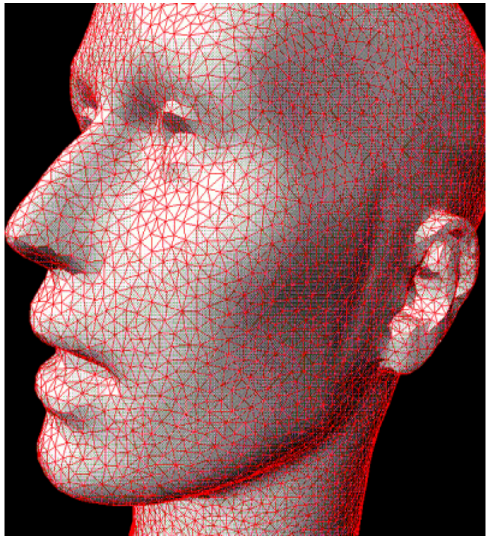
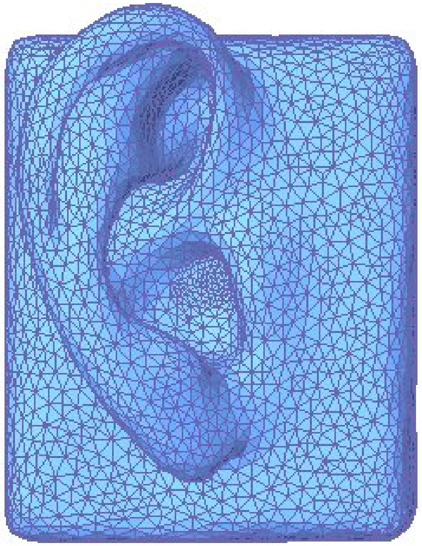
Numerical Modelling of the Head-Related Transfer Function

Yuvi Kahana





27/11/2000

First Question

Can we obtain individualised HRTFs

without a single acoustic measurement ?

Method

Convert the geometry of an object

(head+pinna) into its acoustic response

↓

Solve the wave equation

PREVIOUS WORK

Weinrich (1984)

"The rather complicated shape of the pinna makes a rigorous mathematical treatment very difficult - perhaps impossible"

Shin-Cunningham and Kulkarni (1996)

"Theoretically, it is possible to specify the pressure at the eardrum for a source from any location simply by solving the wave equation…

Needless to say, this is analytically and computationally an intractable problem"

Also Genuit (1986), Katz (1998) and others using simplified techniques

CONTENTS

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- Overview of numerical modelling techniques in acoustics
- HRTFs and the principle of reciprocity (simple/complex models)
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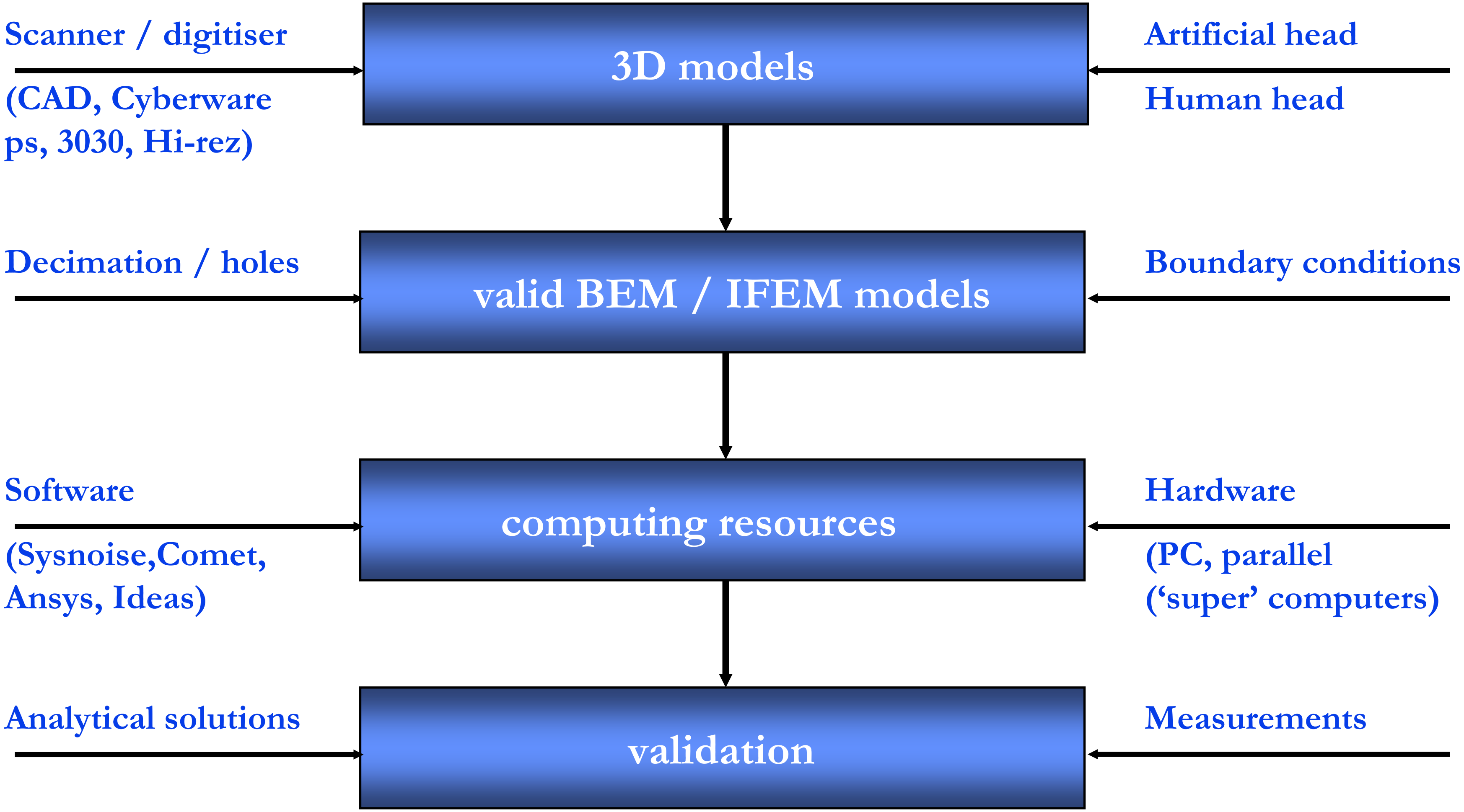
PROJECT "BOTTLE-NECKS"



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NUMERICAL MODELLING OF HRTFs - OBJECTIVES

- Develop a tool for accurate analysis of the physical mechanisms of the external ear
- Analyse pinna-based spectral cues at high frequencies
- Obtain individualised HRTFs by means of an optical sensor
- Use numerical methods for analysis of simple models where analytical solutions cannot be used
- Visualise sound fields for virtual acoustic systems

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METHODS FOR ACOUSTIC CALCULATIONS

Analytical methods

- Closed form solutions
- Only for simple geometry

Geometrical methods

- Ray/beam tracing
- Mirror images

Statistical energy methods (SEA)

- Energy exchanges between system components

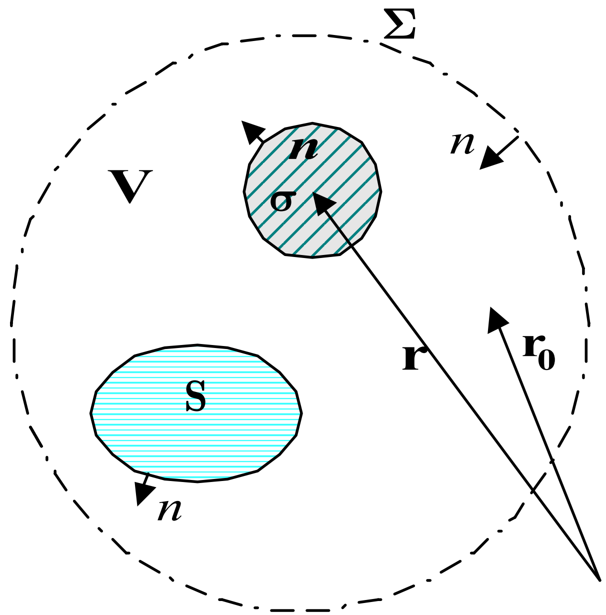
Numerical methods

- Finite Element Method (FEM)
    - Volume discretisation into finite elements
- Boundary Element Method (BEM)
    - Discretisation of bounding surface into boundary elements

BEM - DIRECT BOUNDARY INTEGRAL EQUATION (BIE)

Inhomogeneous Helmholtz equation **(harmonic excitation)**

$$(\nabla^2 + k^2)p(\mathbf{r}) = -Q_{\mathrm{vol}}(\mathbf{r}_0)$$



Free space Green function

$$g(\mathbf{r}\,|\,\mathbf{r}_0) = \frac{e^{-jk|\mathbf{r}-\mathbf{r}_0|}}{4\pi\,|\,\mathbf{r}-\mathbf{r}_0\,|}$$

$$p(\mathbf{r}) = \int_S [g(\mathbf{r}\,|\,\mathbf{r}_0)\nabla_0 p(\mathbf{r}_0) - p(\mathbf{r}_0)\nabla_0 g(\mathbf{r}\,|\,\mathbf{r}_0)].\mathbf{n}\,dS$$

3D ⇨ 2D - computationally inefficient

BEM - PROPERTIES

DBEM (Direct BEM)

- Solves the pressure and particle velocity on the boundary surface
- Exterior *or* interior domains
- Discretisation, collocation, shape functions, nonsymmetric matrices
- Efficient with small to medium size problems

IBEM (Indirect BEM)

- Solves the differences between the outside and inside values of the pressure and particle velocity on the boundary surface
- Exterior *and* interior domains
- Variational formulation, symmetric matrices
- Efficient with large problems

Special formulation: symmetric, axisymmetric, baffled models

Non-uniqueness problem (irregular frequencies) ⇒ regularisation

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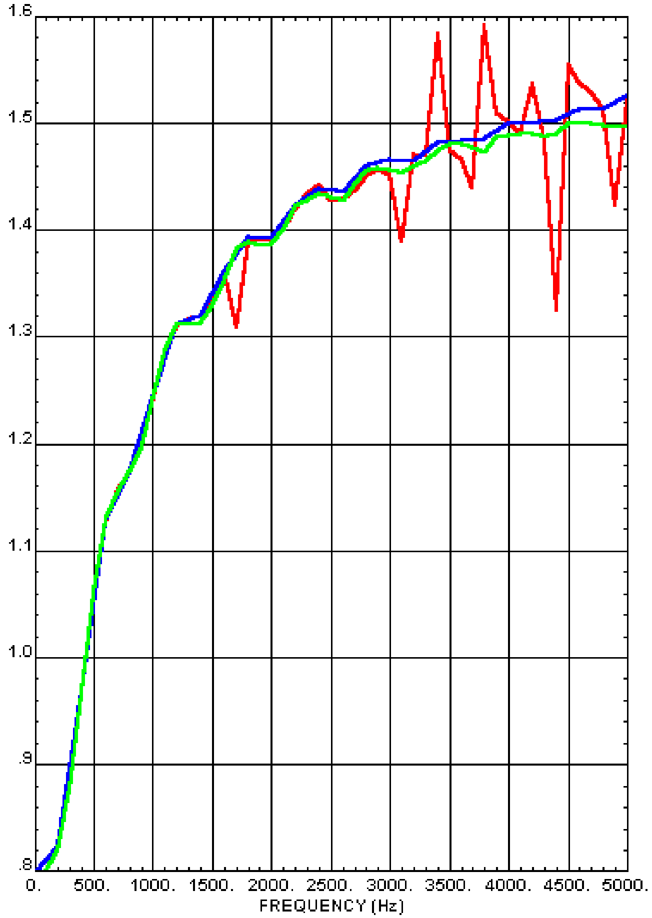
THE "NON-UNIQUNESS" PROBLEM
VALIDATION OF THE SPHERE MODEL

