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An experimental study of the sound emitted from gas bubbles in a liquid

Timothy G Leighton and Alan J Walton

Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, UK

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Abstract An undergraduate-level experiment is described for studying the sound emitted from bubbles formed by blowing gas through nozzles located within the bulk of a liquid. Hydrophone records give the frequency, pressure amplitude, and decay characteristics of the sound produced by a single bubble. The measured frequency agreed with the natural oscillation frequency of the bubble (the Minnaert frequency); the pressure amplitude gives the amplitude of oscillation of the bubble wall; the decay time broadly agrees with the value expected for radiative acoustic damping and thermal damping. The underwater sounds produced by natural brooks were recorded and found predominantly to consist of a succession of Minnaert-like oscillations, thereby allowing the bubble-size distributions to be obtained from the sound spectra.

Zusammenfassung Es wird ein Versuch für Studenten beschrieben, wobei der Laut untersucht wird, den Blasen hervorrufen, die gebildet werden, wenn Gas durch Düsen geblasen wird, die innerhalb der Flüssigkeit angebracht sind. Hydrophonaufnahmen ergeben für die Frequenz, Druckamplitude und den Zerfall die Charakteristiken des von einer einzigen Blase hervorgebrachten Lautes. Die gemessene Frequenz stimmt mit der natürlichen Schwingungsfrequenz der Blase (der Minnaertfrequenz) überein. Die Druckamplitude ergibt die Schwingungsamplitude der Blasenwand. Die Zerfallszeit stimmt grösstenteils mit dem Wert überein, der für akustische Strahlungsdämpfung und thermische Dämpfung zu erwarten ist. Die von natürlichen Bächen hervorgebrachten Unterwasserlaute wurden aufgenommen. Es zeigt sich, dass sie überweigend aus einer Folge von Minnaertartigen Schwingungen bestehen. Hierdurch wird ermöglicht, die Verteilung der Blasengrössen aus den Lautspektren zu ermitteln.

1. Introduction

In his book *The World of Sound* (based on the 1919 Royal Institution childrens' Christmas lectures) Sir William Bragg (1921) suggests that the sounds emitted by running water originate from cavities created by the impact of liquid drops on the water surface. Bragg cites the (previously unpublished) work of Sir Richard Paget who modelled these cavities (as photographed by Worthington (1908)) out of plasticine and found that by blowing across openings in them sounds were produced similar to those heard when objects were dropped into water.

Minnaert (1933) pointed out that additional mechanisms might contribute to the sounds of running water. In particular he suggested that at least some of the sound emission came from spherical gas bubbles undergoing periodic expansions and contractions. Assuming a simple harmonic motion $A_0 \cos \omega t$ about a mean radius a (figure 1), Minnaert equates the kinetic energy of the liquid when the bubble radius is a to the potential energy at an extreme of the motion.

When the bubble radius is R, the kinetic energy of the liquid is $\int_R^\infty \frac{1}{2}(4\pi r^2 \rho \, dr)\dot{r}^2$, where ρ is the liquid density. Assuming the liquid to be incompressible $\dot{r}/\dot{R} = R^2/r^2$, giving the kinetic energy of the liquid as $2\pi\rho R^3 \dot{R}^2$. At $R = a$ and $\dot{R} = A_0\omega$ this has a maximum value

$$E_{\max} = \tfrac{1}{2} m_r (A_0\omega)^2 \qquad (1)$$

where

$$m_r = 4\pi a^3 \rho \qquad (2)$$

is the effective mass (the radiation mass) of the liquid. Assuming the gas (pressure p_g) within the bubble (volume V) to obey the law $p_g V^\kappa = \text{constant}$, where κ is the so called polytropic index which has a value of unity in an isothermal process and a value γ in a reversible adiabatic process (in general, κ can be expected to lie between these limits), it can readily be

