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A Theoretical Study of Factors Affecting the Biot Slow Wave in Cancellous Bone

by

E.R. Hubbuck[1], T.G. Leighton[1], P.R. White[1], G.W. Petley[2]

[1] *Institute of Sound and Vibration Research, University of Southampton*

[2] *Medical Physics and Bioengineering, Southampton University Hospitals NHS Trust*

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Professor P. A. Nelson
Group Chairman

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Abstract

Biot's theory of elastic propagation in fluid-saturated porous media has been previously used to model ultrasonic propagation in cancellous bone. This is in an attempt to improve clinical systems which diagnose the age-related bone disease, osteoporosis. This report studies the interaction between ultrasound and cancellous bone in some detail, focussing on the factors affecting the propagation of the Biot slow wave, such as frequency and mechanical conditions. Cancellous bone is shown to have a relatively "weak frame", which influences the frequencies at which the slow wave may be easily observed. For healthy bone, the slow wave may be subject to viscous and/or scattering losses at all frequencies. In osteoporotic bone, there is a relatively narrow bandwidth where the slow wave may not be subject to non-negligible viscous or scattering losses.

Glossary of Symbols and abbreviations

a	pore radius (m)
BUA	Broadband Ultrasonic Attenuation ($dB.MHz^{-1}.cm^{-1}$)
C_D	Diffusivity in Biot's theory
d_s	viscous skin depth (m)
E_s	Young's modulus of solid (GPa)
E_b	Young's modulus of solid frame (GPa)
$F(\omega)$	Damping term in Biot's theory
f_{crit}	Critical frequency (Hz)
K_f	Fluid bulk modulus (GPa)
K_b	Frame bulk modulus (GPa)
K_s	Solid bulk modulus (GPa)
k_o	d.c. permeability (m^3)
N	Shear modulus (GPa)
n	Index of iteration
P	Generalised elastic coefficient
Q	Generalised elastic coefficient
q	Complex wavenumber
R	Generalised elastic coefficient
s	structure factor
U	Average fluid displacement
u	Average solid displacement
V_{fast}	Fast wave phase velocity (m/s)
V_{fluid}	Fluid velocity (m/s)
V_{shear}	Shear wave phase velocity (m/s)
V_{slow}	Slow wave phase velocity (m/s)

Greek Letters

α	tortuosity
β	porosity fraction
η	shear viscosity (Pa.s)
Λ	dynamic pore size parameter
π	pi
ρ_f	Density of fluid (kg/m^3)
ρ_s	Density of solid (kg/m^3)
ρ_{12}	Cross-mass density term
υ_b	Poisson's ratio of solid frame
υ_s	Poisson's ratio of solid
ς	constant
ω	Frequency in (rads/sec)

I. Introduction

Osteoporosis is a bone wasting disease that contributes to causing thousands of bone fractures in the elderly annually. The disease cannot yet be cured, but if diagnosed sufficiently early, it may be treated effectively, hence reducing the risk of fracture. Recent advances have shown that ultrasonic techniques, such as Broadband Ultrasonic Attenuation (Langton *et al.* 1984), offer cheap and safe bone assessment, and may provide information to complement radiological data. However, little is currently understood about what physical factors affect ultrasonic measurements in clinical tests.

Bone is made up of two distinct types: dense and compact "cortical" bone and porous "cancellous" bone. Cancellous bone is an inhomogeneous matrix of bony strands, known as trabeculae, saturated with fatty marrow (Figure 1), found at the core of load bearing bones. Osteoporosis causes the trabecular structure to be eroded, such that the porosity of the cancellous architecture may increase from around 70 % to 95 % (Mellish *et al.* 1989).