Front

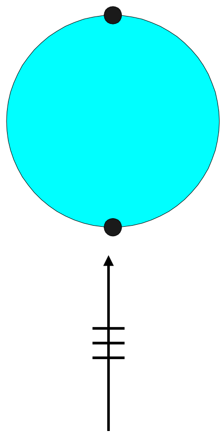
Rear

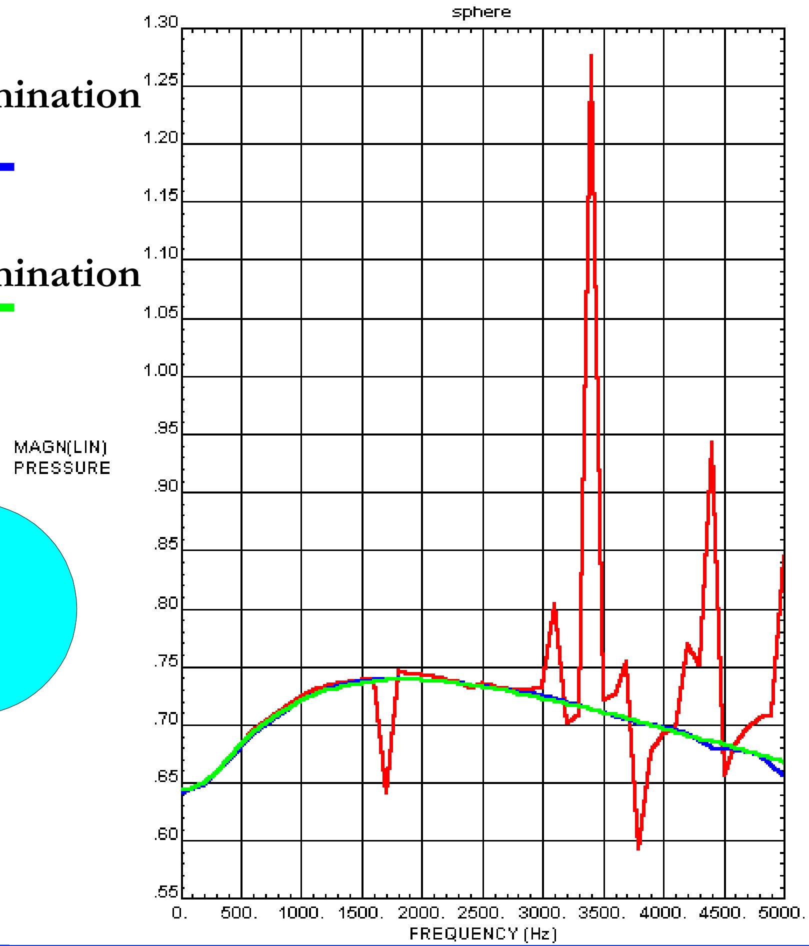


With overdetermination

Without overdetermination

Analytical solution





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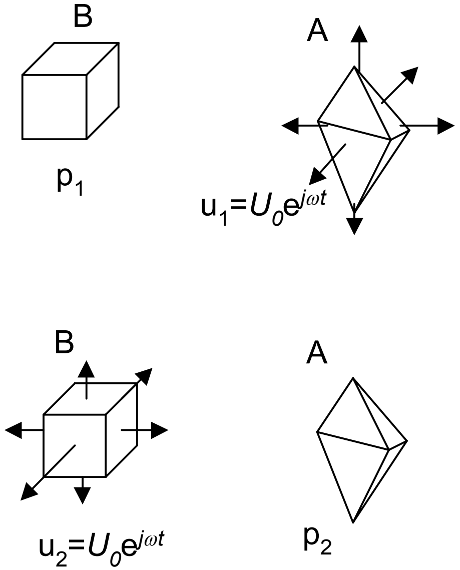
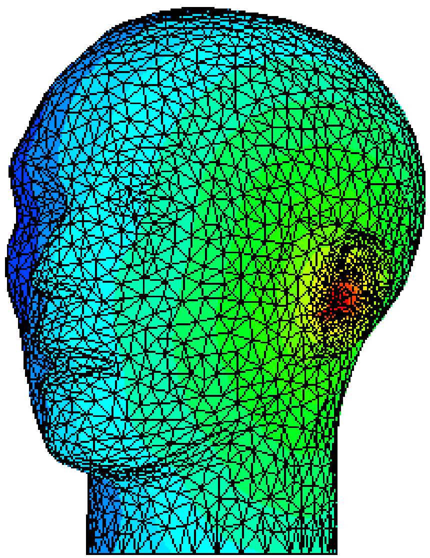
CALCULATION OF HRTFs USING THE PRINCIPLE OF RECIPROCITY



$$\oint_S (\mathbf{p}_1\vec{\mathbf{u}}_2 - \mathbf{p}_2\vec{\mathbf{u}}_1)\cdot\hat{n}\,dS = 0$$

$$\frac{Q_1}{P_1(r)} = \frac{Q_2}{P_2(r)}$$

- Refined ear
- Source positioned close to entrance to ear-canal



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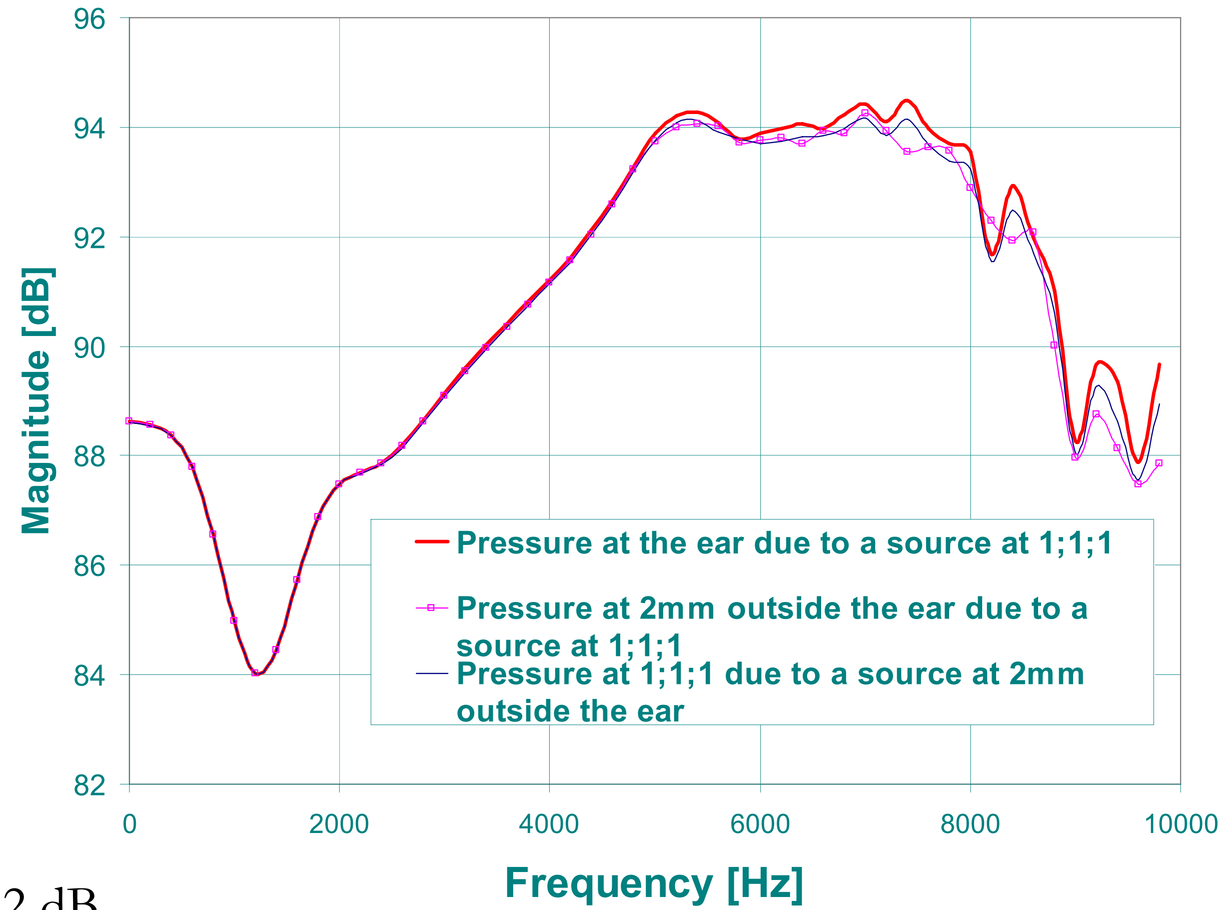
VALIDATION OF THE PRINCIPLE OF RECIPROCITY



- Errors: <0.2 dB
- HRTFs can be calculated with high accuracy for near-field and far-field

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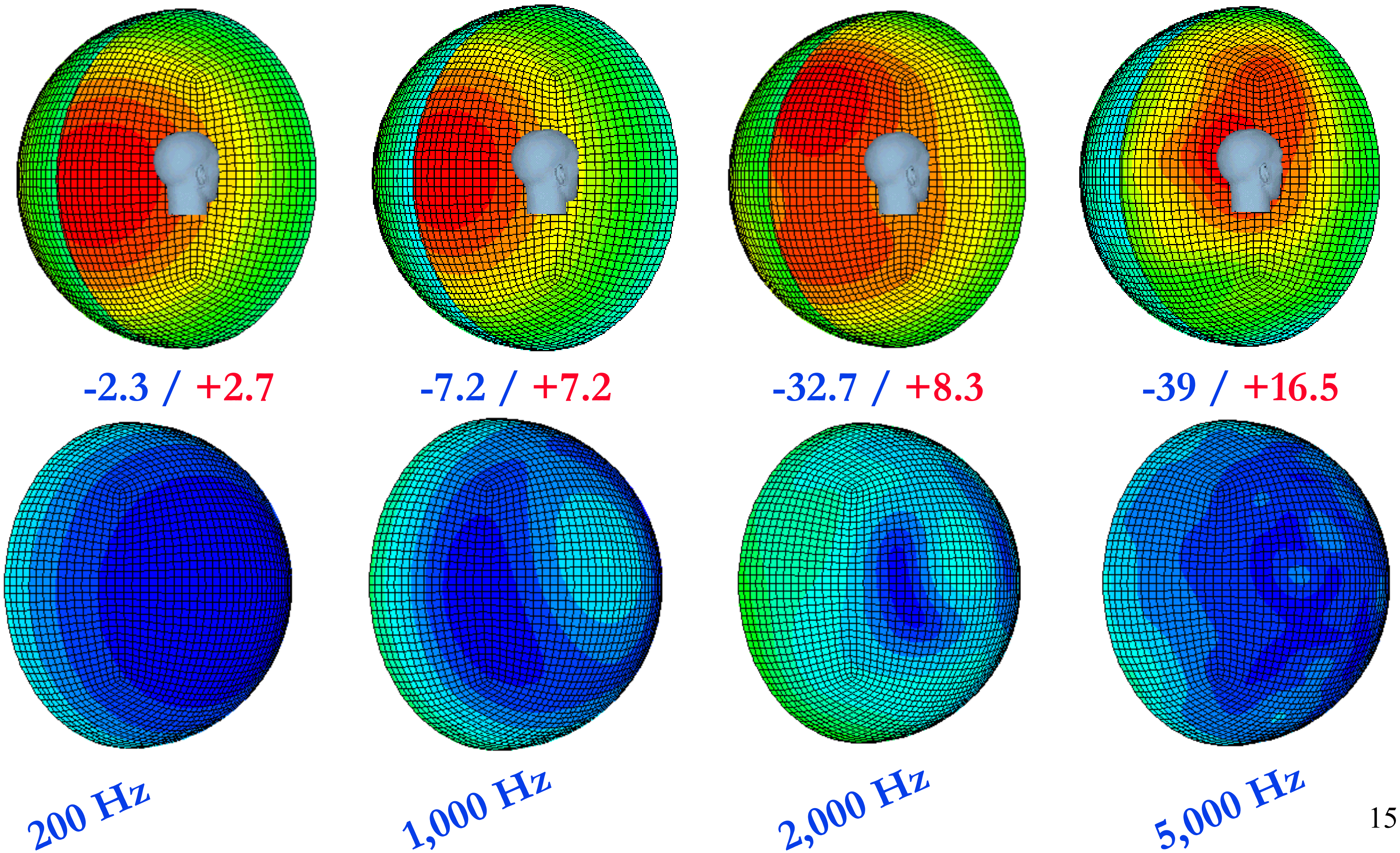
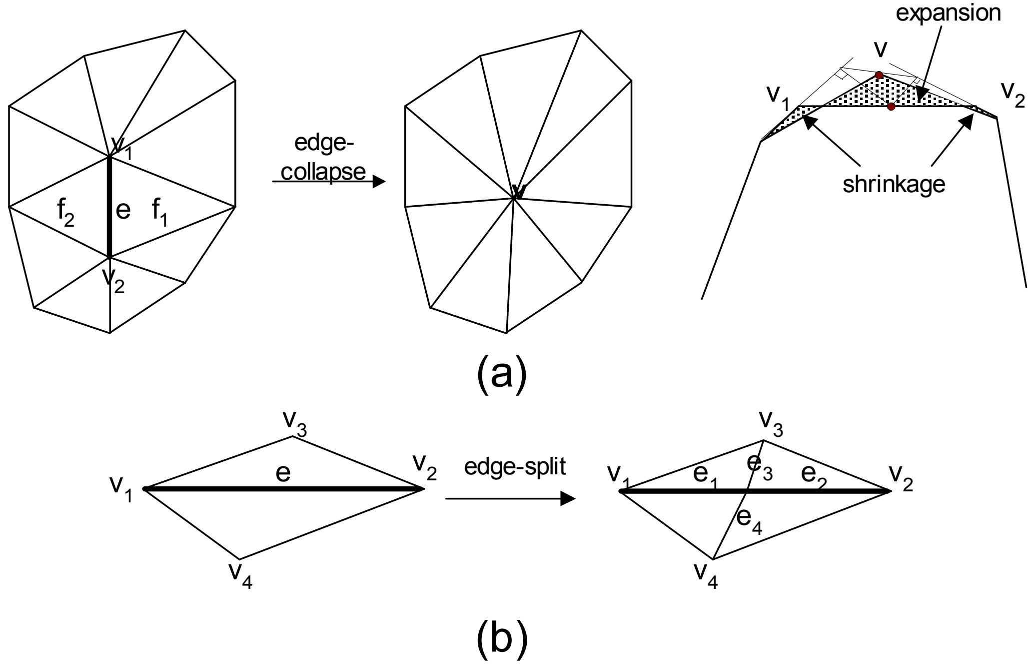
CALCULATION OF HRTFs USING THE PRINCIPLE OF RECIPROCITY (dB scale)

