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EXPLOITATION OF HIGHER ORDER STATISTICS TO COMPUTE BUBBLE CLOUD DENSITIES; EVADING OLBER'S PARADOX

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This paper addresses problems associated with the estimation of bubble size distributions from passive data. Existing methods for computing the rate of entrainment of bubbles, and their size distributions, are based on estimating the acoustic energy within a frequency band, i.e. the spectrum of the received signal. This paper demonstrates how such measurements are subject to an ambiguity. It demonstrates that, by taking into account higher order statistics of signals in a frequency band, this ambiguity can be resolved.

1. INTRODUCTION

Of all natural entities in liquid, gas bubbles are the most acoustically active. This, along with the fact that they can occur in numbers greater than 10^6 m^{-3}, means that their impact on the underwater sound field can be very great. Having natural frequencies inversely proportional to their radii, the typical pulsating oceanic air bubble population (having radii of O(mm) to O(μm)) provide pulsation natural frequencies in the frequency range of at least 1-500 kHz respectively, with commensurate quality factors of roughly 30 to 5.

The acoustic emissions they generate so potently in this frequency range when entrained, may be seen as unwanted noise in some circumstances, and highly desirable in others. The characterisation of bubble populations in acoustical oceanography is not only promoted by their potency in affecting sound fields, but also by their importance to environmentally significant processes: coastal erosion and wave dynamics [1]; methane seeps [2]; and the flux between the ocean and atmosphere of momentum, energy and mass [3]. The top 2.5 m of the ocean has a heat capacity equivalent to the entire atmosphere; and the flux between atmosphere and ocean of carbon alone exceeds 10^9 tonnes/year [4]). Entrainment emissions have been monitored in waterfalls and babbling brooks [5]; under breaking waves; in ships wakes; in industrial sparging; and during rainfall over water [6], where the "lack of surface rainfall data is a recognized problem…especially in oceanic regions… Data are needed to identify occurrence of rain, type of rainfall, and quantification of rainfall amounts… to understand the local, regional and global heat and water budget which control the circulation of the atmosphere and upper ocean" [7].

One of the prime purposes of such monitoring is to estimate the number of bubbles involved, and their size distribution. At low entrainment rates the natural frequencies of individual bubbles are easy to identify [5]. However when they overlap in noisy environments this becomes less easy. Two options are then available. The first is to try to identify these individual emissions using signal processing techniques, such as the Gabor transform [5]. The second is to try to invert a model of the overall emission of the bubble cloud [8]. They might also be used for inversion (estimation of the bubble size distribution from the measured acoustic field), for example to test models of the evolution of bubble clouds under breaking waves [8]. However such spectral techniques may produce ambiguities in the inversion. These and their resolution are discussed below.

2. PRINCIPLES

The goal of this text is to outline the key reasons as to why Higher Order Statistics (HOS) should be considered in this application. To realise this objective we shall consider an illustrative simple geometry: the extensions to realistic geometries are more or less straightforward. Consider an omni-directional hydrophone at the centre of homogeneous medium. In a spherical region, of radius R, centred on the hydrophone bubbles are being formed in a manner that is homogeneous in both time and space; that is to say that the mean rate of entrainment is uniform within the sphere. Outside the sphere it is assumed that no entrainment takes place.

Let $N(\omega_0)$ denote the number of ringing bubbles with natural frequency ω_0 generated per unit volume, in unit time within a 1 Hz frequency band; we shall refer to this as the *entrainment density*. It is assumed that the acoustic pressure signature of any one bubble is

$$y(t;\omega_0,h) = Ae^{-\alpha t}\sin(\omega_0 t) \qquad t>0 \tag{1}$$

where the damping coefficient, α and the natural frequency ω_0 depend on the bubble's radius [11], and the amplitude of excitation A depends on the entrainment depth, h [11]. For the purposes of this paper we shall assume that A is independent of depth and treat it as a constant. Note we explicitly avoid inserting specific models relating the damping and natural frequencies into (1); various forms can be found in the literature [11] and the structure of our method is unaffected by this choice.