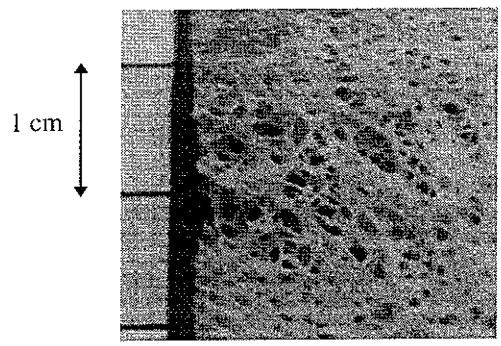

Figure 1 *- Photograph of bovine cancellous bone in the femur*

It is believed that ultrasonic measurements are affected not only by the density of bone, but also by its microstructure (Gluer 1993, Petley 1994). Since the interaction between ultrasound and cancellous bone is not fully understood, examination of an accurate theoretical model is vital if diagnostic ultrasound is to be applied in an optimal way. Biot's theory has been used to study the interaction between ultrasound and cancellous bone by previous authors on a number of occasions. Phase velocity was accurately predicted (Williams (1992), Lauriks *et al.* (1994), Williams *et al.* (1996)), but there was a poor correlation between predicted and measured attenuation (Hosokawa and Otani (1997)), although qualitative changes in attenuation due to simulated osteoporotic effects could be modelled (McKelvie and Palmer (1991)).

This report discusses some aspects of Biot's theory regarding the propagation of waves in cancellous bone. Section II outlines Biot's theory and defines critical parameters. Section III discusses the influence of the contrast between skeletal frame and fluid characteristics and investigates the condition for cancellous bone. Section IV discusses how these results may affect the propagation of the Biot slow wave in cancellous bone.

II. Biot's theory

Biot's theory (1956 a, b) is an established way of predicted ultrasonic propagation in inhomogeneous media, having been extensively applied in the geophysical world. The theory describes elastic wave propagation in fluid-saturated porous media and predicts that three waves propagate simultaneously. These are two compressional waves of differing velocities, known as "fast" and "slow" waves, and one shear wave. The study of ultrasonic propagation in the fluid-filled porous matrix of cancellous bone lends itself to the application of Biot's theory.

In the derivation of Biot's theory (1956 a, b), the average fluid displacement, ***U(x,t)*** and average porous solid displacement, ***u(x,t)***, are considered separately at first, and later, dynamic coupling between the solid and fluid phases is included. This results in two sets of coupled linear equations

$$\nabla^2(Pe + Q\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}e + \rho_{12}\varepsilon)$$
$$\nabla^2(Qe + R\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}e + \rho_{22}\varepsilon) \qquad (1)$$

$$N\nabla^2 w = \frac{\partial^2}{\partial t^2}(\rho_{11}w + \rho_{12}\Omega)$$
$$0 = \frac{\partial^2}{\partial t^2}(\rho_{12}w + \rho_{22}\Omega) \qquad (2)$$

where $e = div\ \boldsymbol{u}$, $\varepsilon = div\ \boldsymbol{U}$, $w = curl\ \boldsymbol{u}$ and $\Omega = curl\ \boldsymbol{U}$. The terms P, Q and R are generalised elastic coefficients, which may be expressed as

$$P=\frac{\beta\left(K_s/K_f-1\right)K_b+\beta^2K_s+(1-2\beta)(K_s-K_b)}{1-\beta-K_b/K_s+\beta\,K_s/K_f}+\frac{4N}{3}, \quad (3)$$

$$Q=\frac{(1-\beta-K_b/K_s)\beta K_s,}{1-\beta-K_b/K_s+\beta K_s/K_f} \quad (4)$$

$$R=\frac{K_s\beta^2}{1-\beta-K_b/K_s+\beta K_s/K_f}, \quad (5)$$

where K_s, K_f and K_b are the bulk moduli of the solid, the fluid and the skeletal frame, respectively, and N is the shear modulus.

The terms ρ_{11} and ρ_{22} in equations (1) and (2) are partial densities, which are given as

$$\rho_{11}+\rho_{12} = (1-\beta)\rho_s, \quad (6)$$

$$\rho_{12}+\rho_{22} = \beta\rho_f, \quad (7)$$

where ρ_s is the solid density, ρ_f is the fluid density and ρ_{12} is a cross-mass density term. The term β is the interconnected porosity.

For plane waves of the form, $\boldsymbol{e} = C_1 \exp(j(\boldsymbol{q}x - \omega t))$ and $\varepsilon = C_2 \exp(j(\boldsymbol{q}x - \omega t))$, where C_1 and C_2 are constants and $\boldsymbol{q}$ is the complex wavenumber, the solutions to equations (1) and (2) give the normal modes of shear and compressional waves through expressions for the velocities of the slow, fast and shear waves

$$V^2_{fast,slow} = \frac{\Delta \pm [\Delta^2 - 4(PR-Q^2)(\rho_{11}\rho_{22}-\rho_{12}^2)]^{1/2}}{2(\rho_{11}\rho_{22}-\rho_{12}^2)}, \quad (8)$$

$$V^2_{shear} = N \;/\; [(1-\beta)\rho_s + (1-1/\alpha)\beta\rho_f], \quad (9)$$

where $\Delta = P\rho_{22} + R\rho_{11} - 2Q\rho_{12}$. The real part of the root provides the phase velocity (m/s), while the attenuation in Np/m, can be found from the imaginary part of the wavenumber, $\boldsymbol{q}_{fast,slow,shear} = \omega / V_{fast,slow\ shear}$.