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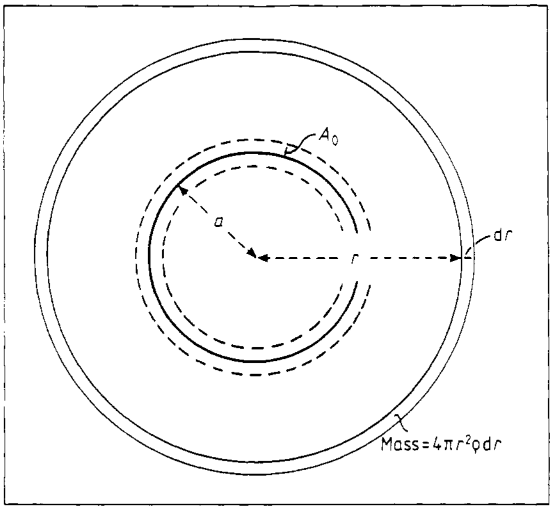


Figure 1 Shows a gas-filled bubble of mean radius a in a liquid of density ρ executing simple harmonic oscillations of amplitude A_0. A spherical shell of liquid of radii r and $r + dr$ has mass $4\pi\rho r^2\, dr$.

shown that the work done in compressing the bubble from a radius a to $a - A_0$ is $6\pi\kappa p_0 a A_0^2$, where p_0 is the total pressure in the surrounding liquid (the sum of the atmospheric and pressure-head terms). Equating this to equation (1) gives

$$\nu = \frac{1}{2\pi a}\left(\frac{3\kappa p_0}{\rho}\right)^{1/2} \tag{3}$$

as the natural frequency of oscillatiton of the bubble. In the case of air-filled bubbles in water at $p_0 = 100$ kPa, equation (3) reduces to the convenient form $\nu a = 3$ when ν is measured in hertz and a in metres. Thus bubbles of the order of a millimetre in size can be expected to produce sound with a frequency of the order of a kilohertz. Equation (1) can also be obtained from the Rayleigh–Plesset equation in the limit of small-amplitude oscillations and with the surface tension and viscous terms ignored (see, for example, Walton and Reynolds 1984).

Minnaert (1933) compared the frequencies of the sounds emitted by bubbles blown from nozzles in the bulk of various liquids with the frequency of sounds from tuning forks (pitched two octaves lower) and thus demonstrated, albeit crudely, the essential correctness of equation (3).

The purpose of this paper is to describe an undergraduate-level experiment aimed at recording and analysing the sounds produced by single gas bubbles in liquids both in the laboratory and in brooks. As will soon become evident, the oscillating bubble provides a good example of the ubiquitous lightly damped simple harmonic oscillator.

2. Experimental techniques

Single bubbles are created as gas is slowly forced through a nozzle immersed in a liquid (figure 2) by the gradual depression of a syringe piston. Bubbles of radii ranging from a few tenths of a millimetre upwards can readily be produced from hand-drawn glass tubing. A micrometer screw pushing against the syringe plunger provides fine control of the gas flow. The filling gas is introduced into the syringe via a three-way tap. The mean volume of a bubble produced by a particular nozzle can most easily be obtained by measuring the total volume of gas from a counted number of bubbles collected in an inverted liquid-filled cylinder. Since the sound emission occurs while the bubble is in the vicinity of the nozzle tip it is, of course, necessary to correct the measured bubble volume for the hydrostatic head.

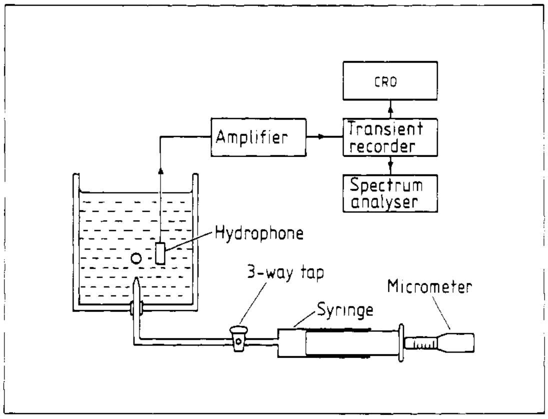


Figure 2 The experimental arrangement for studying the sound emission from a gas-filled bubble in a liquid.

The sound emitted from a bubble was picked up by a calibrated hydrophone (Brüel and Kjaer type 8103). This signal was stored on a transient recorder for display on a cathode-ray oscilloscope (CRO) or for processing in a spectrum analyser. Since the sound emissions studied here were in the audio range they can alternatively be recorded via a low-frequency storage oscilloscope. Although an adequate tank can be made by drilling the base of a 250 ml beaker, a rectangular Perspex tank facilitates bubble observation or photography.