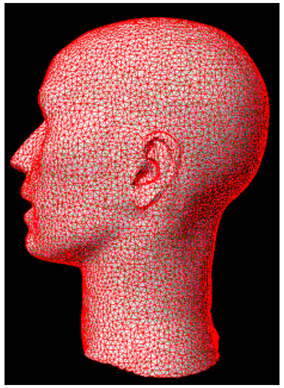


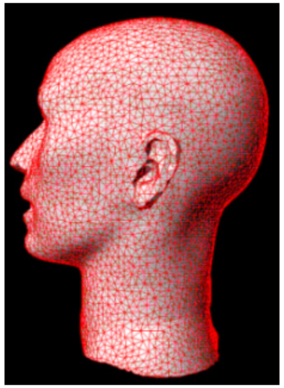
MESH DECIMATION

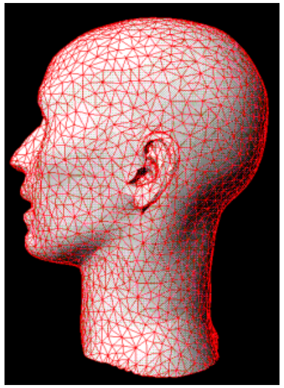
- Preserve shape
- Normalise distances between vertices
- Minimise number of vertices
- Edge split, edge collapse

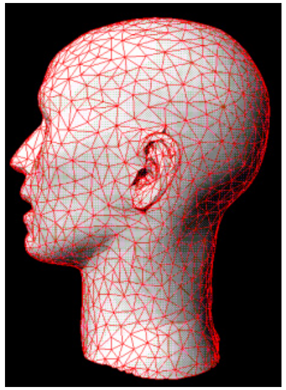


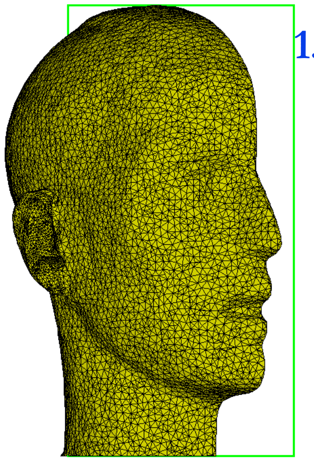
POLYGON REDUCTION / NORMALISED MESH MODELS - FULL AND HALF MODELS OF KEMAR

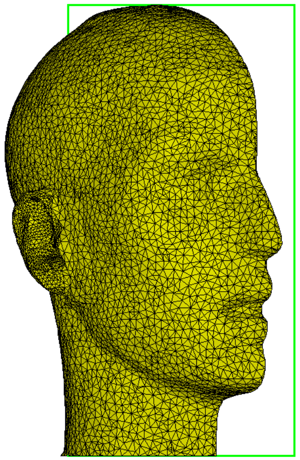

20000


10000


5000


3000


15000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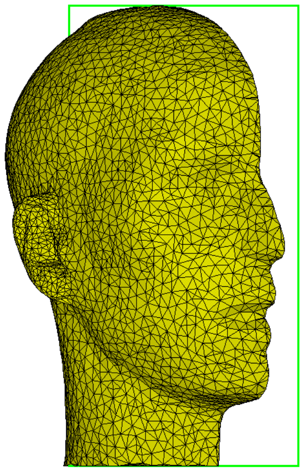
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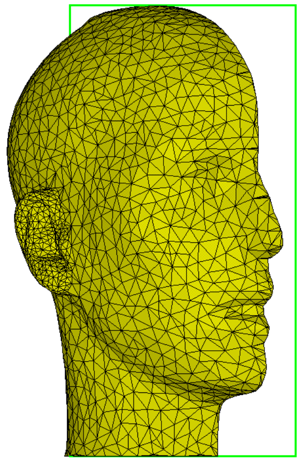
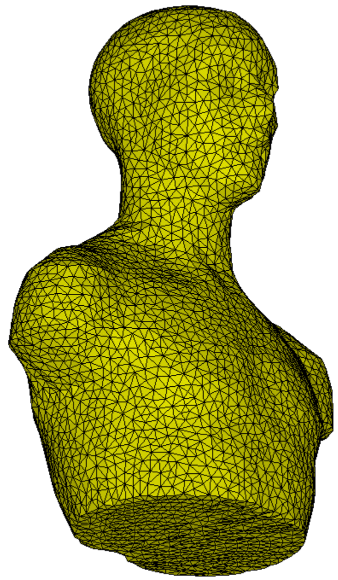

2500

No. of elements

HRTF SIMULATION OF LOW-MEDIUM SIZE SIMPLE MODELS

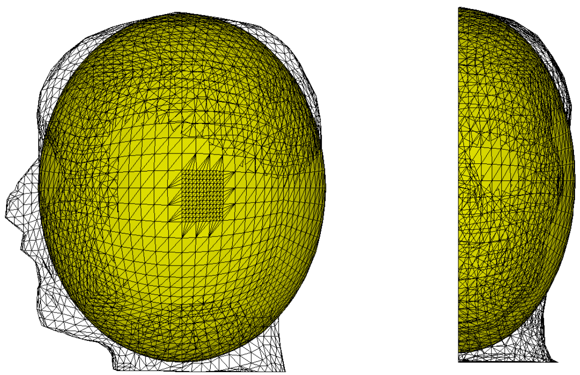


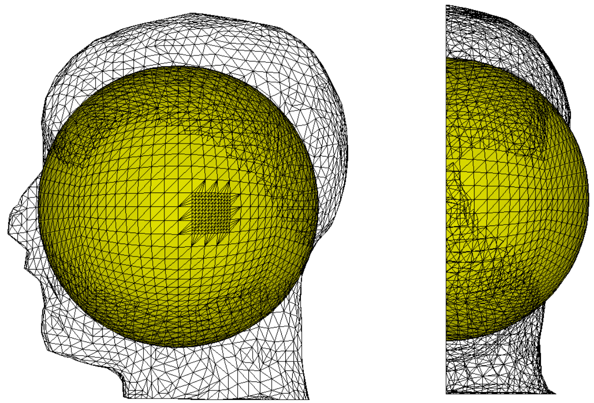
•CORTEX head - with and without torso. Converted from CAD and decimated.

•Sphere - $r = 8.75$ *[cm]*

•Ellipsoid - $r_x=9.6$, $r_y=7.9$, $r_z=11.6$ *[cm]*.

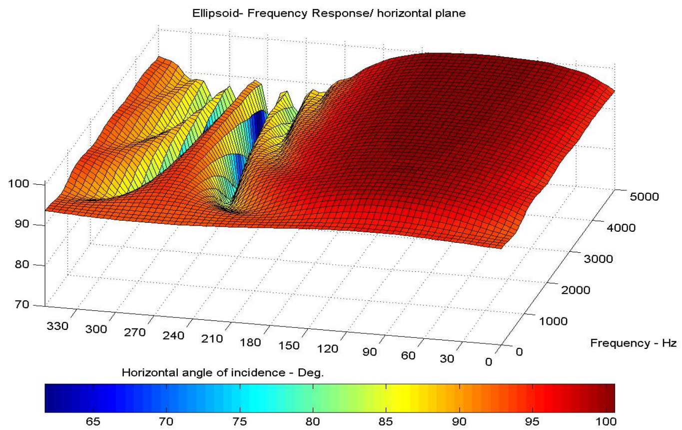
•'Ear' positions optimised for minimal errors, and locally refined for reciprocity.

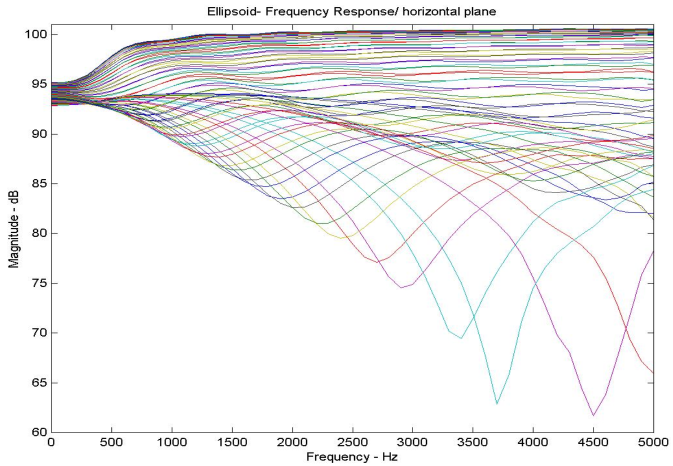




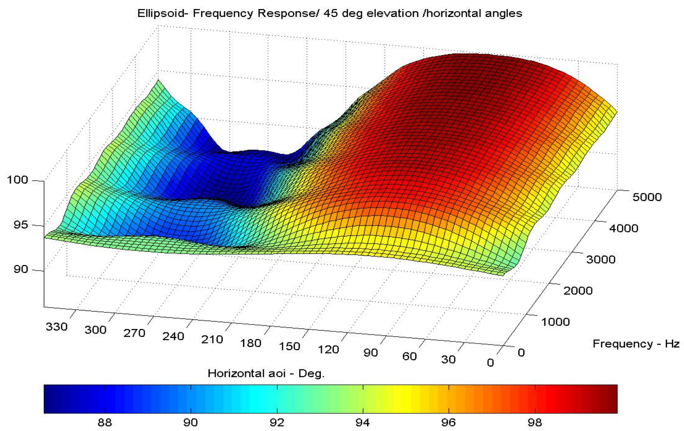
HRTFs OF AN ELLIPSOID

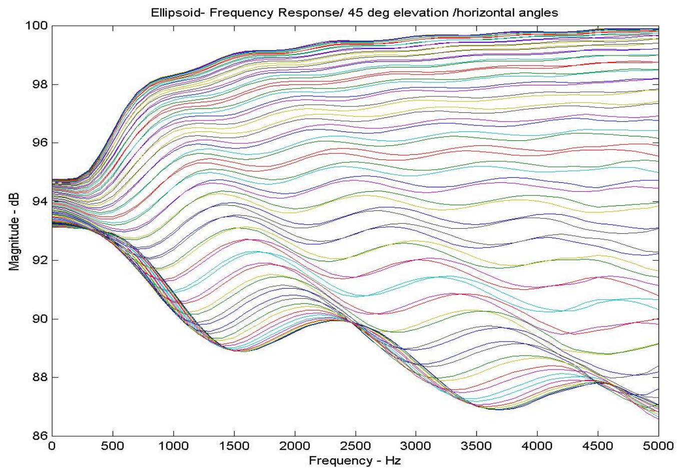
HRTFs at horizontal plane (*el* = 0°, *az* = 0°-355°)