Equation (8) governs the propagation of two compressional waves, known as fast and slow waves. In fast wave motion, the overall macroscopic and fluid movement are in-phase, essentially a bulk wave travelling through the aggregate. The slow wave corresponds to the fluid moving out-of-phase with the solid/fluid combination, that is, there is relative motion between solid and fluid. Equation (9) describes shear wave propagation.

Central to Biot's theory is the dynamic interaction between fluid and solid, incorporated in coupling forces: viscous and inertial coupling. These two forces allow the locking together of the fluid and solid for fast wave motion, but, if large, they prevent the relative motion associated with the slow wave. Biot's theory is divided into low and high frequency ranges and inertial coupling is dominant at high frequencies, while viscous coupling dominates at low frequencies.

Inertial coupling, that is, the inertial drag exerted on the solid by the fluid as one is accelerated relative to the other, is represented by the coefficient, ρ_{12},

$$\rho_{12} = (1 - \alpha)\beta\rho_f\,, \tag{10}$$

where the term α, the tortuosity. In Biot's papers (1956 a, b) the tortuosity was a purely geometric description of solid fused spheres which related α, to a structure factor, s

$$\alpha = 1 - s(1 - 1/\beta)\,. \tag{11}$$

Johnson *et al.* (1987) introduced a frequency-dependent tortuosity, $\alpha(\omega)$, which uses fluid viscosity, η, and density, ρ_f.

$$\alpha(\omega) = \alpha_\infty + j\eta\beta\left(1 - \frac{4j\alpha_\infty^2 k_0^2 \rho_f \omega}{\beta^2 \Lambda^2}\right) / \omega\rho_f k_0 \tag{12}$$

where α_∞ is the experimentally determined tortuosity, Λ is a pore size parameter and k_0 is the permeability (Johnson *et al.* 1987).

The second coupling force in Biot's theory is viscous coupling. This is caused by shearing friction at the fluid/solid interface as the fluid flows relative to the solid. Viscous forces are characterised by a frequency-dependent parameter known as the viscous skin depth, d_s,

$$d_s = (2\eta / \rho_f \omega)^{1/2}. \tag{13}$$

The low and high frequency regions of Biot's theory intercept where the viscous skin depth equals the pore radius, a, at the critical frequency ω_{crit},

$$\omega_{crit} = 2\eta / \rho_f a^2 \ . \tag{14}$$

Below ω_{crit}, the slow wave is diffusive and only the fast wave propagates, but above this frequency both fast and slow waves are propagatory modes.

III. The influence of frame rigidity

As shown above, certain conditions, such as frequency, divide Biot's theory in distinct domains. Another crucial condition is the ratio between the bulk moduli of the fluid and solid frame. The influence of this condition was outlined by Johnson and Plona (1982), whose equations are used in this section to investigate the conditions for cancellous bone.

Propagation in a porous medium exhibits behaviour at the Rigid Frame limit of Biot's theory when $K_b >> K_f$, $K_s > N$, that is, when the bulk modulus of the solid frame is much greater than that of the fluid, and the solid bulk modulus is greater than the shear modulus. At this limit, the motion of the fluid and solid decouple, causing the fast wave to propagate mainly in the solid frame, while the slow wave propagates mainly in the fluid. This allows significant simplification of the equations of motion, such that the wave velocities can be approximated as,

$$V_{fast} \approx \left[\left(K_b + \tfrac{4}{3}N\right) / \left(\left(1-\beta\right)\rho_s + \left(1-\alpha^{-1}\right)\beta\rho_f\right)\right]^{1/2}, \tag{15}$$

$$V_{slow} \approx V_{fluid} / \sqrt{\alpha} \ . \tag{16}$$

It is possible to see in equation (16) that the tortuosity, α, is a crucial parameter for the slow wave velocity in the Rigid Frame limit. Therefore, the slow wave velocity indicates the something of the structure of a porous matrix, if conditions are satisfied.