The cost of the system can be minimised by substituting for the hydrophone a piezoelectric 'tweeter' loudspeaker (e.g. Motorola model KSIV1022A) waterproofed by painting the cone with modelling dope, and by using a simple CRO and camera instead of a transient recorder. Such a system will allow all the experiments described below to be performed with the exception of the measurement of the absolute sound intensities.

3. Laboratory measurements

3.1. The sound period

Figure 3 shows the hydrophone voltage (proportional

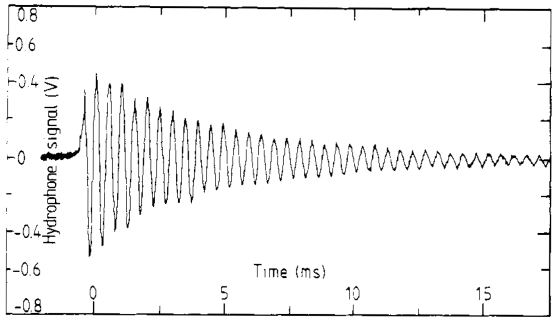


Figure 3 The hydrophone output showing the sound emitted from an air-filled bubble of radius 1.7 mm in water at 106 kPa. The signal from the hydrophone was fed through a broad-band amplifier (gain typically 30 dB) and to a transient recorder and pen recorder.

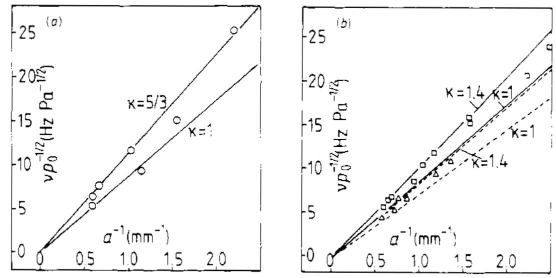


Figure 4 (a) A plot of $\nu/p_0^{1/2}$ against a^{-1} for helium-filled bubbles in water where ν is the frequency of the sound emitted from a bubble of radius a and p_0 is the ambient pressure. The two full curves correspond to the theoretical dependence as predicted by equation (3) with $\kappa=1$ and $\kappa=5/3$. (b) A plot of $\nu/p_0^{1/2}$ against a^{-1} for air-filled bubbles in water (□) and in a saturated $NaClO_3$ solution (△). The full curves indicate the predicted behaviour in water at the limits of $\kappa=1$ and $\kappa=\gamma=1.40$; the broken curves indicate the predicted behaviour in $NaClO_3$ solution at $\kappa=1$ and $\kappa=\gamma=1.40$.

to the sound pressure) record from a typical bubble as captured by a transient recorder. This particular trace is for an air-filled bubble of radius $a=1.7$ mm in water and clearly shows a lightly damped harmonic oscillation of frequency $\nu=2.07$ kHz.

The correctness of equation (3) was investigated by systematically studying the dependence of ν on the bubble radius, the enclosed gas and the liquid density. Figure 4(a) shows $\nu/p_0^{1/2}$ (here p_0 includes both the variable atmospheric and hydrostatic pressure heads) plotted against a^{-1} for helium-filled bubbles in water. The full curves are plots of equation (3) with κ taking the limiting values of unity (the lower line) and $\kappa=\gamma=5/3$ (the upper line). The experimental data indeed lie within these limits; the best straight line fitted to the data has a gradient corresponding to $\kappa=1.46\pm0.09$, indicating that the oscillations might be thought of as 75% adiabatic. Data for propane-filled bubbles in water likewise lie between the limits of $\kappa=1$ and $\kappa=\gamma=1.13$, corresponding to $\kappa=1.10\pm0.02$. Figure 4(b) shows similar plots for air-filled bubbles in water ($\rho=1000$ kg m^{-3}) and in a saturated solution of $NaClO_3$ ($\rho=1376$ kg m^{-3}), confirming the ρ and κ dependence of ν.

Although the majority of the hydrophone records were of the form reproduced in figure 3, a minority showed two such damped oscillations closely following one another. High-speed (8000 frame/s) photographic studies showed that this second oscillation was caused by contact between the just released bubble and its successor growing at the nozzle tip. These studies also showed the presence of surface waves on the bubble, but no trace of a corresponding sound emission was detectable in the spectrum. This is only to be expected since surface waves merely require local liquid movements whereas the Minnaert pulsations in a bubble's volume necessarily lead to long-range liquid movements. (A theoretical discussion of the nature of the sound emission attributable to isochoric oscillations has been given by Stokes (1868)).