HRTFs at elevation (*el* =45°, *az* =0°-355°)





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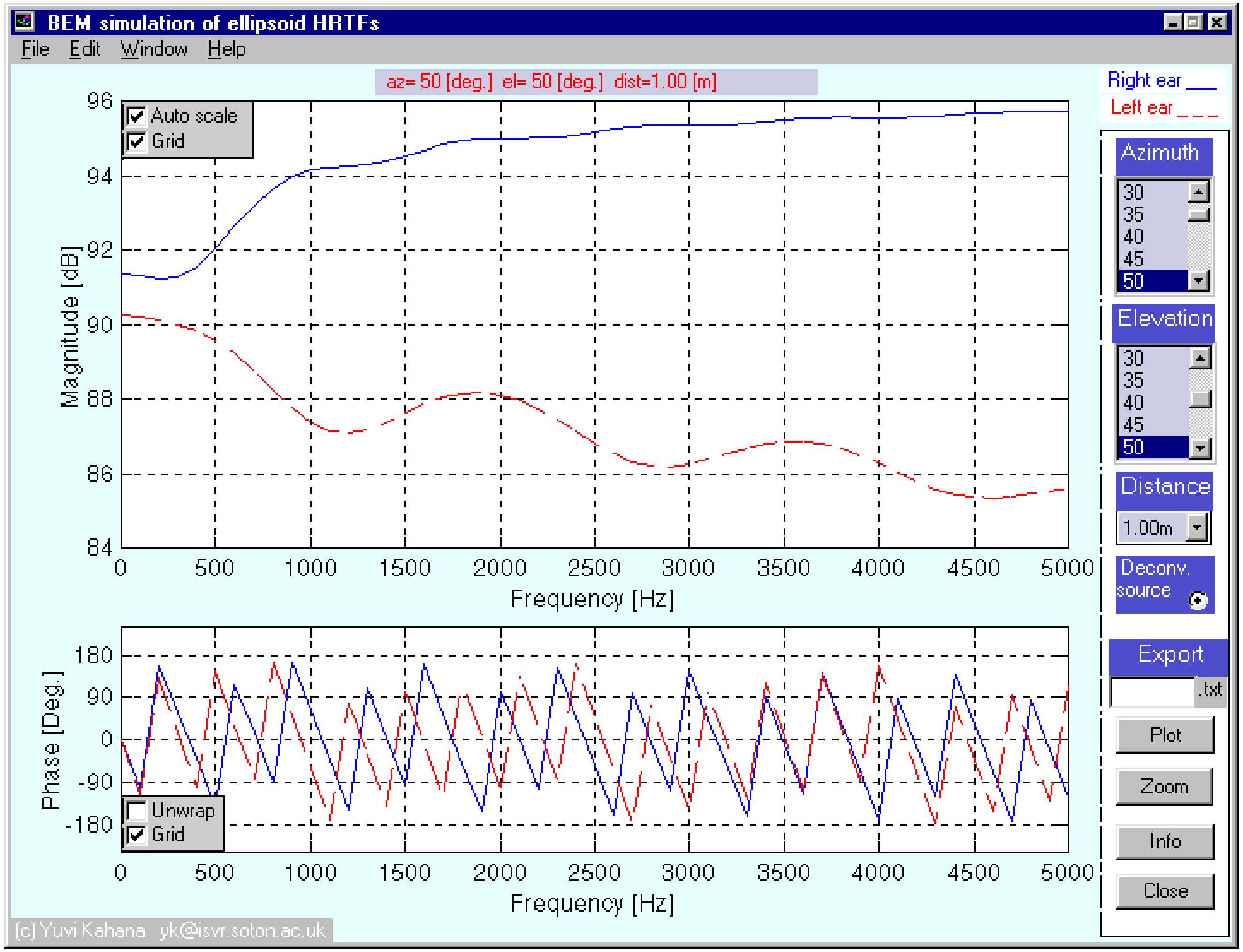
MATLAB GUI OF NUMERICALLY MODELLED HRTFs OF AN ELLIPSOID



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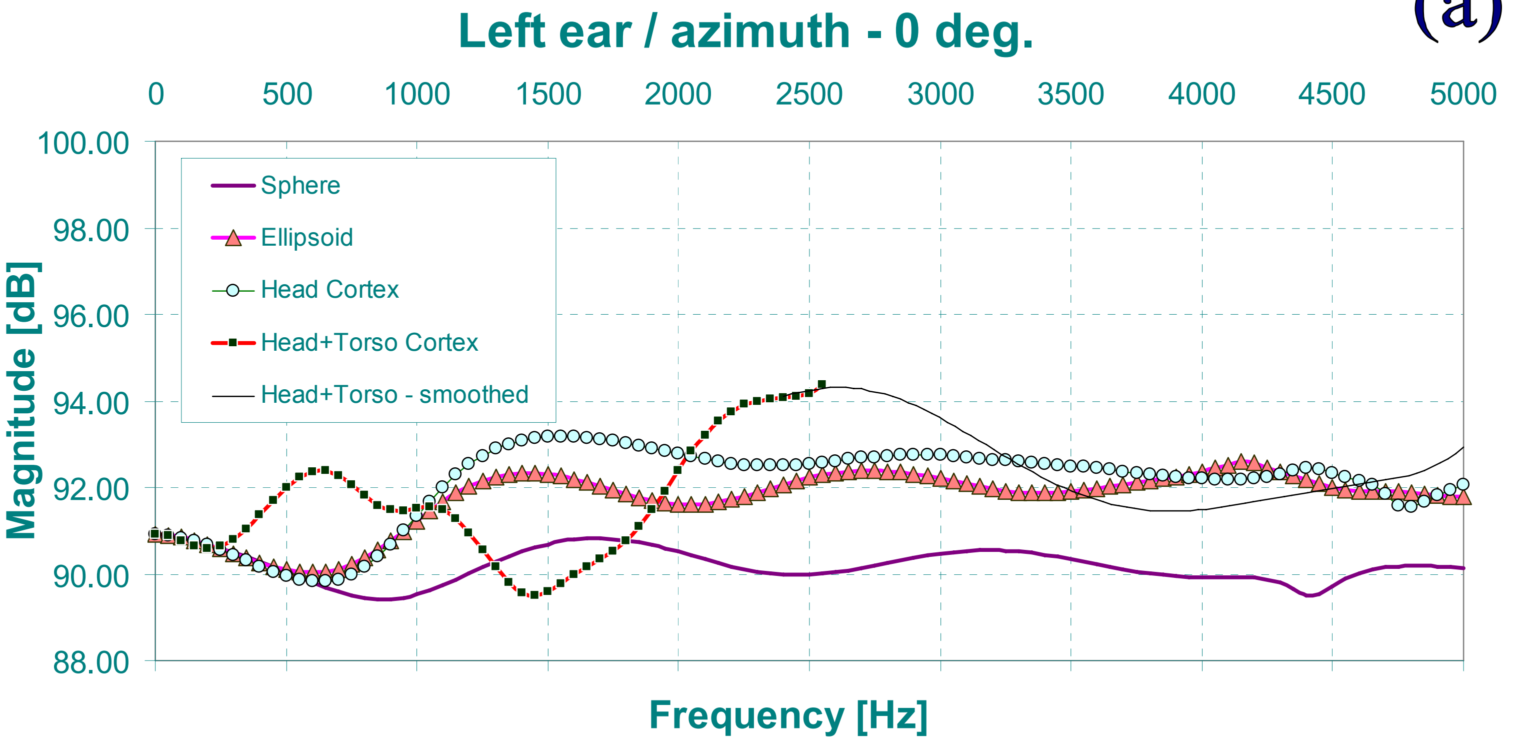
COMPARISON OF HRTFs OF SIMPLE MODELS-HORIZONTAL PLANE

(a)



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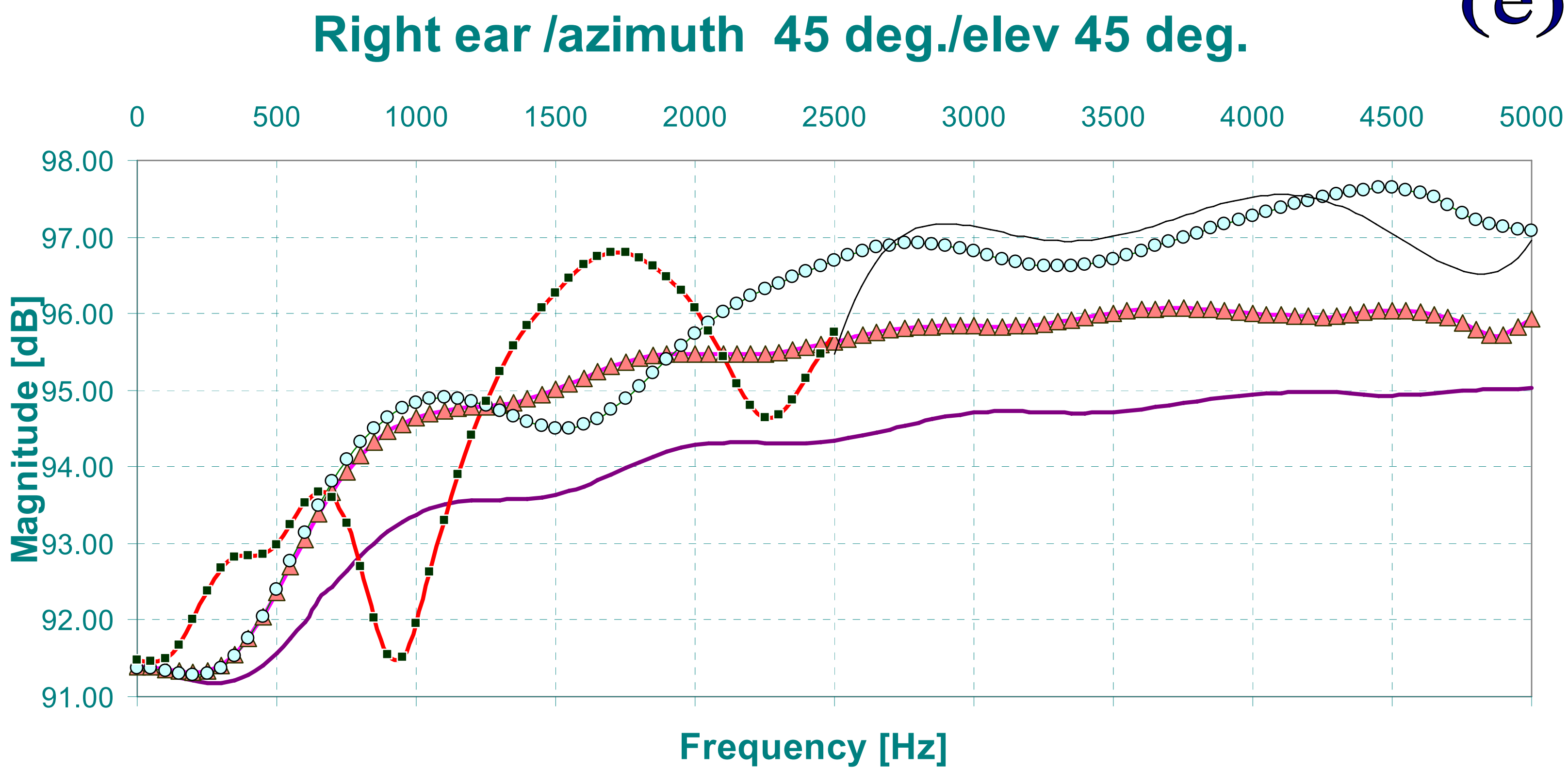
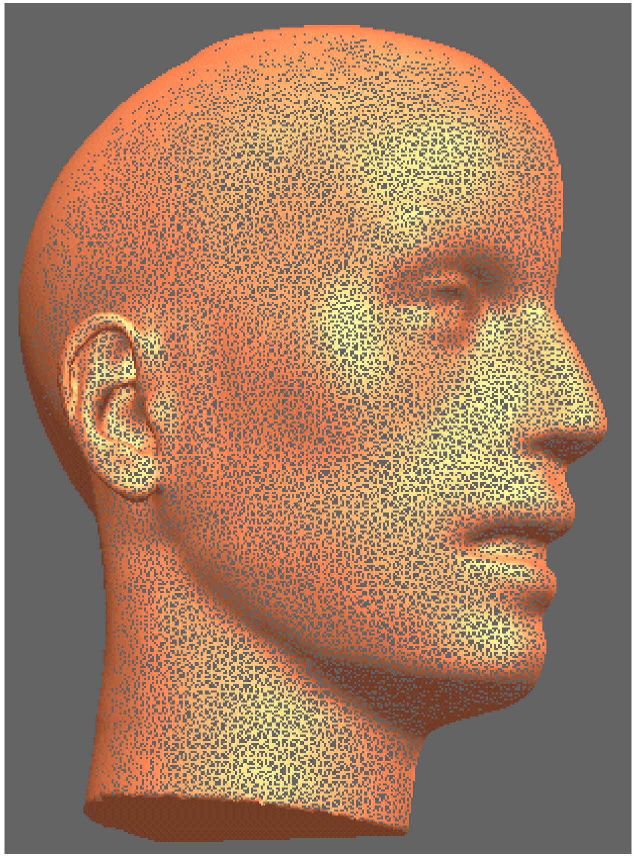
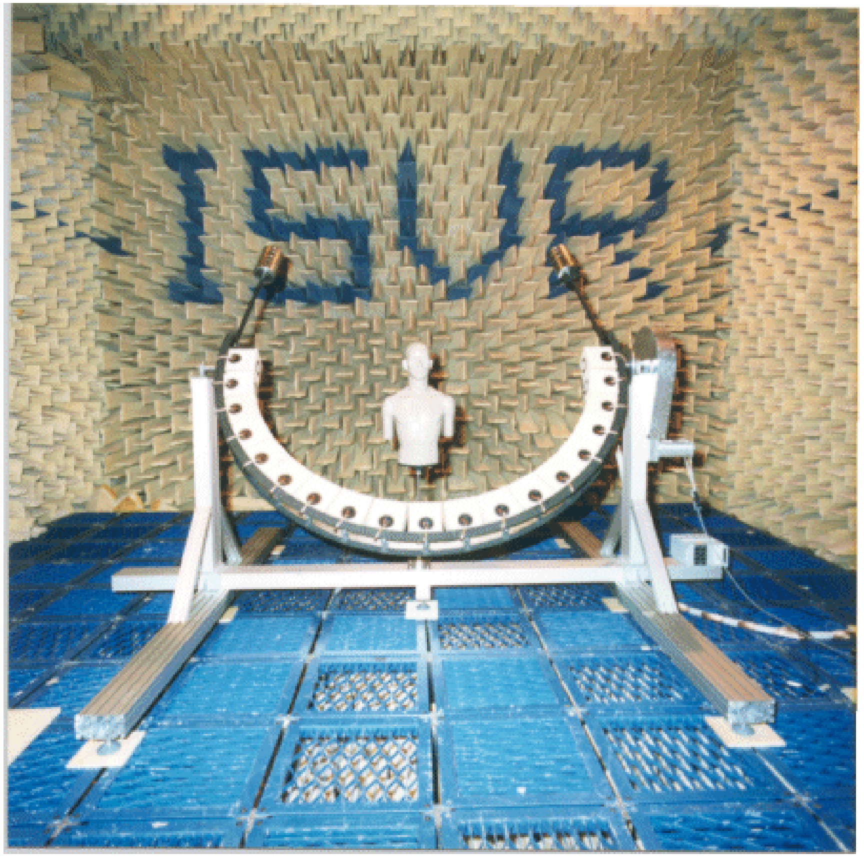
COMPARISON OF HRTFs OF SIMPLE MODELS - AT ELEVATION