Of similar interest is the limit contrary to this, that is, the Weak Frame limit. This exists when $K_b, N << K_f, K_s$, that is, the bulk modulus of the solid frame, and the shear modulus, are much less than the fluid and solid bulk moduli, respectively. An extreme case would be when $K_b = N = 0$, such as grains in a suspension, where $PR - Q^2 = 0$, and, from equation (8) the slow wave speed is found to equal zero and only the fast wave propagates.

The assumption that cancellous bone exhibits behaviour at the Rigid Frame limit has been made (Williams 1992), but there is no evidence for this. If this was a valid assumption, the propagation equations could be simplified, and the slow wave velocity could provide a measure of the tortuosity. This claim can be investigated, as presented here.

From the work of Gibson (1985), the frame bulk modulus and shear modulus of cancellous bone may be calculated from equations (17) to (20), which assume the material is isotropic to avoid complications of an anisotropic matrix. A power law is used to relate the Young's modulus of the frame, E_b, to the porosity, β, and the Young's modulus of the solid, E_s, by,

$$K_b = E_b \;/\; 3(1 - 2\upsilon_b), \tag{17}$$

$$K_s = E_s \;/\; 3(1 - 2\upsilon_s), \tag{18}$$

$$E_b = E_s\,(1 - \beta)^n, \tag{19}$$

$$N = E_b \;/\; 2(1 + 2\upsilon_b), \tag{20}$$

where K_b is the frame bulk modulus, E_b and E_s are the Young's moduli of the frame and solid respectively, υ_b is Poisson's ratio, and β is the porosity, and the factor, n, depends on the alignment of the structure. Isotropy may be assumed in cancellous bone when trabeculae, which grow along lines of stress, are aligned with the direction of propagation. This has been accounted for in many experimental studies (Williams 1992, Hosokawa and Otani 1997).

The bulk modulus of the frame of cancellous bone, K_b, was evaluated from equations (17) and (19), using parameters in Table 1 found *in vitro* from the sources indicated. The index, n, was found by Williams (1992) from iterative curve-fitting procedures, taken to give least-squares convergence, of experimental velocities versus porosity. Here, the value $n = 1.23$ is used for well oriented bone.

Table 2 contains the values of the bulk moduli of the frame, K_b, and solid bone, K_s, and the shear modulus, N, as calculated from equation (17) - (20), for normal and osteoporotic cases, and the bulk modulus of the marrow, as reported in the literature.

Table 1 - Input parameters

Porosity, β - Healthy	0.76 ± 0.03	i
Osteoporotic	0.90 ± 0.04	i
Young's modulus, E_s	20.5 ± 2.6 GPa	ii
Index, n	1.23	iii
Poisson's ratio, υ_b, υ_s	0.32	iv,v

Table 2 - Table of material moduli

Bulk Moduli	Healthy	Osteoporotic
Frame, K_b	3.2 ± 0.6 GPa	1.2 ± 0.5 GPa
Shear, N	1.2 ± 0.2 GPa	0.39 ± 0.18 GPa
Solid, K_s	18.9 ± 2.4 GPa	
Fluid, K_f	2.2 GPa	

[i] *Mellish et al. (1989),* [ii] *Duck (1990),* [iii] *Williams(1992),* [iv] *Katz and Meunier (1970),* [v] *Lang (1987).*

It can be seen from Table 2 that for both healthy and osteoporotic bone the Rigid Frame conditions, are not met. In both cases, the frame bulk modulus, K_b is of the same order of magnitude as that of the marrow, K_f, and the solid bulk modulus, K_s, is greater than the shear modulus, N. It appears that cancellous bone has a relatively weak frame, and clearly the Rigid Frame assumptions of equations (16) cannot be made for propagation in cancellous bone.

IV The slow wave frequency window

IVa The effect of frequency on the slow wave

In practice, the lack of frame rigidity has been found to affect the observation of Biot slow waves (Johnson and Plona 1982). The Biot slow wave is often difficult to observe in practice, since its propagation involves relative motion between fluid and solid frame, which causes high attenuation from viscous losses.