3.2. The oscillation amplitude

Since the hydrophone used in these studies was calibrated, the output signal (figure 3) tells us the sound pressure $P(r,t)$ at the hydrophone lying at radial distance r at time t from the centre of the bubble. Knowing the amplitude $P_0(r,t)$, the corresponding amplitude $A_0(t)$ of the damped oscillation of the bubble wall can be obtained from the expression

$$P_0(r,t)=\frac{A_0(t)\omega_0^2\rho a}{(1+(kr)^2)^{1/2}} \qquad (4)$$

where k is the circular wavenumber of the sound wave of circular frequency ω_0 (as this frequency has been shown to equal that of the bubble oscillation $\omega_0=2\pi\nu$, where ν is given by equation (3)). For a discussion of the derivation of this expression see, for example, Kinsler *et al* (1980) (where their equation (8.5) relates $P_0(r,t)$ to the bubble wall velocity; to obtain equation (4) one merely assumes a simple harmonic motion of the bubble surface). Although equation (4) tends to the expected limit of $U_0(t)za/r$ at large r, where $U_0(t)$ is the particle speed amplitude at the surface of the bubble and $z=\rho\omega_0/k$ is the specific acoustic impedance for plane waves, it is likely that the hydrophone will be located in the near field and students are unlikely therefore to find a $1/r$ dependence of pressure amplitude. It follows from equation (4) that this near field extends to a radius of order $1/k$ from the bubble (typically a few centimetres). When applied to the hydrophone records (such as figure 3) equation (4) shows, for example, that a 1 mm radius air-filled bubble in water has an initial oscillation amplitude $A_0(0)\approx10^{-8}$ m.

3.3. Damping

The sound pressure amplitude $P_0(t)$ decays exponentially with time, as can be seen from the plot of

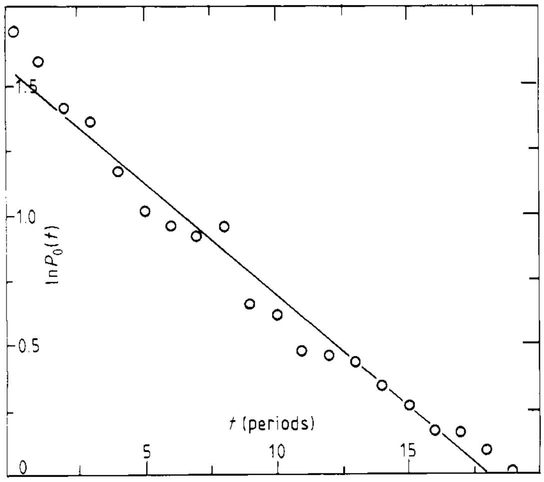


Figure 5. A plot of ln $P_0(t)$ against t, where $P_0(t)$ is the sound pressure amplitude at time t. These data refer to the trace shown in figure 3.

ln $P_0(t)$ against t (figure 5). This indicates that the bubble behaves like a lightly damped simple harmonic oscillator with a characteristic equation of motion:

$$m_r\ddot{\xi} + R_T\dot{\xi} + m_r\omega_0^2\xi = 0$$

where ξ is the bubble wall displacement from the mean value a. R_T is the resistive constant leading to the energy dissipation and m_r is the effective mass of the system (given by equation (2)). This has the familiar solution $\xi = A_0(t)\cos(\omega_0 t + \alpha)$, where $A_0(t) = A_0\,e^{-\gamma t/2}$ with

$$\gamma = R_T/m_r \quad (5)$$

and α is the phase constant. Since $P_0(t)$ is proportional to $A_0(t)$ (equation (4)), ln $P_0(t)$ plotted against t will have a gradient $\gamma/2$, allowing the quality factor

$$Q = \omega_0/\gamma \quad (6)$$

to be obtained. In the case of the air-filled bubble of radius 1.7 mm in water (figures 3 and 5), $Q = 37$. As another example, for a helium-filled bubble of radius 0.87 mm in water, $Q = 6.5$.

