

A tuned damping device for reducing noise from railway track

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Abstract

A promising means to reduce the component of railway rolling noise radiated by the track is to increase the damping of the rail. This increases the attenuation with distance of vibrations transmitted along the rail and thereby reduces the noise radiated. To achieve this, a tuned, damped mass-spring absorber system has been designed. To cover a wide range of frequencies, multiple tuning frequencies are used along with a material with a high damping loss factor. Suitable materials have been found from extensive tests on samples and prototypes of the damper have been built and tested, both in the laboratory and in the field. Results are very promising with reductions of the track component of noise of around 6 dB being measured.

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1. Introduction

The most important source of environmental noise from railway operations in most situations is rolling noise. Reduction of noise at the source can be more attractive than the installation of high trackside barriers, but it requires a thorough understanding of the mechanisms of sound generation.

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Rolling noise is generated by the excitation of the wheels and track by the combined roughness of their running surfaces. The vibration response of the wheels, the rails and the sleepers beneath them all contribute significantly to the overall spectrum of noise. The relative importance of these different sources depends on the particular design of wheels and track, the roughness spectrum and the speed of the train. Except at high speeds, the track is usually at least as important as the wheel for the overall noise level and in many situations the rail is the greatest overall source. A theoretical model, ‘TWINS’, for the generation of rolling noise has been available for some time [1,2]. This has led to the investigation of a number of noise-reducing technologies. This paper describes the development and testing of a rail damping device for reducing the vibration of the rail.

Fig. 1 shows a typical one-third octave band spectrum of the sound power predicted for a single wheel and the associated track vibration. This prediction is for a train speed of 100 km/h and a roughness spectrum typical of tread-braked rolling stock with cast-iron brake blocks, which are typical for freight traffic in Europe. In this case, the sleeper is the dominant source of noise below 400 Hz and the wheel is the major source in the 1600 Hz band and above. Overall, however, the rail is the greatest source of noise in this case, being responsible for the broad peak seen in the spectrum from the 500 Hz to the 2000 Hz band. This also coincides with the peak in the roughness excitation. It can be seen that the sound radiation from the vertical vibration of the rail is greater than that from the lateral component.

For the noise from the track, it has been shown that the stiffness of the rail pads, located between the rail and sleepers, has a strong influence on the levels of noise. It controls the balance of the vibration response of the rail and sleeper and the rate of decay of vibration along the rail [3]. This has been demonstrated in earlier experiments [4]. The sound power produced by the rail in a particular frequency band is inversely proportional to the decay rate Δ in dB/m, since a high decay rate implies a short radiating length. Conversely a stiff pad increases coupling to the sleeper and hence leads to greater noise from the sleeper. An optimum stiffness of rail pad exists at which the rail and sleeper components of sound are

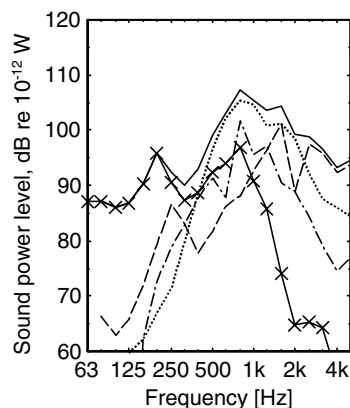


Fig. 1. Predicted rolling sound power generated per wheel for a standard 920 mm freight wheel running on a standard track at 100 km/h. (—) total, 112.8 dB(A), (---) wheel, 106.3 dB(A), ... rail vertical, 110.6 dB(A), (-.-.) rail lateral, 104.2 dB(A), (x—x) sleepers, 99.6 dB(A).

Table 1
Parameters for baseline track

		Vertical	Lateral
Rail bending stiffness	EI (Nm ²)	6.42×10^6	1.07×10^6
Rail mass per unit length	ρA (kg/m)	60.4	60.4
Rail shear stiffness	GA (N)	6.17×10^8	2.47×10^8
Rail shear parameter	κ	0.4	0.4
Rail rotational inertia	ρI	0.240	0.0396
Pad stiffness	K_p (N/m)	2.0×10^8	2.5×10^7
Pad damping loss factor	η_p	0.16	0.16
Sleeper mass (modal model used)	m_s (kg)	150	150
Sleeper spacing	d (m)	0.6	0.6
Ballast stiffness	K_b (N/m)	5.0×10^7	4.0×10^7
Ballast loss factor	η_b	1.0	1.0

equal [3]. However, this is very stiff compared with the pad stiffnesses now commonly adopted. Low stiffness pads have become standard in the rail industry as they reduce high frequency forces transmitted to the sleepers and to the ballast. This reduces track component damage and the tamping-related costs of track maintenance. Furthermore there is evidence that stiff pads may be counter-productive for noise by leading to an increase in the roughness of the rail head [5].

