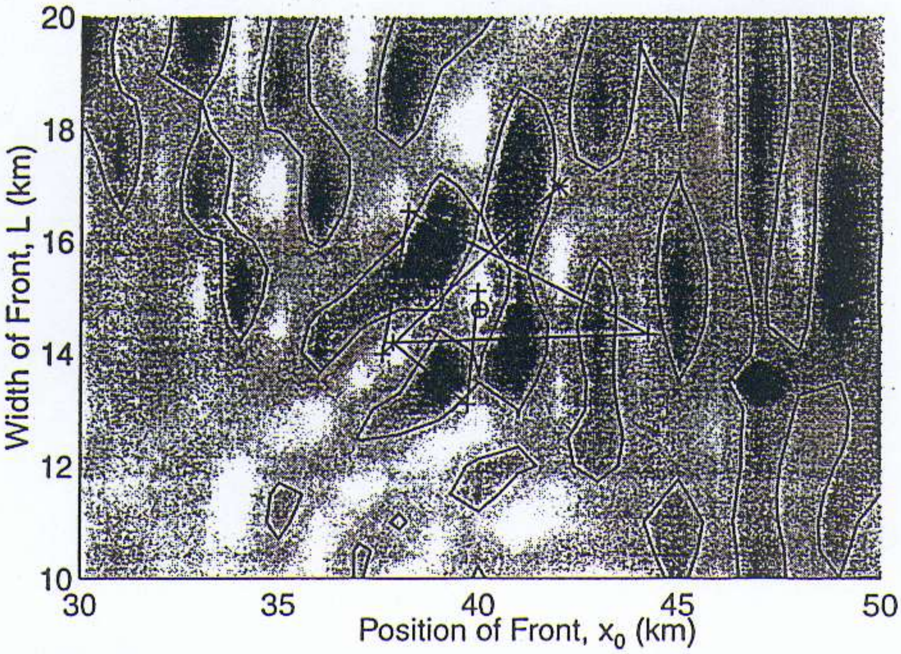

Figure 4: *The results of a simulated annealing search overlaid onto the corresponding cost function surface. The correct parameters are a range of 40 km and a width of 15 km. The asterisk shows where the search started and the circle is the optimum set of parameters found after 52 iterations.*

minimum.

4. CONCLUSIONS

It has been shown that a simulated annealing search method can be employed to identify the parameters used to describe an ocean front. However, this only works if the cost function surface is conducive to such a search.

The features that can be seen on the plot of the cost function as the front's position and width are varied indicate their relative importance. The main features of the surface are long narrow hills and valleys almost aligned with one of the axes. This shows that a change in the front's range from the source will have a greater effect than a change in its width. This is what would be expected as the position of the front relative to the source will influence the position of convergence zones. The surface is frequency dependent and by using several frequencies the surface is smoothed. A broadband source would reduce the possibility of the simulated annealing search becoming trapped in a local minimum still further.

In this paper the effects of changing two frontal parameters have been discussed. Simulated annealing is a search method that allows many parameters to be manipulated and preliminary studies have been performed using all of the frontal parameters with a degree of success. Using more parameters means longer computation times but will still be successful.

Future work on this project will involve using oceanographic data to provide sound speed profiles through which acoustic propagation will be modelled to give the measured field. Since the estimated fields will be produced using the mathematical front, the cost function will show different characteristics. In this study when the parameters were correctly guessed, the 'estimated' field matched the 'measured' field exactly. This will not happen when a computational ocean front is compared to a real ocean front.

If a Minimum Variance Processor (MVP) is used instead of a Linear Processor then the cost function surface is very different. When used in this study the MVP cost function surface is of the order 10^{-8} everywhere except at the location of the true parameters where it has a value of unity. In a case where the best fit to the measured field will not be exact it may be found that the MVP provides a more acceptable method of calculating the cost function.

5. ACKNOWLEDGMENTS

This project is funded by the Natural Environment Research Council as part of the UK World Ocean Circulation Experiment. Our thanks are extended to the Institute of Oceanographic Sciences for advice on formulating an ocean front, and to Mike Collins at NRL for the use of RAM.

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