There are three main mechanisms for damping of the bubble oscillations. The first is viscous damping but Devin (1959) showed that this is usually negligible in bubbles of the size studied here (amounting to only 1% of the total damping). This conclusion was checked by measuring γ in a variety of different-viscosity liquids ranging from 3.3×10^{-4} N s m^{-2} (acetone) to 0.94 N s m^{-2} (castor oil). Throughout this range γ remained constant to within 10%.

The second damping mechanism arises from the energy radiated as sound. Kinsler *et al* (1980) calculated the impedance for this process, the real part of which is the acoustic radiation resistance

$$R_r = 4\pi a^2 \rho c(ka)^2 \quad (7)$$

where c is the speed of sound in the liquid. It is this resistance which is responsible for the acoustic damping.

The third damping mechanism arises from the conduction away of heat produced as the gas within the bubble is periodically compressed. Devin (1959) has shown that these losses can also be thought of as arising from a thermal radiation resistance

$$R_t = Cm_r\omega_0^{3/2} \quad (8)$$

where the constant C has the value of 1.6×10^{-4} s$^{1/2}$ for the range of bubble sizes considered here.

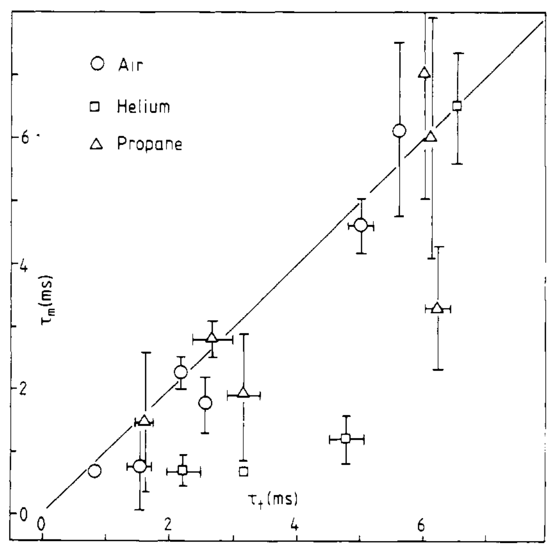
The overall radiation resistance is thus $R_T = R_r + R_t$. It therefore follows from equations (5), (6), (7) and (8) that

$$Q = \frac{\omega_0 m_r}{R_r + R_t} = \frac{1}{(\omega_0 a/c) + C\omega_0^{1/2}}. \quad (9)$$

Since ω_0 follows from equation (3), the Q (or alternatively, the decay time $\tau = 2Q/\omega_0$) can thus be calculated for any bubble. Figure 6 compares the measured decay times τ_m with the theoretical decay times τ_t calculated via equation (9). Although there is reasonable agreement between experiment and theory for air and propane, the measured decay times for helium are significantly less than the predicted values. The cause of this discrepancy is not immediately clear. Similar discrepancies have been noted by other workers (Devin 1959).

To obtain the magnitude of the energy loss per oscillation we recall that a lightly damped system

Figure 6 The measured decay times τ_m for gas-filled bubbles in water plotted against the corresponding theoretical decay times τ_t as calculated via equations (9) and (3). The values of τ_m were obtained directly from hydrophone traces (such as figure 3) by measuring the time taken for the amplitude to decay to 1/e of its initial value.



loses approximately $2\pi/Q$ of its energy – namely $\frac{1}{2}m_r(A_0\omega_0)^2$ – during every cycle. By way of example, an air-filled bubble of radius $a = 1.9$ mm in water with $\omega_0 = 1.15 \times 10^4$ rad s^{-1} and $A_0 = 2.1 \times 10^{-8}$ m (deduced from the calibrated hydrophone signal) has an oscillatory energy of 2.5 pJ. Since $Q = 25$ it therefore follows that the energy loss per oscillation is 0.63 pJ. This loss is, of course, divided between radiative and thermal mechanisms in the ratio of R_r/R_t. It follows immediately from equations (7) and (8) that $R_r/R_t = Da^{1/2}$, where the constant D has a value of 18.3 m$^{-1/2}$ for air bubbles in water. Thus the thermal losses predominate in bubbles of less than 3 mm radius.