The starting point for reducing the rail noise has therefore been taken as a typical ballasted track with UIC 60 rails, monobloc concrete sleepers and a rail pad stiffness (at high frequency and low amplitude) of 200 MN/m per pad. The spectra presented in Fig. 1 have been calculated for this track. Parameters describing this track are listed in Table 1.

A preliminary study using finite element calculations showed that constrained layer damping treatments on a rail would only be effective above 2 kHz, due to the long wavelengths in the rail. In order to obtain high damping of the rail at lower frequencies, a damped mass-spring absorber system has been considered (see also [4]). An initial parameter study is used to show its effectiveness and to identify the optimal configuration. This is followed by more detailed analysis of a practical implementation.

It is found that temperature is an important parameter. The rail temperature usually follows the ambient temperature, although in strong sunlight it can be several degrees warmer than the surrounding air. In Europe, therefore, the range of temperatures to be considered can be between approximately -20 and $+40$ °C depending on the location. The noise reduction solution should ideally be effective over this range of temperatures. However, within the work described here, the effect of temperature has only been partially considered.

2. Initial studies of tuned absorbers

2.1. Methodology

Initially the concept of a tuned absorber system was studied using a simple track model to identify appropriate parameters. For these initial studies, a track model is used in which the rail is represented as a Timoshenko beam on a continuous spring–mass–spring foundation representing the rail pad, sleeper and ballast [6], derived from a model developed by Grassie [7].

The effect of damping devices on the rate of decay of vibration along the rail has been calculated using a two step approach. Firstly, the effect of the damping treatment on the rail vibration is calculated for an unsupported rail. This frequency-dependent decay rate, $\Delta_{d, \text{free}}$ is then added to that for the untreated rail located in the track, Δ_0 , to give an estimate for the decay rate of the damped track, Δ_d . The sound power is calculated by modifying the component associated with each direction, vertical and lateral, as predicted using the TWINS model, according to the difference in combined decay rate. Thus the sound power level of the damped track, L_d , in a given frequency band, is given by the level for the untreated track, L_0 , modified by the change in decay rate

$$L_d = L_0 + 10 \log_{10} \left(\frac{\Delta_0}{\Delta_d} \right) = L_0 + 10 \log_{10} \left(\frac{\Delta_0}{\Delta_0 + \Delta_{d, \text{free}}} \right) \quad (1)$$

Fig. 2 shows the effect of a single continuous mass/spring system on the decay rate. This has a mass per unit length of 17.5 kg/m and a damping loss factor of 0.35. The stiffness has been selected so that it is tuned to a frequency of 800 Hz. The result is compared with decay rates measured on a rail in situ for a track corresponding to the parameters of Table 1. The approach of Eq. (1) is approximate, but gives results that are close to the correct one provided that the mass of the absorber is much less than the mass of the rail.

For the untreated rail in track, at low frequencies, the waves in the rail have a high rate of attenuation, around 10 dB/m, because of the stiffness of the foundation. Around 400 Hz, a resonance of the track occurs in which the rail vibrates vertically on the stiffness of the pad. Above this frequency, wave propagation occurs freely in the rail and the decay rate for the untreated track declines rapidly. This broad dip in the decay rate corresponds to the peak in the rail noise spectrum shown in Fig. 1. An effective rail damper therefore has to increase the decay rate in this frequency region.

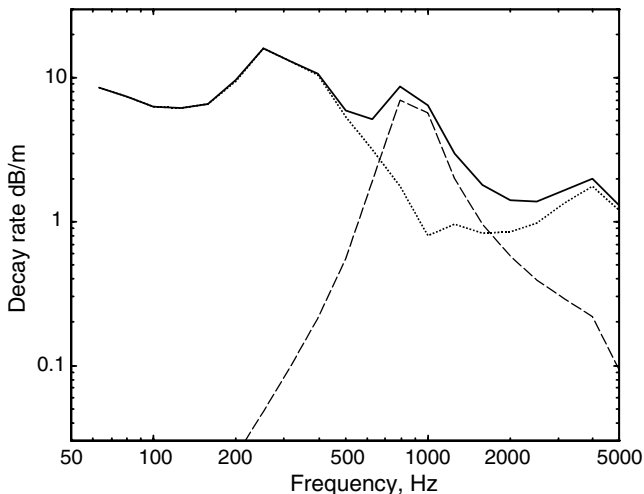


Fig. 2. Track decay rates for vertical vibration of the reference track with a continuous tuned absorber system added, tuning frequency 800 Hz, mass 17.5 kg/m, loss factor 0.35. (—) Track with absorber, (---) absorber only, ··· measured in the track with no absorber (at about 5 °C).