(e)



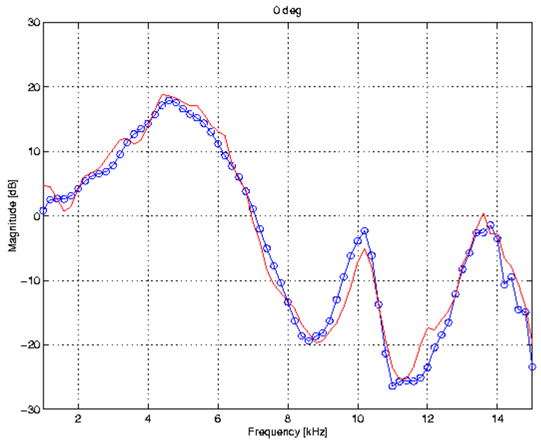
HRTF SIMULATION AND MEASUREMENT ARRANGEMENTS



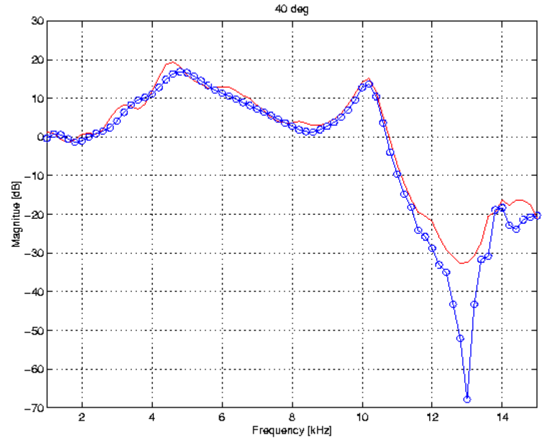


HRTFs OF KEMAR (WITH DB60)
MEDIAN PLANE MEASUREMENT AND SIMULATION

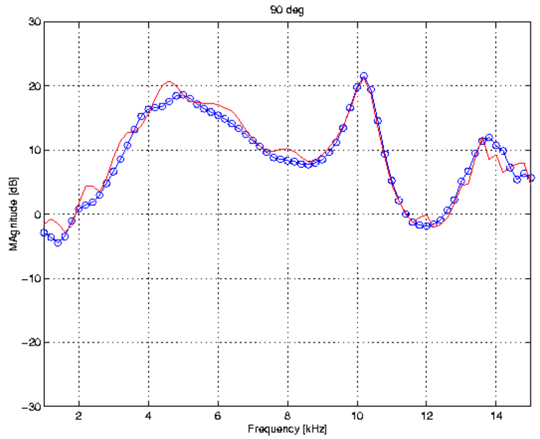
0 deg.



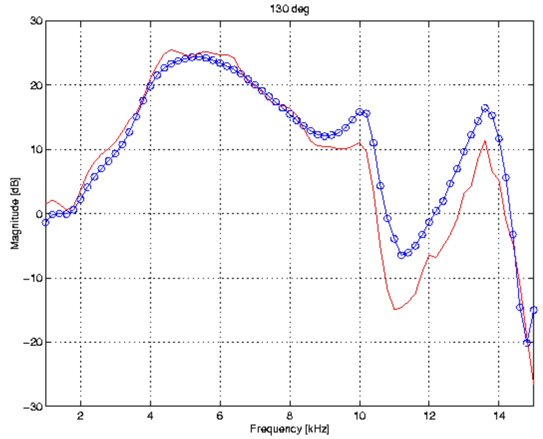
40 deg.



90 deg.

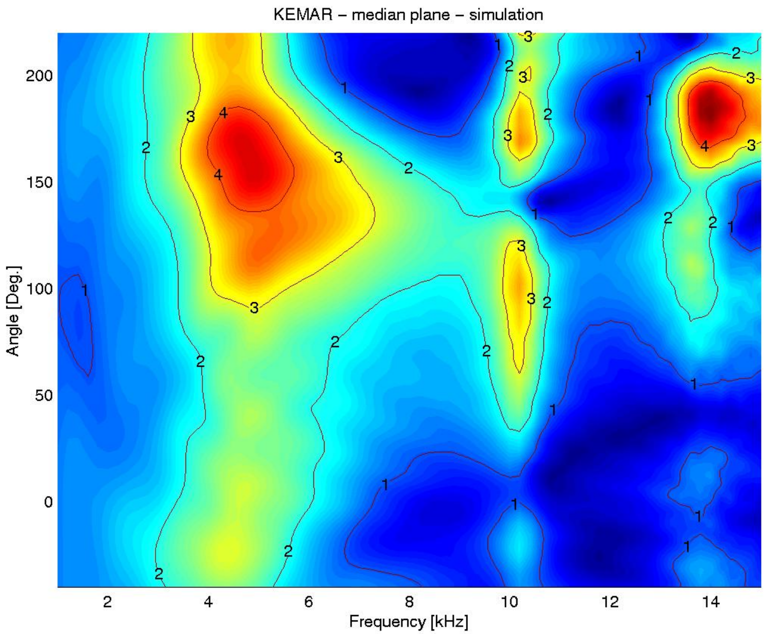


130 deg.

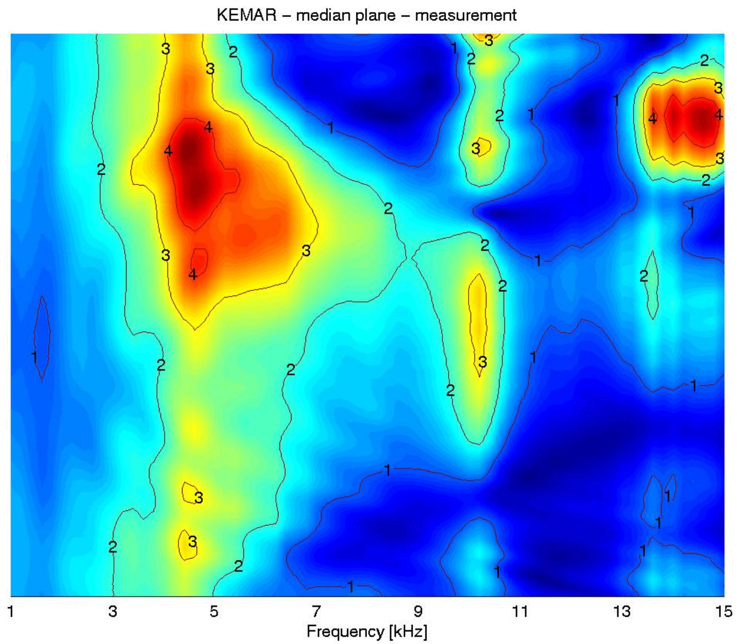


COLOUR MAPS OF SIMULATION AND MEASUREMENT OF THE HRTFs OF KEMAR - MEDIAN PLANE

SIMULATION



MEASUREMENT



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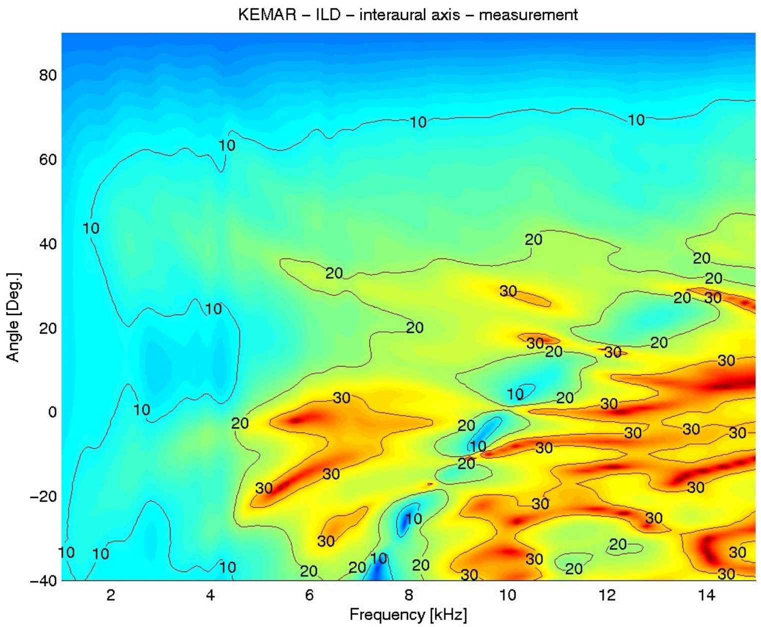
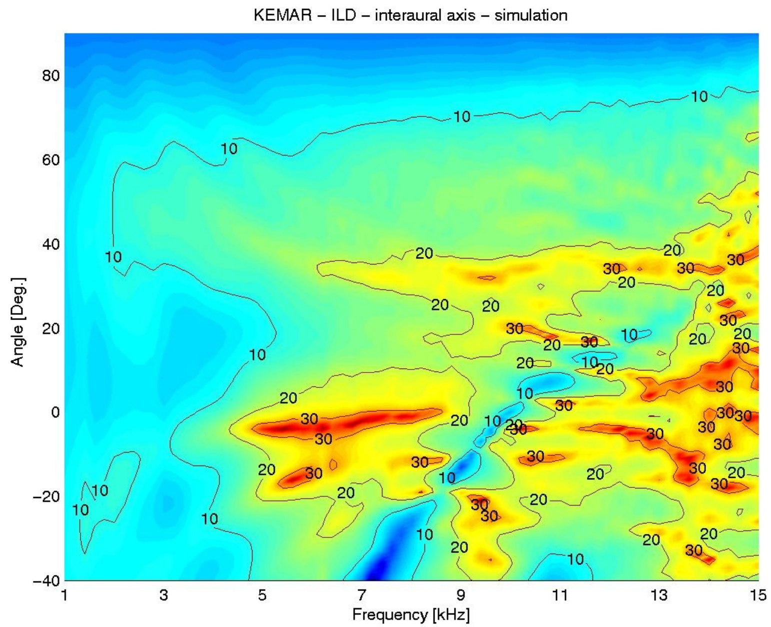
COMPARISON OF SIMULATION AND MEASUREMENT OF THE ILD IN THE LATERAL VERTICAL PLANE