The power spectral density (PSD), $S(\omega)$, of the signal received at the hydrophone can (assuming a spherical spreading model) be shown to be:

$$S(\omega) = \int_{vol}\int_{freq} N(\omega_0)\frac{Y(\omega;\omega_0)}{r^2}d\omega_0 dV \tag{2}$$

where the two integrals are taken over the volume of the bubble cloud and the frequency band of interest respectively. Here r is the distance from an elemental volume dV, to the hydrophone; and $Y(\omega;\omega_0)$ is the squared magnitude of the Fourier transform of (1). The notation aims to make clear the distinction between the natural frequency ω_0 and the frequency resulting from the Fourier transformation ω. Integrating over the assumed spherical bubble cloud gives:

$$S(\omega) = 4\pi R \int N(\omega_0) Y(\omega;\omega_0)\, d\omega_0 \qquad (3)$$

The key observation from (3) is that the measured PSD is proportional to the overall radius of the bubble cloud. This formulation predicts that the acoustic energy at the hydrophone grows without bound as the size of the cloud increases. This is a form of Olber's Paradox [12] (the observation that the night sky should be uniformly bright, if the universe were infinite and the distributions of stars homogeneous). If one considers spherical shells of radius δr then each such shell adds to the integral (3) a quantity that is independent of r, so that distant and nearby shells make equal contributions. Hence if one seeks to employ (3) to estimate the entrainment density, $N(\omega_0)$, it is necessary to obtain an estimate of the effective radius of the bubble cloud, R, or to impose constraints upon the geometry. In many applications it is difficult to obtain a reasonable estimate for R.

Estimates of the entrainment density are obtained by discretising (3) as

$$S(\omega_n) = 4\pi R \sum_m N(\omega_{0,m}) Y(\omega_n;\omega_{0,m})\, \delta\omega_{0,m} \approx 4\pi R N(\omega_{0,n}) Y(\omega_n;\omega_{0,n})\, \delta\omega_{0,n} \qquad (4)$$

where $N(\omega_{0,n})$ is the mean entrainment density in the n^{th} frequency bin; $Y(\omega_0;\omega_{0,n})$ is the energy integrated over the n^{th} frequency bin from a bubble in the centre of bin m; and $\delta\omega_{0,n}$ is the width of the n^{th} frequency bin. The exact form in (4) can be solved using matrix methods. The approximation neglects the contributions that bubbles with natural frequencies $\omega_{0,n}$ make to adjacent frequency bins, i.e. it removes coupling between data in different frequency bins. This approximation is reasonable if the frequency bin widths $\delta\omega_{0,n}$ are large in comparison to bandwidths of the individual bubble signatures (1), but sufficiently small so that the value entrainment density can be regarded as being constant within a bin. Under these conditions the solution to (4) can be expressed as

$$N(\omega_{0,n}) = \frac{S(\omega_n)}{4\pi R Y(\omega_n)\, \delta\omega_{0,n}}; \qquad Y(\omega_n) \quad Y(\omega_n;\omega_{0,n}) \qquad (5)$$

In order to remove the dependence of the results on the parameter R we shall exploit HOS. In particular we shall seek to compute the fourth order cumulant C_x of the received waveform, $x(t)$, defined as:

$$C_x = E\left[x(t)^4\right] - 3E\left[x(t)^2\right]^2 \qquad (6)$$

where $E[]$ denotes the expectation operator. The use of cumulants (rather than moments) is motivated by two factors [13,14]. Firstly cumulants obey a super-position principle for independent random processes; greatly simplifying the subsequent theory. Secondly cumulants (of order greater than two) are unaffected by additive Gaussian noise.

For the sake of brevity we shall assume there is no coupling between frequency bins. The extension to include coupling is straightforward. In which case our method is direct analogous to that defined by (5). In the absence of such coupling one can write

$$C_x(\omega_n) = \int\limits_{vol} N(\omega_n)\frac{K(\omega_n)}{r^4}dV; \qquad K(\omega_n) \quad E\left[y(t;r,h)^4\right] \tag{7}$$

where $C_x(\omega)$ is fourth order cumulant computed in the n^{th} frequency bin. Expressing the volume integration in (7) using spherical polar co-ordinates leads to

$$C_x(\omega_n) = 4\pi K(\omega_n)N(\omega_n)\int\limits_{r=0}^{R} \frac{1}{r^2}dr \tag{8}$$