The tuned absorber introduces a high decay rate around its tuning frequency, in this case 800 Hz. The width of this peak depends on the damping loss factor. When this decay rate is added to that of the rail in the track it is found that the decay rate is increased in the region 630–2000 Hz, the largest increase occurring around the tuning frequency. The difference between the dotted and solid lines corresponds to the expected reduction in noise, see Eq. (1).

2.2. Single-frequency system for vertical and lateral motion

The computation represented by Fig. 2 for the decay rates of vertical bending waves in the track is relatively efficient. This, and the corresponding calculation for lateral waves, have been implemented within an automatic procedure to vary the parameters of tuning frequency and damping of the absorber. The dynamic model of the track, which is a relatively quick part of the overall noise calculation procedure, was used to calculate the decay rates for a large number of parameter combinations. The corresponding components of the rail sound power were predicted using the TWINS model for the baseline case of Fig. 1, and the change in each has then been estimated from the change in their decay rates obtained from the track model, using Eq. (1). Only mass/spring systems are studied that operate at the same frequency for both vertical and lateral waves.

Fig. 3 shows the variation of predicted rail noise reduction as a function of the tuning frequency of the absorber for different train speeds. Here a mass of 17.5 kg per metre of rail has been used. The graph shows that the best tuning frequency is around 900 Hz for 100 km/h and is not strongly dependent on the train speed.

Fig. 4(a) shows the effect of the damping loss factor of the absorber for a train speed of 100 km/h and a tuning frequency of 900 Hz in terms of the reduction in overall A-weighted noise from the rail. The benefit increases with increasing loss factor for values up to about 0.35 but, above this, little extra noise reduction is predicted. The variation of the sound reduction with the active mass of the absorber is shown in Fig. 4(b). It can be seen that there is no optimum mass, but improved noise reduction is obtained as the mass

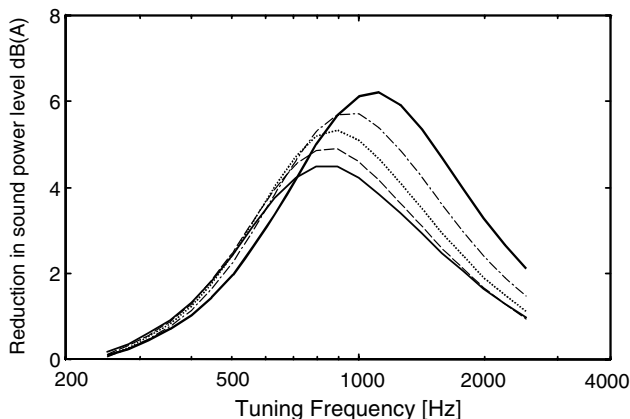


Fig. 3. Reduction in rail noise from the reference track for an absorber of mass 17.5 kg/m and loss factor 0.35 as a function of tuning frequency. (—) 60 km/h, (---) 80 km/h, (···) 100 km/h, (-·-·) 125 km/h, — (thick line) 160 km/h.

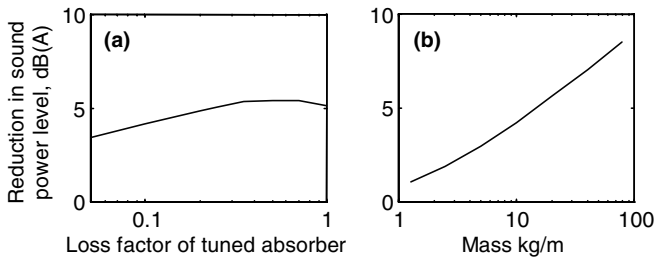


Fig. 4. Effect of parameters of single tuned absorber on reduction in rail noise for tuning frequency of 900 Hz. (a) Effect of damping loss factor for mass of 17.5 kg/m. (b) Effect of absorber mass for a damping loss factor of 0.35.

is increased. However, the practical limit for the mass of the absorber is about 17.5 kg/m due to space constraints, as will be seen later. This gives a reduction of 5.3 dB.

2.3. Two-frequency systems

Increasing the damping loss factor broadens the peak in the decay rate with frequency seen in Fig. 2, but also reduces its height. Another method of broadening the peak is to use multiple tuning frequencies. The same procedure as above has been used to study the noise reduction for a system with two tuning frequencies. The total added mass allowable is assumed to remain constant at 17.5 kg/m; thus this mass is now divided between the two tuning frequencies. The system is described by three parameters that are varied independently: its first natural frequency, the ratio of the two tuning frequencies and the ratio of the two masses. As before, identical tuning frequencies are assumed to act in the vertical and lateral directions. Fig. 5 indicates the combinations of parameter values considered. Where a noise reduction greater than 5.5 dB is found, this is indicated by a plus sign and the optimum configuration is shown by a circle. The optimum configuration gave a noise reduction of 5.9 dB(A), a gain of 0.6 dB over a single-frequency absorber. Moreover, a two-frequency system is more robust for variations in train speed, train type or temperature (which will affect the stiffnesses), since the damping is effective over a broader frequency range.