SIMULATION



MEASUREMENT



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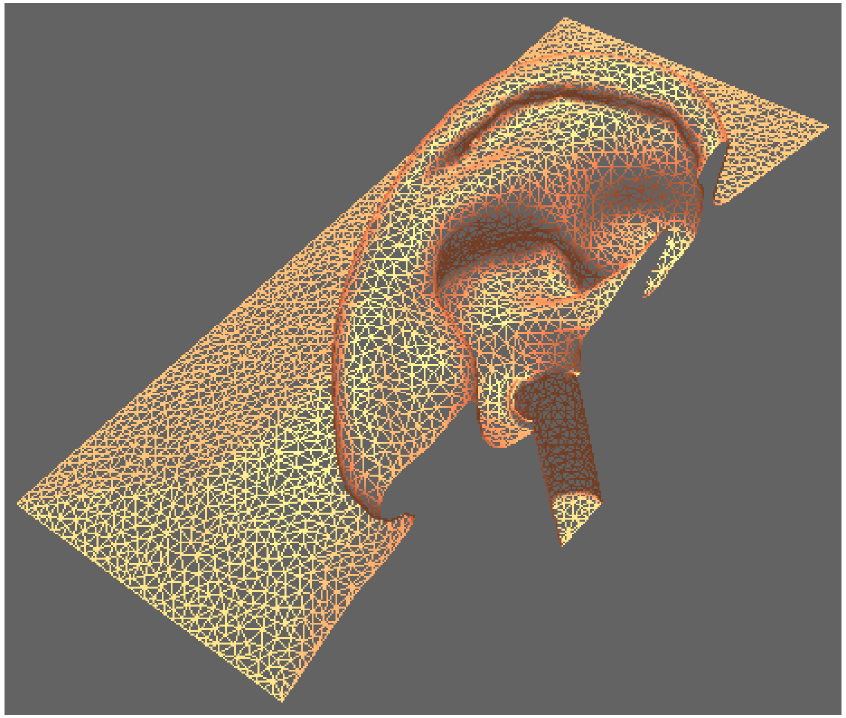
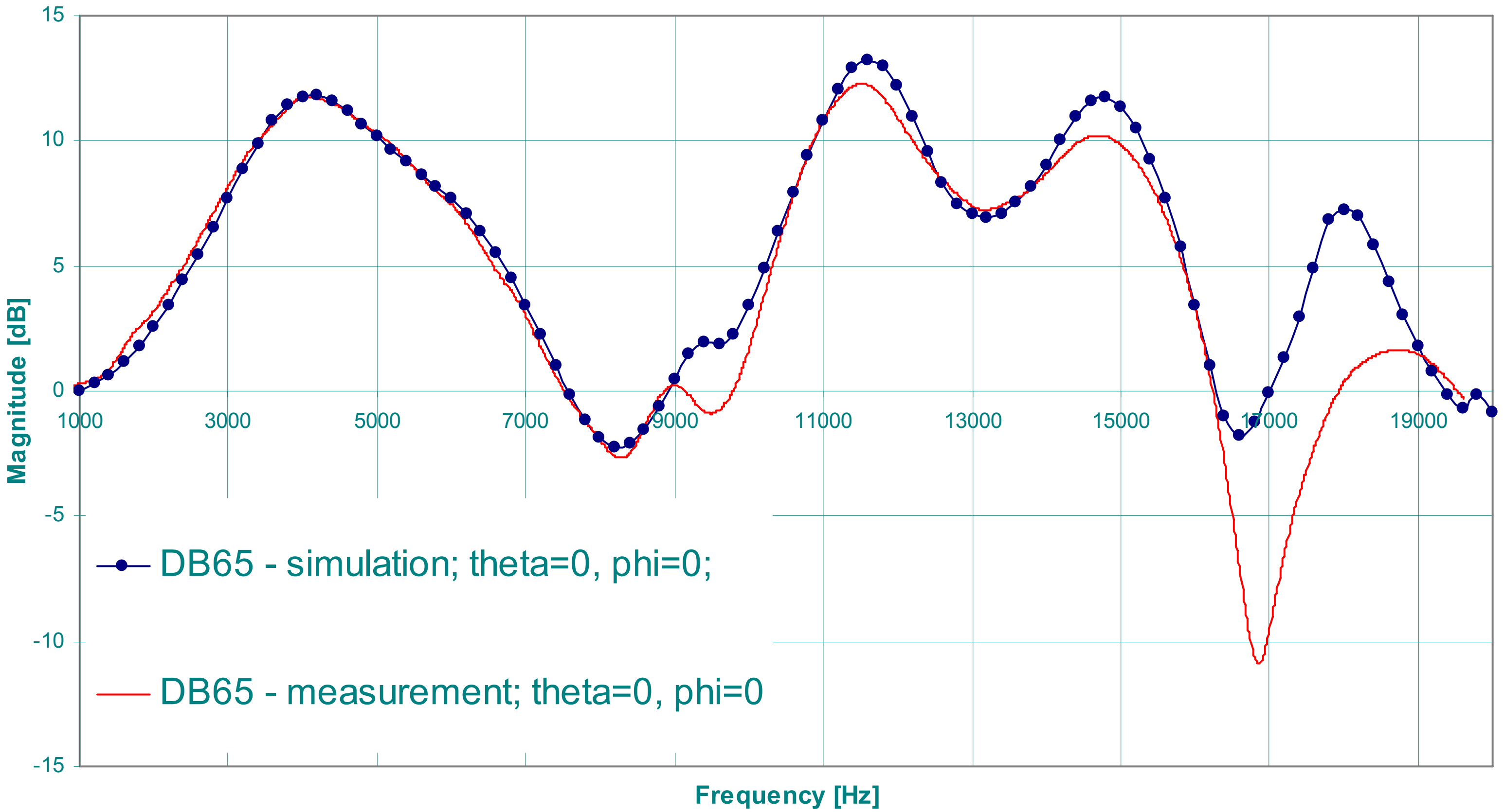
THE RESPONSE OF THE EXTERNAL EAR - SIMULATION MODEL AND MEASUREMENT APPARATUS



SIMULATION AND MEASUREMENT OF THE RESPONSE OF A BAFFLED DB65 PINNA - θ=0



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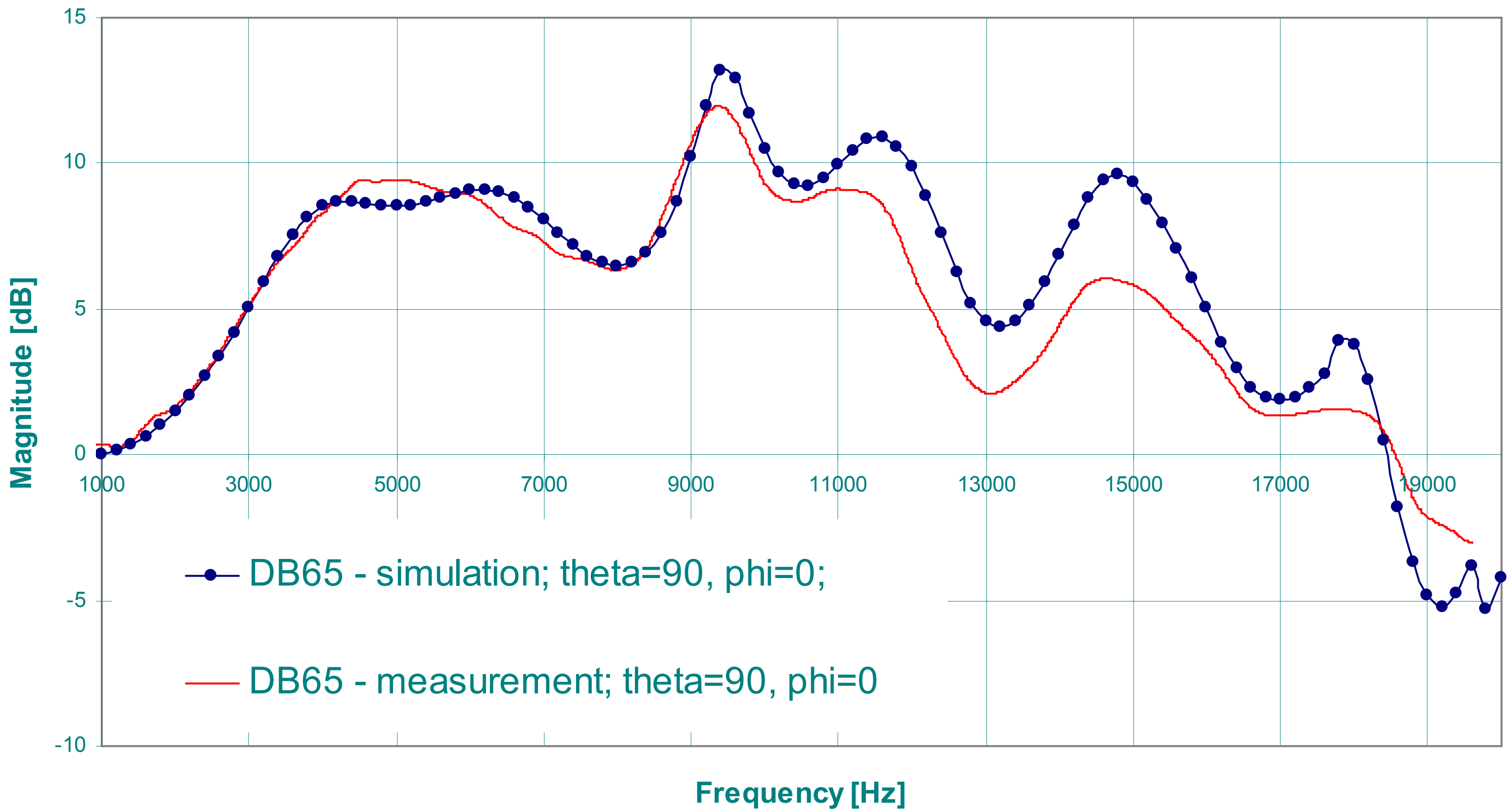
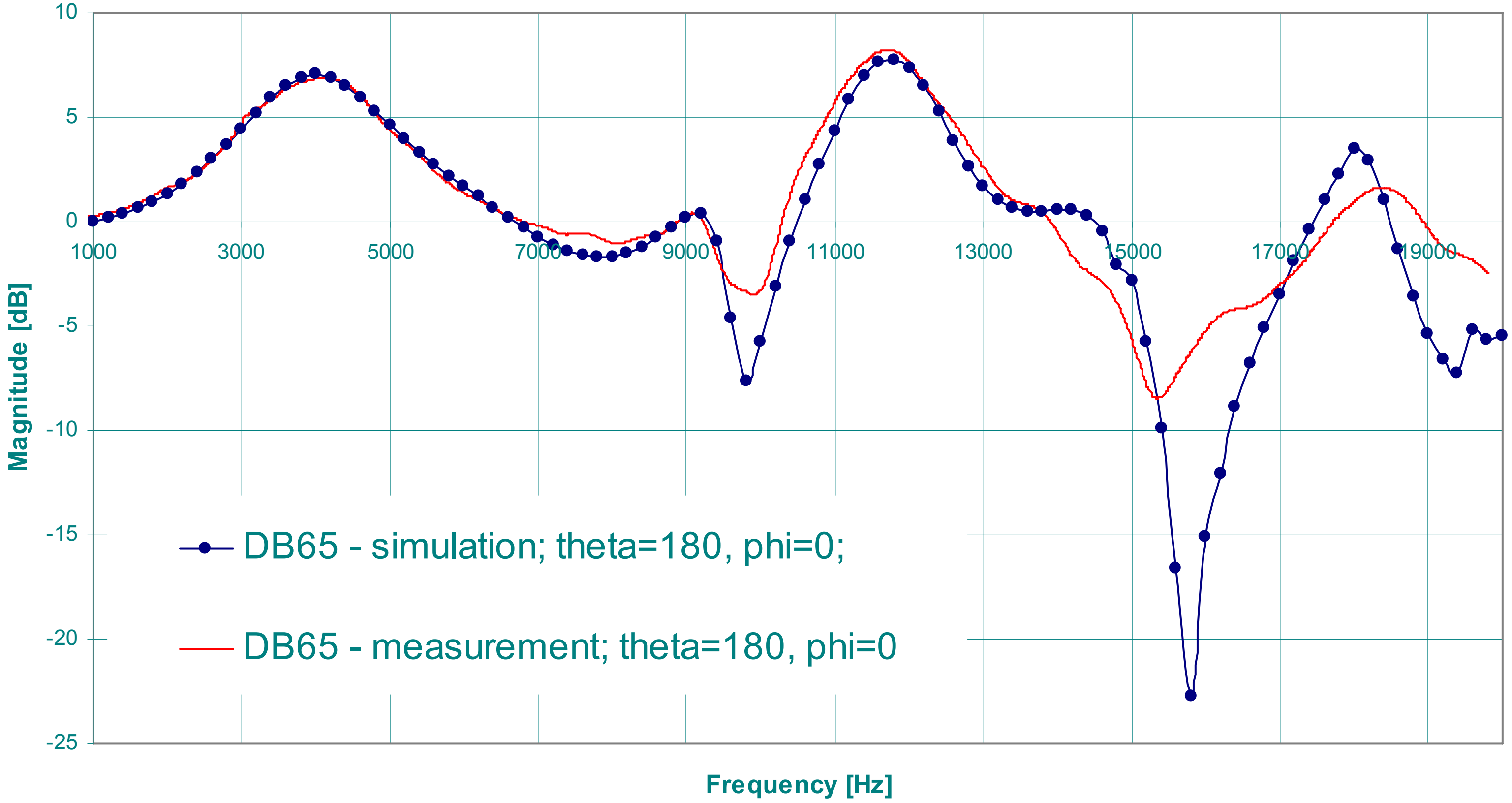
SIMULATION AND MEASUREMENT OF THE RESPONSE OF A BAFFLED DB65 PINNA - θ=90



