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The frequency analysis of transients

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Abstract When a short-lived signal is recorded by a transient recorder which then relays it to a spectrum analyser to resolve the frequency components of that signal, distortions may occur which suggest the presence of spurious resonances in the spectrum. The source and nature of these distortions are discussed here.

Zusammenfassung Wenn ein kurzfristiges Signal von einem Transientenspeicher registriert wird, der es dann auf einen Spektralanalysator überträgt, um die Frequenzkomponenten dieses Signals aufzulösen, können Verzerrungen auftreten, die die Anwesenheit von unechten Resonanzen im Spektrum vortäuschen. Die Quelle und Natur dieser Verzerrungen werden hier untersucht.

A convenient and common way of resolving the frequency components of a short-lived signal is to first capture that signal in a transient recorder, which then continuously replays it into a spectrum analyser. This it must do because the analyser takes time to scan across the frequency regime; it is this process which causes the output of the analyser to misrepresent somewhat the true frequency spectrum of the original signal. The purpose of this article is to point out the nature of the misrepresentation. It is prompted by several cases (such as occurred at the 1986 Institute of Acoustics Conference at Bath, England) in which the misrepresentation led to the reporting of misleading results.

As an illustration of the problem we will consider the case of the sound emitted from the radial oscillations of a gas-filled bubble located within the body of a liquid. The sound emission (recorded via a hydrophone) has the form of an exponentially decaying sinusoidal oscillation (Leighton and Walton 1987). This is normally stored on a transient recorder which is triggered at the start of the signal and records, say, for 20 ms. Thus the signal captured is a truncated, exponentially decaying sinusoid (shown schematically in figure 1(*a*)). This truncated transient is then replayed repeatedly into the spectrum analyser. Here the signal is distorted in two ways. Firstly, what was a 20 ms recording is condensed into the repetition time of the recorder, so all frequencies will be multiplied by the ratio of the signal length to the repetition time. For example, the DL 922 (Datalab) transient recorder has a repetition time of 4 ms, so all frequencies in the 20 ms trace analysed by the spectrum analyser will be a factor of five too great. Such frequency shifts are, of course, easily corrected.

A second and more subtle alteration arises because the signal now received by the analyser is no longer a single truncated decaying sinusoid, but a continuously repeated version of the same (figure 1(*b*)). This can be represented by the original signal (condensed into the repetition time of the recorder) convoluted with a train of delta functions (the separation of which is also the repetition time). The spectrum analyser takes a Fourier transform of the entire input signal, so this convolution will lead to a delta-function train appearing as a multiplier in the analyser's output spectrum.

To formalise the argument, the Fourier transform of the sinusoid (labelled A) is a delta function positioned at the frequency of the sinusoid (labelled A')

$$\mathcal{F}(\sin\omega_0 t) \propto \frac{1}{2\pi}\int_{-\infty}^{\infty} (e^{i\omega_0 t} - e^{-i\omega_0 t})\,e^{-i\omega t}\,dt$$
$$= \delta(\omega - \omega_0) - \delta(\omega_0 + \omega)$$

or

$$\mathcal{F}(A) = A'.$$

Similarly the Fourier transform of a decaying exponential (B) is a Lorentzian (B')

$$\mathcal{F}(e^{-t/\tau}) \propto \int_0^{\infty} e^{-t/\tau}\,e^{-i\omega t}\,dt = (1/\tau + i\omega)^{-1}$$