A three-frequency system was also studied in the same way, with variations allowed in five parameters. However, it was found to give much smaller improvements over the two-

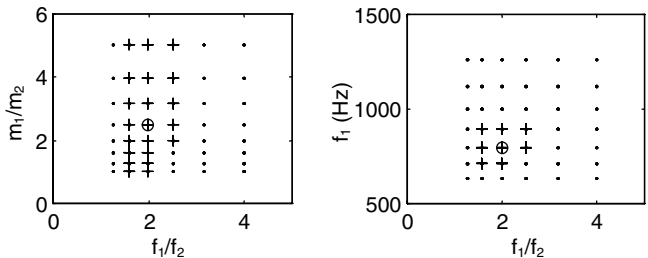


Fig. 5. Range of parameter study for two separate absorbers: ‘+’ indicates noise reductions greater than 5.5 dB(A), ‘⊕’ indicates optimum configuration.

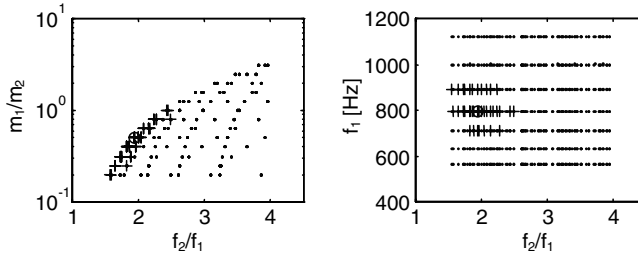


Fig. 6. Range of parameter study for stacked two-frequency absorbers. ‘+’ indicates noise reductions greater than 5.5 dB(A), ‘⊕’ indicates optimum configuration.

frequency system for the same overall mass. This benefit was considered to be insufficient to justify the added cost and complication of manufacture of a three-frequency system.

Although the two-frequency system could be produced, as in [4], using two separate single-degree-of-freedom (SDOF) systems, an alternative was explored in which a two-degree-of-freedom system is added to the rail. Not all combinations of frequency and mass ratios are possible, as shown in Fig. 6, but the optimum system that was found corresponds closely to the optimum of the two SDOF systems, and the noise reduction is identical.

In summary, the parameter study indicates an optimum system with tuning frequencies at 800 Hz and 1600 Hz operating both for vertical and lateral directions. However, it was then realised that as the decay rates used for the track (Fig. 2) were based on a measurement taken at about 5 °C, for a higher temperature the pad stiffness would be lower. In order for the solution to be robust for a range of temperatures, a slightly modified system was considered with tuning frequencies at 630 Hz and 1350 Hz.

3. Implementation

Railway track is designed with the criteria of safety, low cost of maintenance and standardisation as the highest priorities. For new methods of achieving noise reduction to be accepted in the short or medium term, they must be capable of being fitted to the established infrastructure design with minimal change to railway practice. Therefore, the present development has concentrated on a design of rail damper that may be fitted to the standard UIC60 rail section, either at manufacture or as a retro-fit, as a continuous system along the whole length of the rail. Variants, in which discrete dampers are attached between sleepers, have also been investigated but are not presented here. The absorber is confined to a space that does not interfere with the handling machinery for track laying and maintenance, nor with the rail clips. The allowable envelope assumed for a continuous absorber system is shown in Fig. 7.

A rail absorber was designed having a total mass per unit length of 17.5 kg/m, with two steel masses on each side of the rail, separated from each other and from the rail by layers of an elastomeric material, as shown in Fig. 7. The design was made symmetrical in order to avoid introducing cross-coupling between the vertical and lateral dynamics of the rail. The system has two resonances in the vertical direction in the desired ratio, but this was not possible for the frequencies in the lateral direction. However, the vertical rail vibration

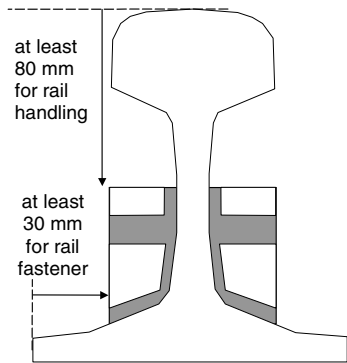


Fig. 7. The UIC 60 rail section, showing the space occupied by the absorber that allows normal track construction and maintenance and the form of the rail absorber.

is the dominant noise source for this track (see Fig. 1). This is generally the case for tracks with soft rail pads [3].