SIMULATION AND MEASUREMENT OF THE RESPONSE OF A BAFFLED DB65 PINNA - θ=180



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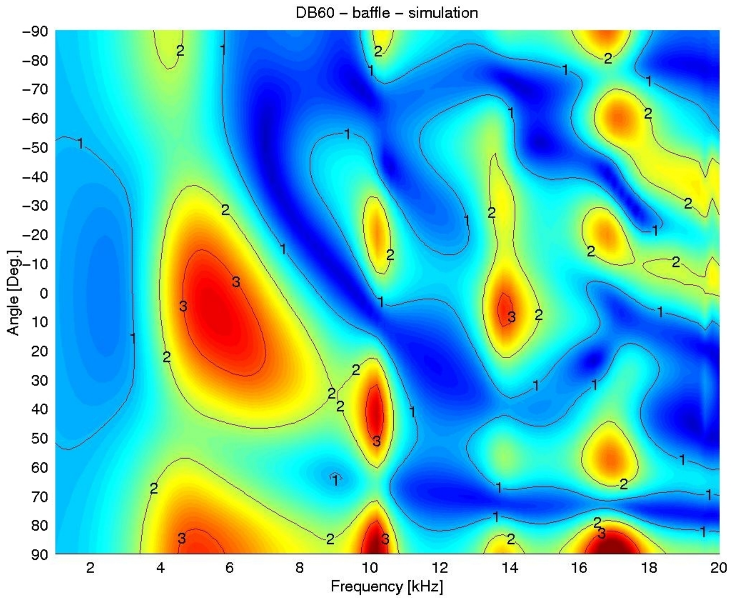
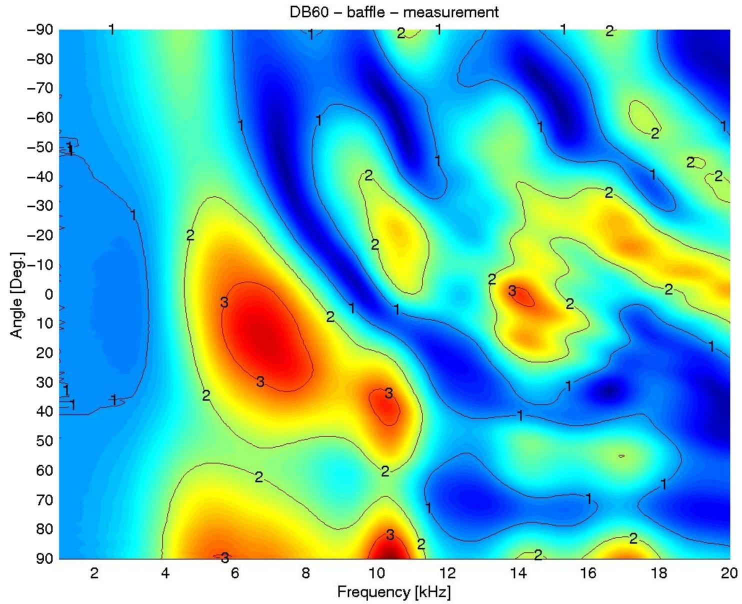
SIMULATION AND MEASUREMENT OF THE RESPONSE OF A BAFFLED DB60 PINNA

SIMULATION



MEASUREMENT



- Lateral vertical (frontal) plane
- Resolution of 1 degree on a linear scale
- High accuracy up to 20 kHz

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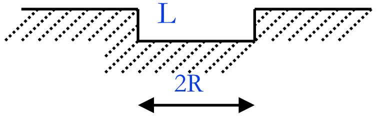
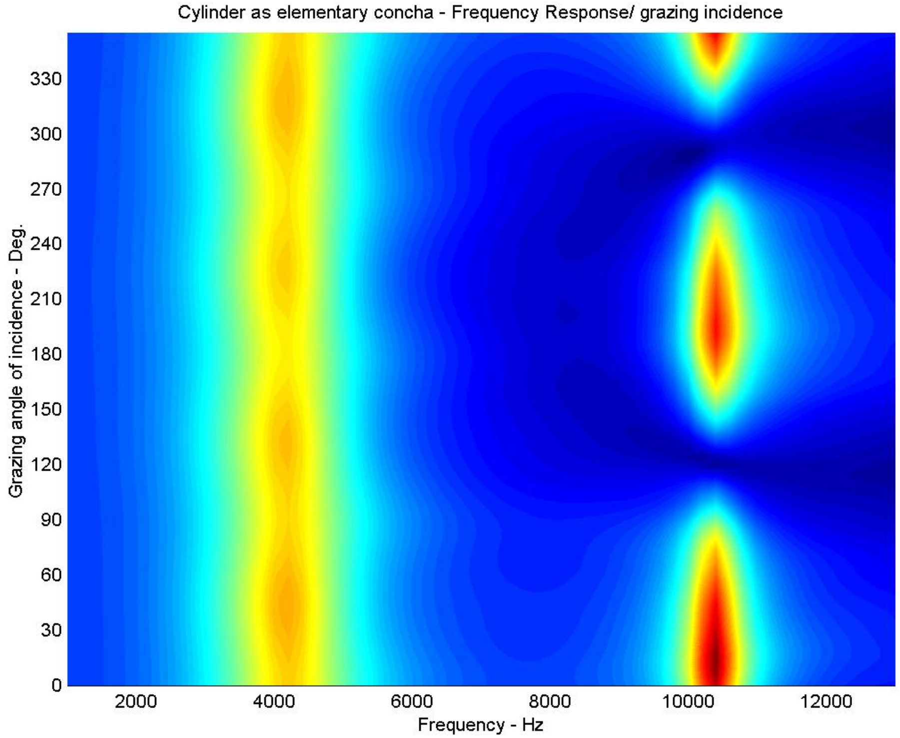
MODES OF A CYLINDER IN AN INFINITE BAFFLE



L=10mm, 2R=22mm

- Simple theory

$$\begin{cases} \dfrac{\lambda_{max}}{4R} = \dfrac{L}{R} + 0.822 \\ \dfrac{P}{P_0} = 1 + 7.86\left(\dfrac{L}{R}\right) \quad [\text{dB}] \end{cases}$$



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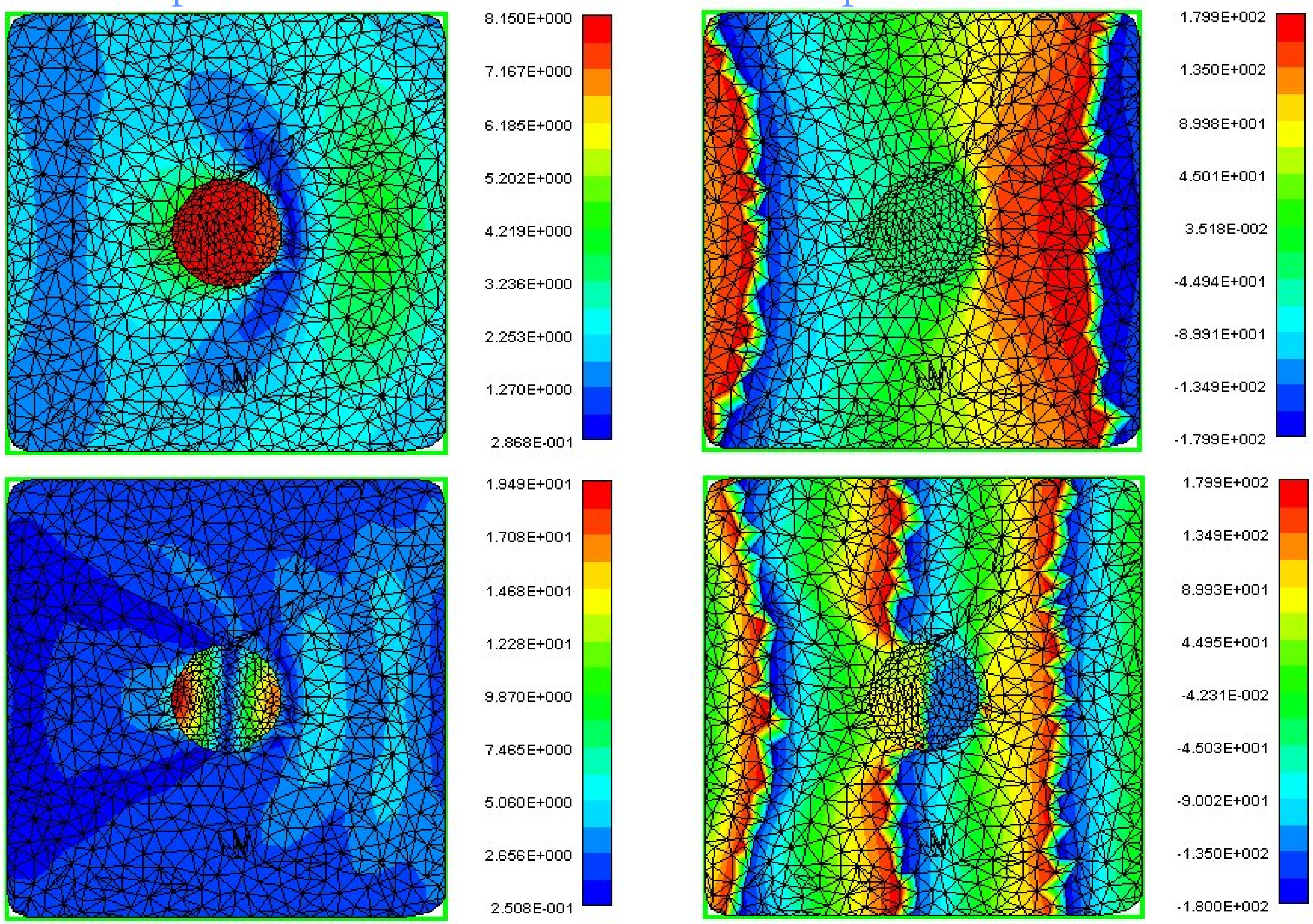
MODES OF AN INCLINED CYLINDER IN AN INFINITE BAFFLE (cont.)