Equation (8) is unbounded because of its behaviour close to r=0. The physical reason for this is that our analysis so far is based on a far-field approximation for the amplitude of the received pressure waveform. It is evident that as r approaches 0 a far-field approximation is inappropriate. In order to ensure the convergence of the integral in (8) one needs to adopt a more realistic model of how the pressure amplitude of a bubble signature varies as a function of range to the transducer. We shall assume that the transducer is small in comparison to the wavelength, implying an omni-directional beam-pattern for the transducer – more formally ka<<1, where a is the radius of transducer's face plate. Under these circumstances one assumes that the off-axis transducer response is equal to the on-axis response. This is equivalent to replacing the integral in (9) by [15]

$$\int\limits_{r=0}^{R} r^2\left(\frac{4c}{\omega a^2}\right)^4 \sin\left(\frac{\omega r}{2c}\sqrt{1+\left(\frac{a}{r}\right)^2}-1\right)^4 dr = I_R(\omega,a) \tag{9}$$

This integral is not analytically tractable but can be readily be approximated using numerical methods (since this can be performed off-line, the computational burden this imposes is of little practical importance). This integral converges to a finite limit as $R\to\infty$, so for large bubble clouds we can employ the value $I_\infty(\omega,a)$. This leads to an estimate of the entrainment density given by

$$N(\omega_0) = \frac{C_x(\omega_0)}{4\pi I_\infty(\omega_0,a)K(\omega_n)} \tag{10}$$

In contrast to (5) the above does not depend on the size of the bubble cloud, other than to assume that the cloud is sufficiently large as to allow one to replace $I_R(\omega,a)$ with $I_\infty(\omega,a)$. Figure 1 plots the values of $I_R(\omega,a)$ for a particular configuration. In this case the integral converges for values of R in the region of 10cm.

3. RESULTS

In order to demonstrate the utility of this method we present the results of a simulation studies. Random realisations of homogeneous, spherical, ringing bubble fields are constructed, with controllable entrainment densities, radii and controllable transducer surface area. The numerical model computes the time series at the hydrophone output. Two examples of such time series are shown in Figure 1 – these data are constructed so that there PSDs are equal, i.e. so that $RN(\omega)$ is constant. From this Figure one can see that the two time series are very different: the data from the more dense bubble cloud (Figure 2a) appears much less impulsive that the data from more disparate bubble cloud (Figure 2b). By computing the fourth order cumulant we are constructing a measure of this impulsiveness.

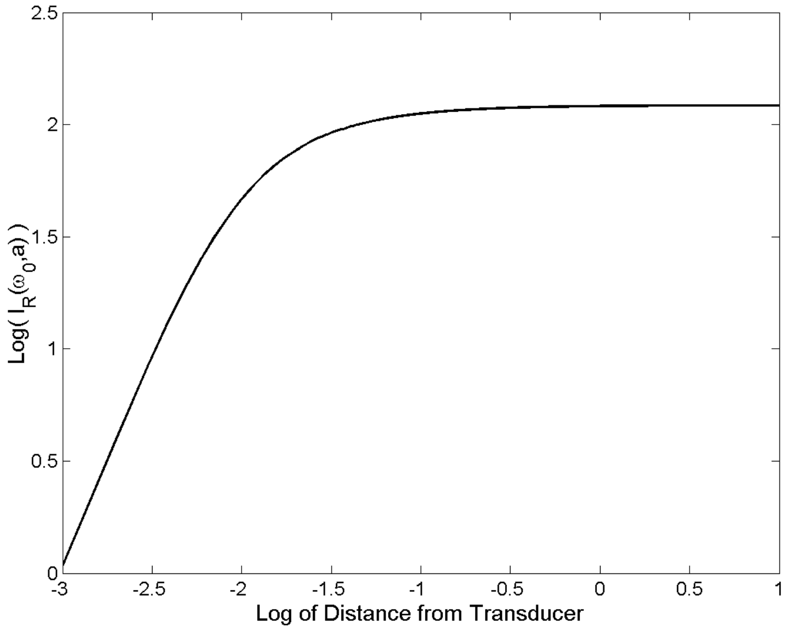

Figure 1: Plot of the value of $I_R(\omega, a)$ for a=1cm at 3kHz.