In a porous material, the existence of the slow wave as an observable propagatory mode depends on certain factors. Conditions enabling propagation include the continuity of the fluid phase, a low fluid viscosity, a large pore radius and high permeability. Theoretically, the slow wave may only propagation in the high frequency region of Biot's theory (section II), where the viscous skin depth, d_s, is considerably less than the pore radius. Due to the fact that viscous effects do not "cut-off" abruptly at the critical frequency, viscosity still affects propagation up to a practical limit, known as the "viscous frequency", $\omega_{viscous}$, which using equation (13) and for the pore size, a, can be expressed as

$$\omega_{viscous} = (2\eta \; / \; \rho_f a^2 \zeta^2) \; . \qquad (21)$$

The constant, ζ , which represents the restriction of motion due to the viscosity being greater for the slow wave than the fast wave, has been found experimentally to be approximately 0.01(Johnson and Plona 1982).

In practice, there is also an upper frequency limit above which observation of slow waves becomes difficult because of losses due to scattering. Scattering losses increase as the wavelength becomes close to the size of the structural scale, and, at a given frequency, the slow wave tends to suffer more scattering than the fast wave. Scattering sets in at the frequency where $q_{slow}.a = 1$, the wavenumber of the slow wave, with speed V_{slow}, where $q_{slow} = \omega \; / \; V_{slow}$, of equation (8).

Between viscous and scattering frequency limits there is a frequency window where the slow wave may easily be observed, which is specified by

$$(\omega_{crit}) \; < \omega_{viscous} \; < \; \omega \; < \; V_{slow} \; / \; a \; . \qquad (22)$$

The slow wave propagates above the critical frequency ω_{crit} , but it is only above $\omega_{viscous}$, the viscous frequency, that it will be easily observed. Similarly, this wave will propagate above the scattering frequency V_{slow} / a, but attenuation due to scattering makes it difficult to observe.

Viscous and scattering losses arise gradually with decreasing and increasing frequency, respectively, as in Figure 2. This figure is intended simply to illustrate the decreasing viscous effects and increasing scattering losses with increasing frequency, and the rates of decay are not evaluated. Therefore, equation (22) actually represents the bandwidth where the slow wave will be most effective, and is a guide established from practical experience.

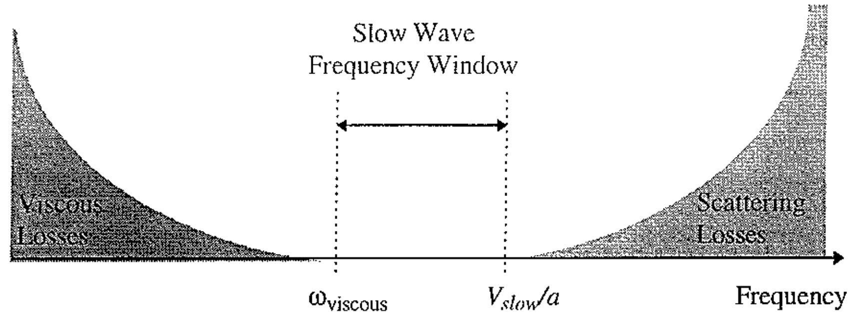

Figure 2 *- The slow wave frequency window*

IVb The effect of frame rigidity on the slow wave

To observe a slow wave in cancellous bone the ratio of the slow wave wavenumber, q_{slow}, to the pore radius, a, is an important factor. In the high frequency region, the slow wave propagates with speed V_{slow}, which can be represented as a function of the frame and shear moduli in combination (Johnson and Plona 1982),

$$V_{slow} \;\alpha\; K_b + {}^4/_3 N . \tag{23}$$

By writing the slow wave velocity in this way, the influence of the frame and shear moduli on the width of the frequency window can be examined. Ensuring $q_{slow}.a > 1$, to avoid scattering, q_{slow} must be considerably smaller than $1/a$, where a is constant. This implies that V_{slow} must be large to avoid scattering, since $q_{slow} = \omega / V_{slow}$, and ω is initially large. As the material approaches the weak frame limit, the term $(K_b + {}^4/_3N)$ tends to zero, V_{slow} decreases, and the frequency keeping $q_{slow}.a > 1$, must also decrease. For small enough values of $(K_b + {}^4/_3N)$, the frequency at which scattering sets in can be very low, possibly lower than the frequency of the viscous limit, $\omega_{viscous}$. Hence, for a medium where the solid frame is "weak", there is no frequency window in which a strong slow wave may be observed.