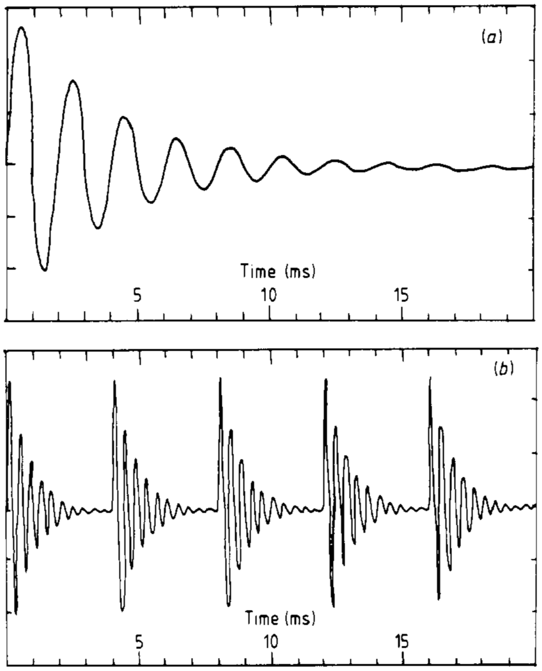

Figure 1 (*a*) Schematic drawing of the exponentially decaying sinusoid, truncated after 20 ms, as recorded by the transient recorder. It has a frequency of 500 Hz and a 4 ms time constant of decay. (*b*) The output of a transient recorder of repetition time 4 ms, which has recorded the signal shown in figure 1(*a*) (schematic drawing).

or

$$\mathcal{F}(B) = B'.$$

The transform of a train of delta functions, spacing t_1, (C), is a similar train in ω-space (C'), of spacing $2\pi/t_1$

$$\mathcal{F}\left(\sum_{n=-\infty}^{\infty} \delta(t - nt_1)\right) = \sum_{n=-\infty}^{\infty} \delta(\omega - 2n\pi/t_1)$$

or

$$\mathcal{F}(C) = C'.$$

So, ignoring any errors due to the truncating of the original decay—these distort the width of the Lorentzian by less than 5% if the truncation occurs after a time equal to three times the time constant of decay—the signal recorded can be thought of as being

$$A \times B$$

and thus the input to the analyser is

$$C*(A \times B).$$

The spectrum analyser takes the Fourier transform of this. Applying the convolution theorem tells us that the analyser's output is

$$C' \times (A'*B')$$

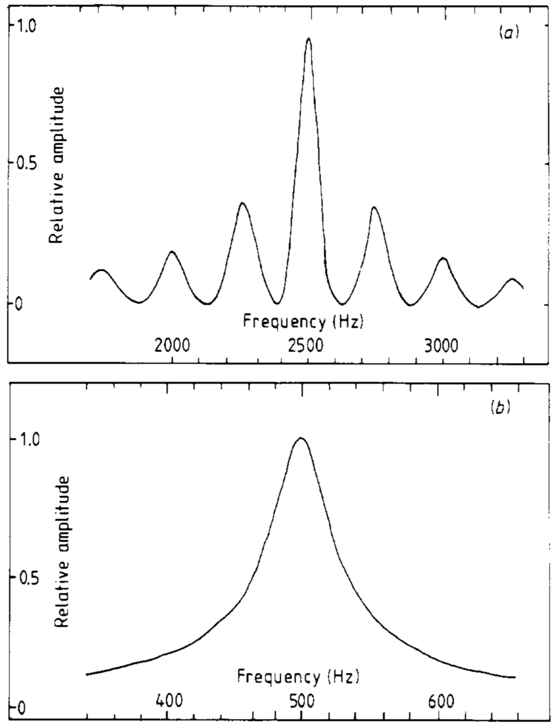

Figure 2 (*a*) The distorted spectral analysis of the signal shown in figure 1(*a*), using the transient-recorder–spectrum-analyser system. It is actually the frequency spectrum of the trace shown in figure 1(*b*). Note that because the spectrum analyser must have a finite window, the delta functions have been broadened (schematic drawing). (*b*) The true frequency spectrum of the signal shown in figure 1(*a*) (schematic drawing).

that is, an array of delta functions under a Lorentzian envelope which is centred on the frequency of the sinusoid (actually, the frequency of the original sinusoid multiplied by the ratio of the trace length to the repetition time of the recorder). This can be seen in figure 2(*a*).

The true representation of the spectrum would be

$$A'*B'$$

that is, a Lorentzian centred on the oscillatory frequency (figure 2(*b*)). It can readily be seen how the misrepresented spectrum could mistakenly suggest the presence of resonances to the unwary experimenter.

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Reference

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