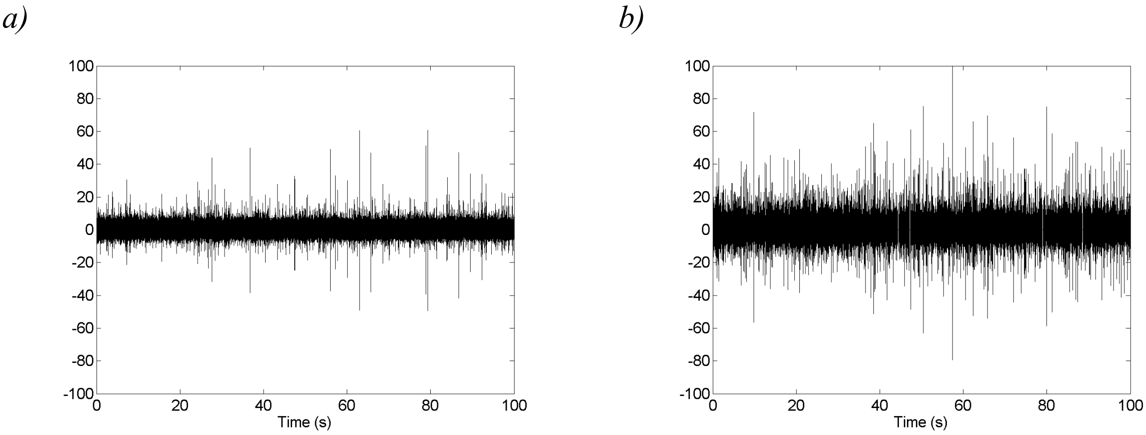

Figure 2: Simulated time series for bubble clouds with different entrainment densities (see Figure 2 for details) a) Low density cloud; b) High density cloud

In Figure 3 one can see a comparison of the energy and the fourth order cumulant measured in 1kHz frequency bands. This demonstrates that for these two cases shown in Figure 1 the energy curves are very close, whereas the cumulant curves are very different. This is a simple illustration of how the fourth order cumulant differentiates cases for which the energy based methods give an ambiguous result.

a) *b)*

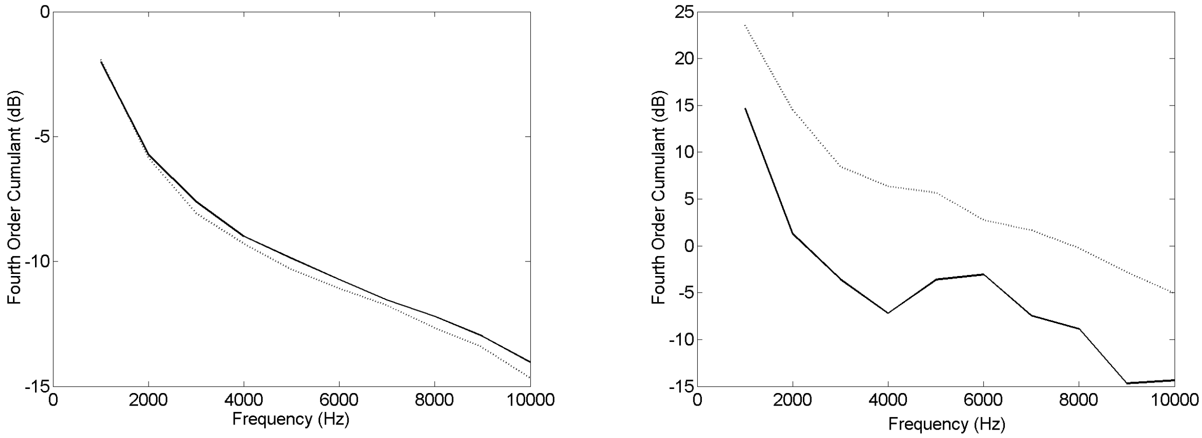

Figure 3: Energy and Fourth order cumulants computed in energy bands for two bubble clouds (radii 1 and 0.25) of entrainment densities: $N(\omega_0)=1\ Hz^{-1}s^{-1}m^{-3}$ and $4\ Hz^{-1}s^{-1}m^{-3}$

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