4. Babbling brooks

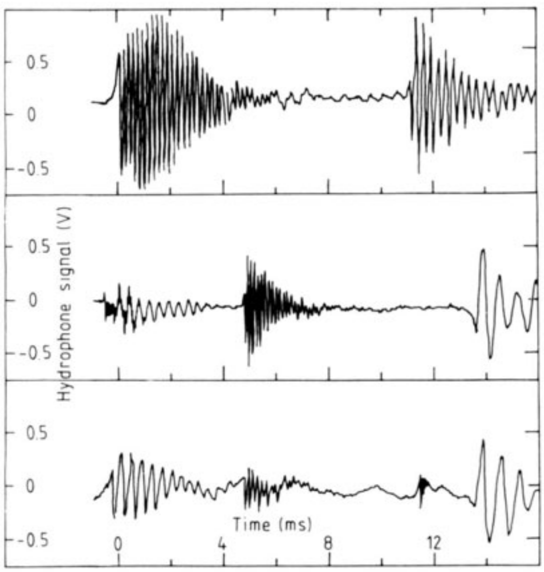
Using the calibrated hydrophone and a portable recorder (Sony Walkman model WM-D6, frequency response 40–15 000 Hz ± 3 dB) recordings were made from various brooks. Figure 7 shows one of the authors (TGL) making such a recording from a brook running down from Kinder Scout in the Peak District (Derbyshire, UK). This shows the hydrophone (mounted at the end of a paxolin tube and lying at some 5 cm below the mean water level) placed within a well localised bubble field. Monitored on headphones, the sound was a series of sharp metallic 'pings' against a much fainter 'white noise' background. When the hydrophone was moved to clear water outside the bubble field (see figure 7) the 'pings' disappeared, suggesting that they arose from bubbles. Back in the laboratory, the recordings were played back into a storage CRO while being monitored on headphones. Figure 8 shows three representative selections of the recording made at the site of figure 7 and it clearly shows a series of Minnaert-like damped oscillations (cf figure 3), each one of which coincides with the presence of a 'ping'. During a sample time of around 30 s, the frequencies of such recorded oscillations were measured directly from the CRO and the results plotted as a histogram. Figure 9 shows these histograms for four of the sites studied by us. Because the sound emission is predominantly due to Minnaert-like oscillations of discrete bubbles it is possible to apply equation (3) to deduce corresponding bubble sizes and these are also indicated in figure 9. Figure 9(a) is for the site illustrated in figure 7; details of the other sites are given in the figure caption.

The broad range of Minnaert frequencies evident in figure 9(a) accords with our observation that the bubble field at this site contained a broad range of bubble sizes. In figure 9(b) the narrow distribution in bubble sound frequencies (and hence in bubble radii) again agrees with our subjective impression as to the range of bubble sizes present. At this site water flowed smoothly over a large submerged stone, creating a narrow stream of bubbles at its trailing edge, where the hydrophone was located. The same narrow spread in bubble radii, but shifted to smaller radii, is also evident in figure 9(c); here the hydrophone was located in a rockpool fed by a single falling stream (this may be compared with the many different streamlets feeding the site of figure 9(a)). The broad range of bubble sizes seen in figure 9(d) is what one might expect at the base of a classic waterfall.

It is worth pointing out that had the audio tape been fed into a spectrum analyser instead of a CRO, the Minnaert-like character of the emitted sound would have been missed and no certain correlation could

Figure 7 One of the authors (TGL) recording sound emitted from a brook running from Kinder Scout in the Peak District (Derbyshire, UK). The hydrophone is mounted at the end of the paxolin rod. The lead runs to the recorder (not shown) where the sound signals are monitored by the second author.

Figure 8. Three selections from the sound recording made at the site shown in figure 7. This trace was obtained by playing the output from the recorder into a storage oscilloscope and pen recorder.