IVc The slow wave window for cancellous bone

The slow wave bandwidth was calculated for human cancellous bone in normal and osteoporotic states. The lower limit, $f_{viscous}$, was calculated using equation (21) and the upper limit, V_{slow}/a, was calculated using equations (3) - (8), (10) and (12), in Hertz. The value of the critical frequency, f_{crit}, of equation (13) and the effect of skin depth with frequency were studied, using the values in Table 3.

Figure 3 shows a graph of viscous skin depth versus, for a viscosity of $\eta = 1.495$ Pa.s and $\rho_f = 970$ kg/m^3. The average pore radii for healthy and osteoporotic bone are shown in dashed lines, taken from the work of Mellish *et al.* (1989), including error bars indicating the standard deviation in the population. This is to indicate the frequency at which the two intersect in each case, that is, the critical frequency, f_{crit} that divides the two regions of Biot's theory.

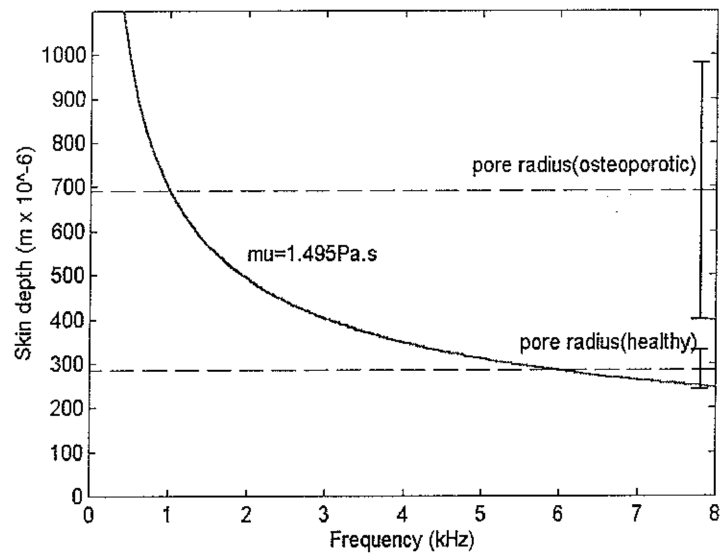

Figure 3 *- Graph showing viscous skin depth versus frequency*

Figure 3 shows how the viscous skin depth decreases with increasing frequency. The critical frequency, f_{crit}, where the curve and dashed lines intersect, has a mean value for healthy bone of 6 kHz, and 1.1 kHz, for osteoporotic bone. Clinical systems use the frequency range 200 kHz - 600 kHz (Langton *et al.* 1984), and, from the above results it can be concluded that they operate in the high frequency region of Biot's theory, where two compressional modes propagate.

Table 3 - Table of input parameters for the Biot model

Density of solid bone, ρ_s		1940 ± 85 kg/m^3	i	
Density of marrow, ρ_f		983 ± 50 kg/m^3	i	
Young's modulus of bone, E_s		20.5 ± 2.6 GPa	i	
Bulk modulus of marrow, K_f		2.2 GPa	ii	
Poisson's ratio, υ_s, υ_b		0.32	iii	
Fluid viscosity, η		1.495 Pa.s	iv	
Index, n		1.23	ii	
Tortuosity, α		1.58	v	
		Healthy		Osteoporotic
Porosity, β	vi,vii	0.76 ± 0.03		0.90 ± 0.04
Pore radius, a	vi,vii	285 ± 46 μm		690 ± 290 μm
Permeability, k_o	iv	5×10^{-9} m^3		5×10^{-7} m^3

[i] *Duck (1990),* [ii] *Williams (1992),* [iii] *Lang (1987),* [iv] *McKelvie and Palmer (1991),* [v] *Williams et al. (1996),* [vi] *Croucher et al. (1994),* [vii] *Mellish et al. (1989).*

The parameter values in Table 3 were taken for human cancellous bone. The solid was taken as cortical bone, and the fluid was assumed to be equivalent to fat. It is assumed that material densities and the fluid viscosity do not change with osteoporosis. The least well defined, but most influential parameter in the model, is the viscosity of the marrow, η, which affects ω_{crit}, $\omega_{viscous}$ and the calculation of velocities, which will depend, among other things, on temperature.