Finite element models of the rail and absorber cross-section were used to study the design in detail. These led to the specification for an elastomer in terms of its Young’s modulus and its loss factor which should be at least 0.35. The desired material properties should be achieved over a suitable working temperature range. The specification was sent to suppliers and the dynamic Young’s modulus and loss factor of a number of material samples was measured over the frequency range from 300 Hz to 3000 Hz. This information was used to enable the suppliers to develop the most suitable materials. In addition, a number of test samples of 0.15 m lengths of rail were assembled with the absorber using the different materials. These were tested to ensure that the expected tuning frequencies were obtained in practice.

Fig. 8 shows the temperature dependence of the material parameters for one of the materials found to be acceptable and which was used in subsequent prototypes. It can

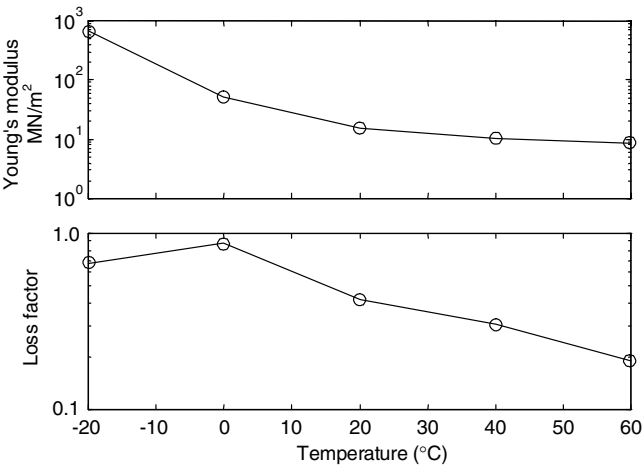


Fig. 8. The temperature dependent Young’s modulus and damping loss factor of the elastomer at 1 kHz.

be seen that the transition between the glassy and rubbery phases of the elastomer occurs in the working temperature range. In fact this is necessary in order to meet the requirement for a high loss factor [8]. Unavoidably, there is, therefore, a high dependence of both parameters with frequency and temperature.

4. Detailed modelling

For the detailed evaluation of the specific design, a periodic structure model of the track is used [9]. In this, a 10 mm slice of the rail cross-section and damper are modelled using finite elements. The slice is then assumed to be repeated to form an infinite structure on which the modes of vibration of the cross-section propagate as waves with complex wave numbers.

For this purpose the finite element mass, stiffness and damping matrices for the rail cross-section with the absorber were obtained using 8-noded hexahedral elements. The damping matrix was assembled using the element stiffness matrices, each multiplied by an appropriate element loss factor. In this way the model was able to account for the different material damping of the elastomeric and steel components of the composite cross-section.

The decay rates were calculated using three sets of material properties of the elastomer, that are valid at different frequencies (300, 1000 and 3000 Hz). An interpolation scheme was then used between the three curves obtained to give the decay rates corresponding to the stiffness that applies at intermediate frequencies, the results at 300 or 3000 Hz being used outside this range. The resulting predicted curves for the vertical and lateral decay rates of the free rail with absorbers at 20 °C are presented in Fig. 9.

A 4 m length of rail was fitted with the absorber and measurements of the decay of vibration in the rail were made at 20 °C. At low frequencies the decay rate was measured indirectly by modal analysis, using a method described in Appendix A. At frequencies above about 700 Hz, where this was no longer possible due to very high modal damping,

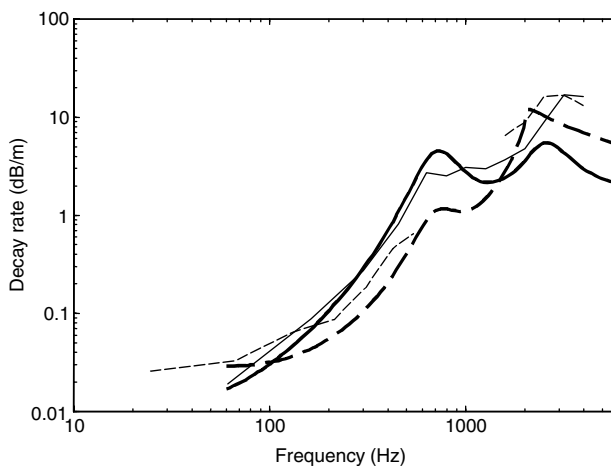


Fig. 9. Predicted decay rates of rail fitted with absorbers compared with measurements on a 4 m length of freely suspended rail. (—) vertical, (---) lateral. Thin lines: measured; thick lines: predicted.