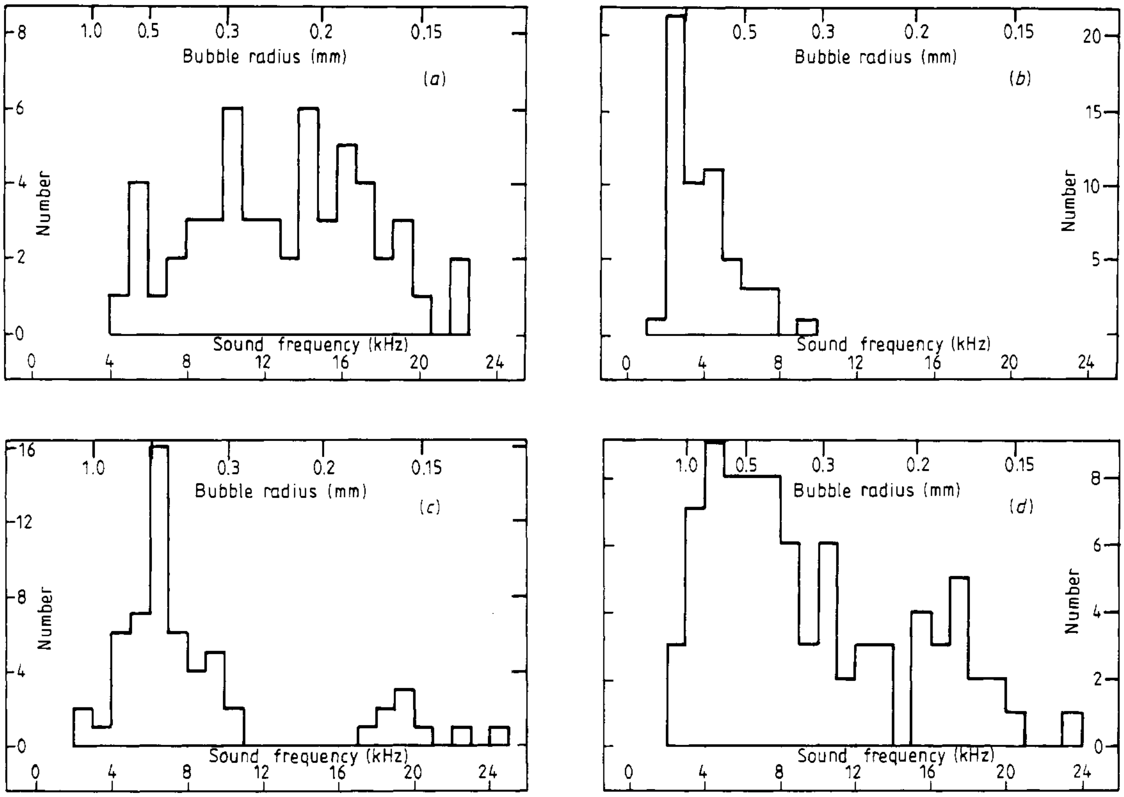


Figure 9 Histograms showing the numbers of bubbles having sound emission frequencies within 1 kHz bandwidths at various sites. The corresponding bubble radii were calculated from equation (3). Site (*a*) is shown in figure 7. At site (*b*) water flowing over a large rounded stone created a narrow stream of bubbles at its trailing edge in which the hydrophone was located; there was no surface splashing. At site (*c*) water flows smoothly, without splashing, over a 30 cm drop into a rock pool containing the hydrophone. At site (*d*) the hydrophone was placed near the base of a 1 m high, fast-flowing waterfall at a point at which the bubbles emerged at the surface.

have been made between the shape of the spectrum and the distribution of bubble sizes. In a recent hydrophone study of the underwater sound produced when various types of precipitation fell on a lake, Scrimger (1985) fed his recordings directly into a spectrum analyser and thus failed to associate the peaks in the observed spectrum with the possible presence of oscillating bubbles. Of course, once an initial CRO study has been made it does make sense to use a spectrum analyser.

5. Conclusions

A gas-filled bubble in a liquid provides a good system for studying a lightly damped simple harmonic oscillator. Furthermore, it encourages students to seek an explanation for 'the murmur of the brook, the roar of the cataract, or the humming of the sea' (to quote Minnaert (1933)). In our view, much current physics teaching, in the lecture theatre and the laboratory, fails to encourage students to look for physics in action in the natural world. This project allows physics students to get out of the laboratory, to make measurements of a natural process about which comparatively little is known, and to analyse their data in terms of familiar undergraduate concepts. In an introductory course the analysis might simply centre around Minnaert's relation; in more advanced courses a study of the absolute sound pressure amplitudes and of the damping processes at work in the bubble can enhance a student's understanding of acoustics and thermal physics.

Acknowledgments

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