Table 4 contains the frequencies of the viscous and scattering limits for healthy and osteoporotic bone. Figures 4a and b illustrate that viscous effects decrease with increasing frequency, scattering losses begin to increase.

Table 4 - Slow Wave frequency window limits

	Viscous Limit (MHz)	Scattering Limit (MHz)
Healthy Bone	1.59 ± 0.42	0.73 ± 0.09
Osteoporotic Bone	0.27 ± 0.14	0.33 ± 0.13

In the case of healthy bone, Table 4 shows that $f_{scat} < f_{viscous}$, that is, the scattering frequency is lower than the viscous frequency, that is, the slow wave will be subject to viscous and/or scattering losses at all frequencies. For osteoporotic bone there is a relatively narrow band of frequencies where the slow wave will not be subject to non-negligible viscous or scattering losses. However, errors associated with the frequency limits suggest the window should be noted.

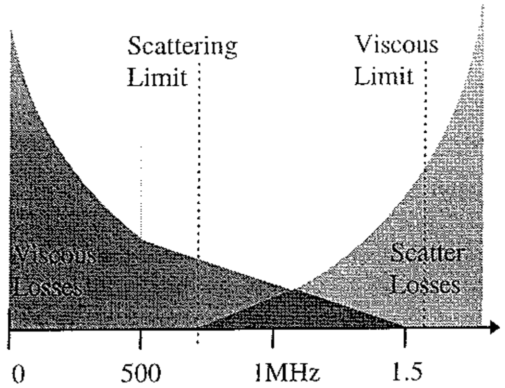

Figure 4a - Frequency effects for healthy bone

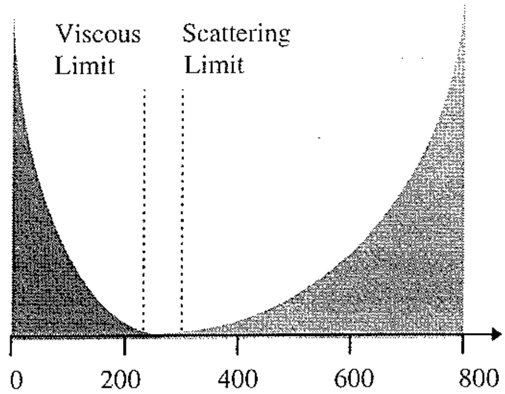

Figure 4b - Slow wave window for osteoporotic bone

V. Discussion and Conclusions

Current ultrasonic bone assessment systems may not use ultrasonic information in an optimal way since the interaction between ultrasound and cancellous bone is not understood. A thorough understanding may lead to more sensitive diagnostic techniques. Biot's theory has been used to study ultrasonic propagation in cancellous bone. This report has discussed points relating to the specific material characteristics of bone and marrow, and their effect on the propagation of the Biot slow wave.

Using data taken from the literature for human bone, it has been shown that cancellous bone has a relatively weak frame. That is, the solid frame bulk modulus is of the same order as that of the marrow It can be concluded that it is not valid to apply the Rigid Frame approximation of equations (15) and (16) to ultrasonic propagation in cancellous bone.

It was shown for healthy cancellous bone that the slow wave may be subject to viscous and/or scattering losses at all frequencies. For osteoporotic bone, there is a relatively narrow band of frequencies where it is predicted that the slow wave will not be subject to non-negligible these losses. This may be related to the lack of cancellous frame rigidity.

One of the least well defined, but most influential parameter in the model, is the viscosity of the marrow, η. Estimates for η from the literature vary from 0.04 Pa.s (Bryant *et al.* 1989) to 1.495 Pa.s (McKelvie and Palmer 1991), which would clearly give vastly different values of ω_{crit}, $\omega_{viscous}$ and phase velocity. The general effects outlined here may therefore occur in different frequency ranges with different viscosities.

One of the first reports of two compressional waves in cancellous bone was recently published by Hosokawa and Otani (1997). At frequencies of around 1 MHz they observed both compressional waves, and these results were reproduced by the authors (to be published elsewhere). These observations suggest that the viscous and scattering losses may not attenuate the slow wave to the extent that it would be obscured in experimental tests for the type of bone used here.

This report has concluded that the rigid frame approximation cannot be made for cancellous bone. It has predicted that current clinical systems operate in the high frequency region of Biot's theory, where slow wave propagation may be subject to viscous and/or scattering losses at most, if not all frequencies.

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