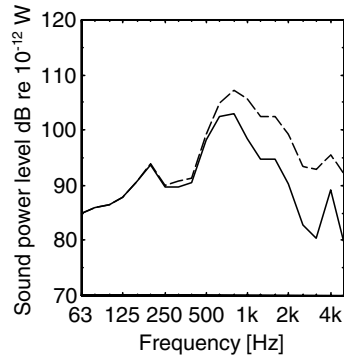


Fig. 10. Predicted track sound power with and without the absorbers. (—) with absorber, 106.9 dB(A), (---) untreated, 112.3 dB(A).

the decay rate was sufficient for it to be measured directly within the 4 m length. The measured vertical and lateral decay rates are given in Fig. 9 for comparison with the predictions. Very good agreement is found, although the measurements exceed the predictions above about 2 kHz, possibly due to the neglect of frequency dependence of the material properties above 3 kHz.

The predicted noise reduction using the decay rates measured on the 4 m samples can be seen in Fig. 10. Thus, at a rail temperature of 20 °C, a reduction of the rail noise of 5.4 dB(A) is calculated. Using the predicted decay rates at this temperature from Fig. 9, the calculated reduction is similar at 5.6 dB(A).

The vibration of the absorber masses will also radiate some sound. In a lightly damped absorber this may be significant. The track model was therefore used to calculate this vibration during a train pass-by. From this, the sound power radiated by the active masses themselves was calculated using the same model of sound radiation as used for the rail. These results indicate that, although the noise reduction would be compromised slightly,

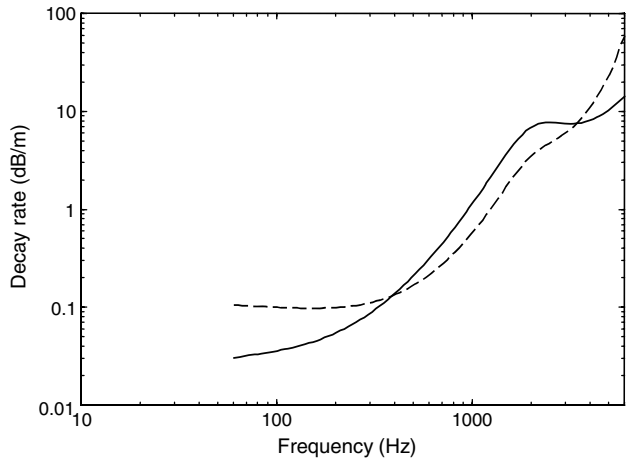


Fig. 11. Predicted decay rates of rail fitted with absorbers at 0 °C. (—) vertical, (---) lateral.

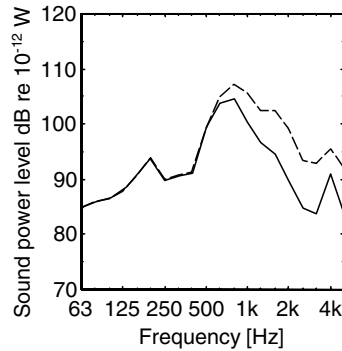


Fig. 12. Predicted track sound power with and without the absorbers, based on decay rates with absorber at 0 °C from Fig. 11. (—) with absorber, 108.5 dB(A), (---) untreated, 112.3 dB(A).

the effect would be less than 1 dB(A). This can be understood to be due to the high loss factor and small surface area of the absorber masses. The effect has not been included in the predictions of Fig. 10.

A prediction of the decay rates for the elastomer properties at 0 °C has also been made. Fig. 11 shows the predictions of decay rate for a bare rail fitted with the absorber at 0 °C. From this it can be seen that, due to the stiffening of the material, the operating frequencies of the absorber have increased and the curve has shifted towards higher frequencies compared with Fig. 9. The benefit of the absorber would therefore be reduced, as shown in Fig. 12, which gives the predicted track noise for an absorber at 0 °C. However, this should also be offset against the changes with temperature in the decay rate of the untreated track, due to stiffening of the pad. This latter effect has not been calculated due to the absence of data on the temperature dependence of the pad stiffness.

5. Field measurements

Field measurements on a track fitted with a prototype rail absorber were conducted in May 1999 as part of the ‘Silent Track’ project [10]. The decay rates in the track were measured using impact excitation and rolling noise measurements were made using a test train. The rail temperature was in excess of 30 °C, which meant that the rail pads were softer than was the case in the previous decay rate measurements (Fig. 2), and the rail noise component would be higher. On the other hand, the rail damper elastomer also had a lower stiffness and therefore a lower tuning frequency. From Fig. 3 it can be seen that this may reduce the performance of a single-frequency absorber, and the same is the case for the two-frequency absorber.

The measured decay rates for vertical bending waves at the field test site are shown in Fig. 13. The decay rate for the track without absorbers is lower than that in Fig. 2 due to the higher temperature. It is used here as the basis Δ_0 to which either the predictions or laboratory measurements of decay rate for the freely supported rail, $\Delta_{d,free}$ (Fig. 9) have been added to obtain predicted decay rates of the damped track, see Eq. (1). In both cases, absorber results corresponding to 20 °C have been used. The decay rates predicted using either method agree quite well with those measured. However, the first frequency of the absorber can be seen to occur at a lower frequency in the track measurements (around

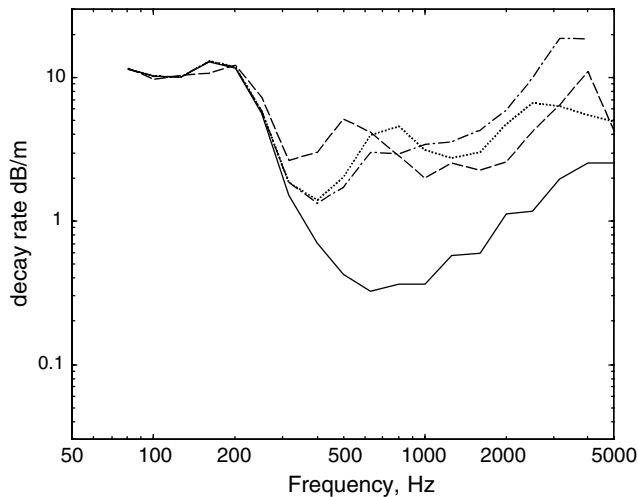


Fig. 13. Measured decay rates in track compared to predictions and laboratory measurements. (—) measured, untreated, (---) measured, with absorbers, (···) estimated from predictions, (- · - ·) estimated from laboratory measurements.

500 Hz) than in the predictions (630–800 Hz) due to the higher temperature during the tests.

For the noise measurements, a weighted average of the sound pressure at three microphones, 3 m from the near rail at different heights, was used. The one-third octave band spectra measured from a test train passing at 100 km/h with and without the absorbers are shown in Fig. 14. The measurement shows an overall reduction of 5.6 dB(A). In order to show the reduction in track noise most clearly, these results are for a low-noise wheel [11,12]. The design of wheel had no effect on the track component of noise. However, even for this wheel, the overall measured noise is influenced significantly by the noise from the wheel in the 1600 Hz band and above. The full reduction of the track noise is therefore not seen in these bands, as could be verified by track vibration measurements. Estimates in

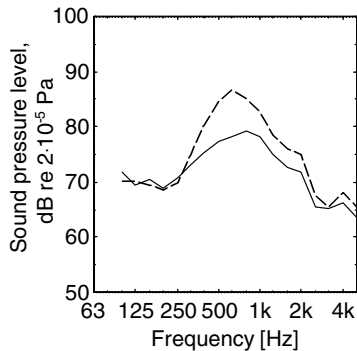


Fig. 14. Results of measurements of rolling noise at 100 km/h using a vehicle fitted with a noise reducing wheel. Average from three microphones at 3 m from near rail. (—) with absorber, 85.1 dB(A), (---) untreated reference track, 90.5 dB(A).

which the wheel noise is removed indicate that the track noise is reduced by 6.0–6.2 dB(A) due to the absorber. These results are consistent with the predictions described in Section 4.

6. Concluding remarks

The stages in the development of a damping device to reduce the rail component of rolling noise have been described. A damped tuned absorber has been chosen to give a broad-band reduction in noise over the region 500–2000 Hz. An initial parameter study using an efficient method was used to obtain the main parameters which were realised in a design using steel masses and an elastomer formulated to a specification. Samples were tested in the laboratory using a combination of modal analysis and direct decay rate measurements. This confirmed the results of detailed modelling, which takes account of the temperature- and frequency-dependence of the material. These models can therefore be used as an accurate design tool for further development and refinement of the absorber. Field tests of a prototype absorber have demonstrated that it reduces rail noise by about 6 dB(A) for a modern track design with soft rail pads. At 0 °C the benefit is expected to be about 4 dB(A), but it is possible that this can be improved by alterations to the material specification. For a track with a stiffer pad the reduction will inevitably be much smaller, but the design has been optimised for soft pads as they are likely to become more widely used for other reasons. Apart from the additional weight, a rail fitted with the absorber can still be handled using conventional machinery. Further work is required to understand fully the effect of temperature on track noise and on the performance of the absorber.

Since the work described here, practical implementations of the rail absorber have been installed in the Netherlands, France, Germany and Sweden.

Acknowledgement

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Appendix A. Procedure for finding rail decay rates from measurements on a finite length of rail

In frequency response measurements on a 4 m length of rail, the modes of the finite length can be clearly seen in the response at low frequencies, whilst the decay along the length of the 4 m section is small. A procedure is therefore required based on measuring the modal damping (expressed as a damping loss factor) and converting this to an equivalent decay rate. The modal damping loss factor η_n for mode n can be found from the half-power bandwidth, $\eta_n = \Delta f_n / f_n$ where f_n is the resonance frequency, and Δf_n is the bandwidth at which the input mobility is 3 dB below the resonance peak. To relate modal damping to decay rate it is necessary to determine the wavenumbers and the group velocity.

The n th mode of a free-free beam corresponds to a wavenumber k_n , given by

$$k_n \approx \frac{(n + \frac{1}{2})\pi}{L}, \quad \text{for } n = 1, 2, 3, \dots \quad (\text{A1})$$

where L is the length of the beam. For an Euler–Bernoulli beam this expression for wavenumber is correct to within 0.4% for the first mode, the accuracy increasing rapidly with increasing n [13]. For a Timoshenko beam, however, the wavenumbers associated with propagating and evanescent waves differ slightly from each other, and the boundary conditions are more complex than for an Euler–Bernoulli beam. Nevertheless, from numerical calculations it is found that, for the parameters corresponding to a rail in vertical bending, Eq. (A1) gives the propagating wavenumber at mode n of a free-free Timoshenko beam to within 1% for all modes up to 5000 Hz (the cut-on frequency for higher order modes). Therefore Eq. (A1) may be used to estimate the wavenumbers associated with each mode.

These wavenumbers can be plotted against the modal frequencies determined by experiment, as shown in Fig. A1(a) for vertical and lateral motion of a 4 m section of undamped rail. Also shown are theoretical dispersion curves predicted using the parameters for a rail in vertical and lateral bending, listed in Table 1, in a Timoshenko beam model. For vertical bending the agreement with measurements is very good whereas, for lateral bending, coupling to higher order cross-sectional motion results in deviation of the measured dispersion curve from the theoretical one, as the latter only includes bending.

The gradient of these dispersion plots can be used to estimate the group velocity, $c_g = d\omega/dk$ where ω is the circular frequency and k is the wavenumber. c_g is also required to determine the decay rate. From the definition of loss factor, the energy E reduces with time t at a rate

$$E(t) = E_0 e^{-\omega\eta t} \quad (\text{A2})$$

where E_0 is the value of E at $t = 0$ and η is the damping loss factor. The time taken for the energy to travel 1 m is $t = 1/c_g$. Therefore the energy in a propagating wave reduces by a factor $e^{-\omega\eta/c_g}$ over 1 m. Thus the decay rate of a propagating wave in dB/m is given by

$$\Delta = -10 \log_{10} e^{-\omega\eta/c_g} = 4.343 \frac{\omega\eta}{c_g} \quad (\text{A3})$$

For an Euler–Bernoulli beam the group velocity is simply $c_g = 2\omega/k$, and Eq. (A3) becomes $\Delta = 8.686 (k\eta/4)$ where the term $k\eta/4$ is the imaginary part of the wavenumber. However, for a Timoshenko beam c_g tends from $2\omega/k$ at low frequency to ω/k at high frequency as shear deformation becomes important. Fig. A1(b) shows this in the form of the

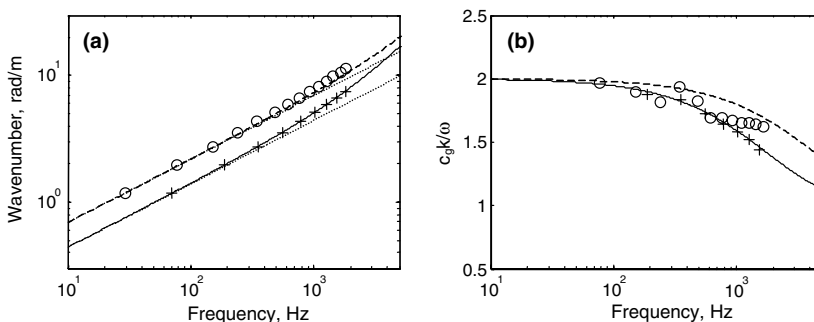


Fig. A1. (a) Dispersion curve for rail. (b) Ratio of group velocity to phase velocity. — vertical bending, Timoshenko beam theory, --- lateral bending, Timoshenko beam theory, ... Euler–Bernoulli beam theory, '+' vertical, measured on 4 m sample, 'O' lateral, measured on 4 m sample.

ratio $c_g k/\omega$ derived from the measured dispersion plots of Fig. A1(a), and predicted using the parameters for vertical and lateral bending. For vertical bending the agreement is again very good whereas for lateral motion some deviation again occurs due to coupling with other wave types.

Finally, decay rates can be obtained from Eq. (A3) for each resonance frequency using the measured modal loss factors, the wavenumbers from Eq. (A1) and the group velocities from the slope of the measured dispersion curves. Note that this procedure must be carried out separately for the damped rail as the presence of the rail absorber modifies the natural frequencies of the rail as well as its damping.

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