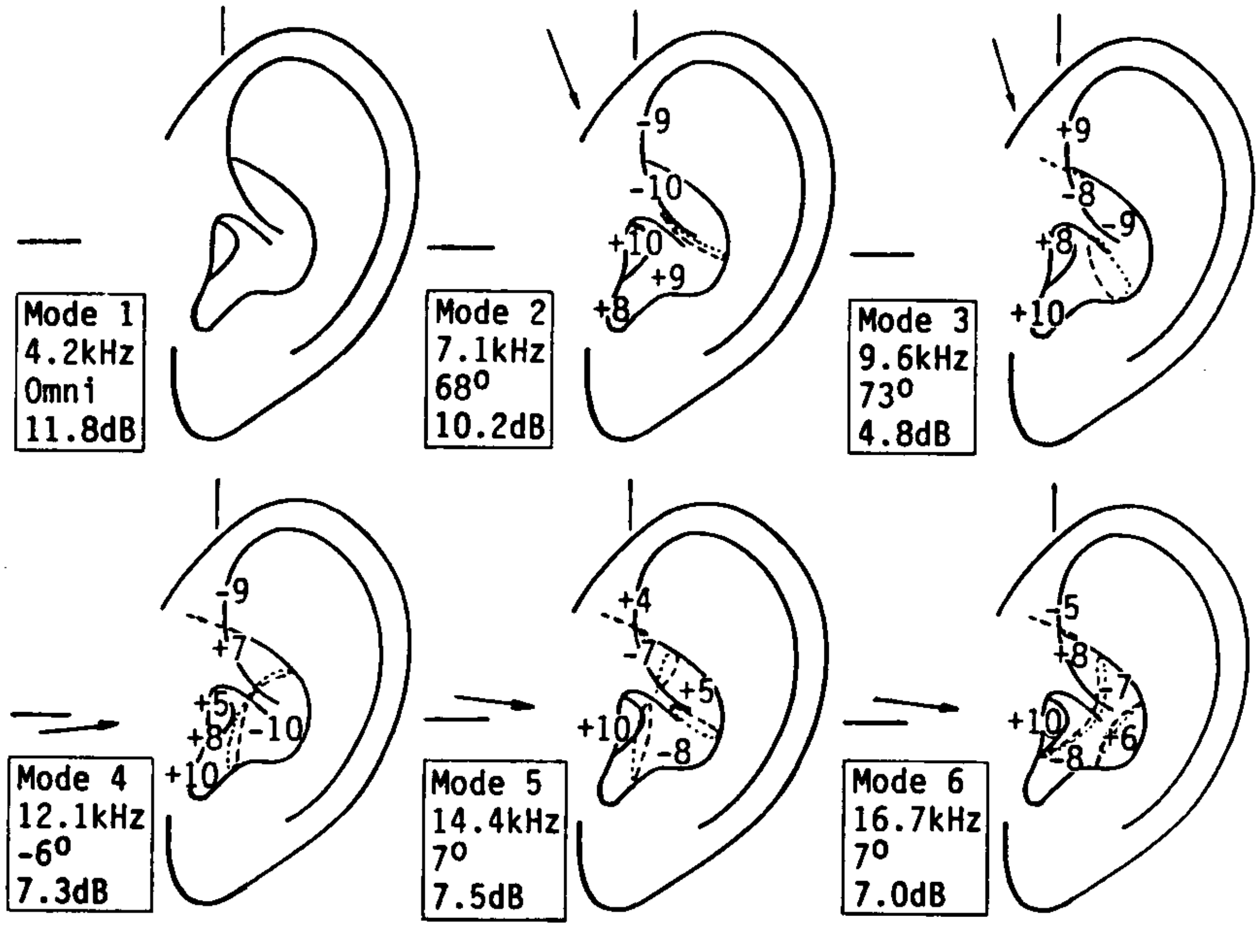
amplitude phase

4.1 kHz

11 kHz



MODE SHAPES OF THE EXTERNAL EAR
(AVERGAE OF 10 PINNAE)
EXCITATION AT GRAZING INCIDENCE (AFTER SHAW 1997)



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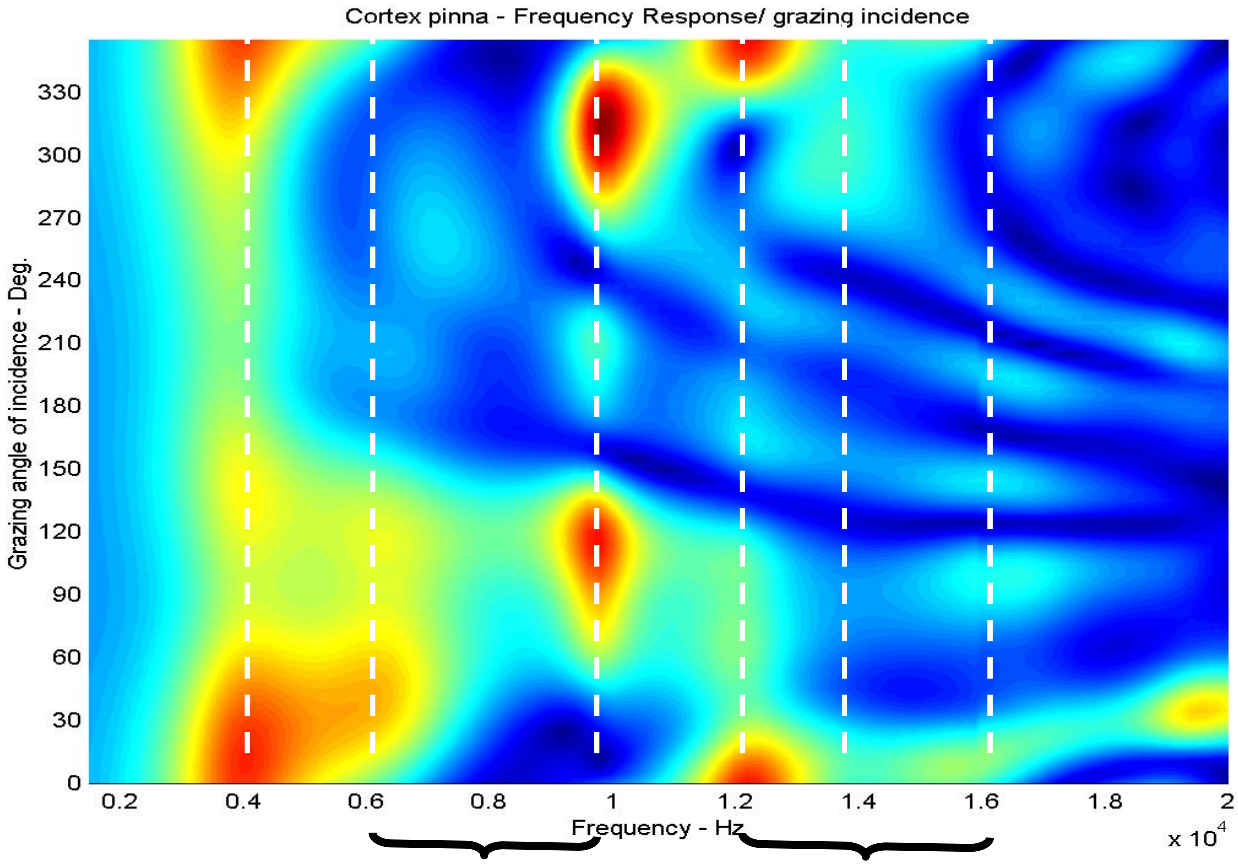
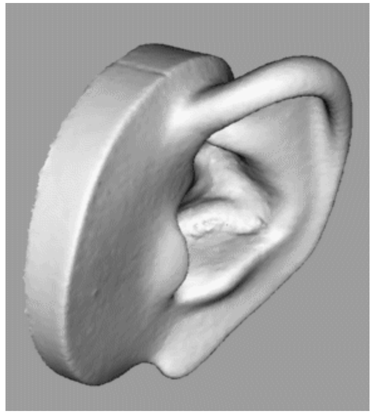
FREQUENCY RESPONSE OF THE CORTEX PINNA IN AN INFINITE BAFFLE - GRAZING INCIDENCE ANGLES





Omni-directional

'Vertical' transverse modes

'Horizontal' transverse modes

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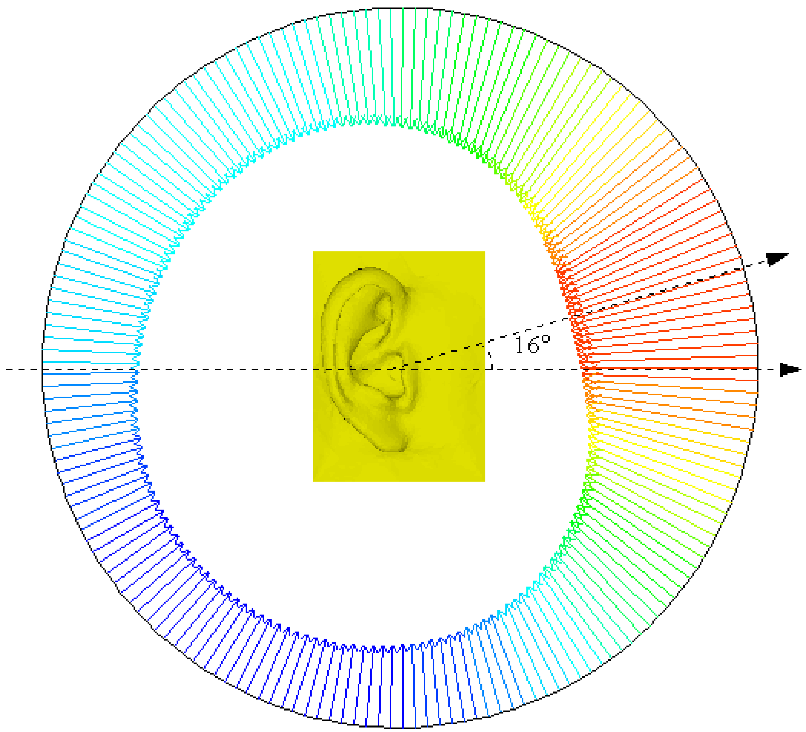
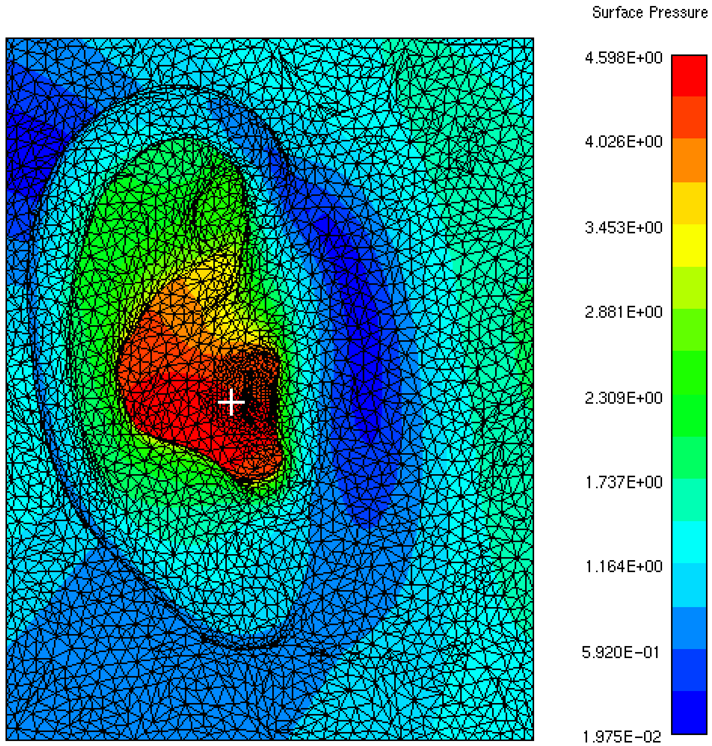
BEM SIMULATION OF THE MODE SHAPES OF THE DB65 PINNA –THE FIRST MODE

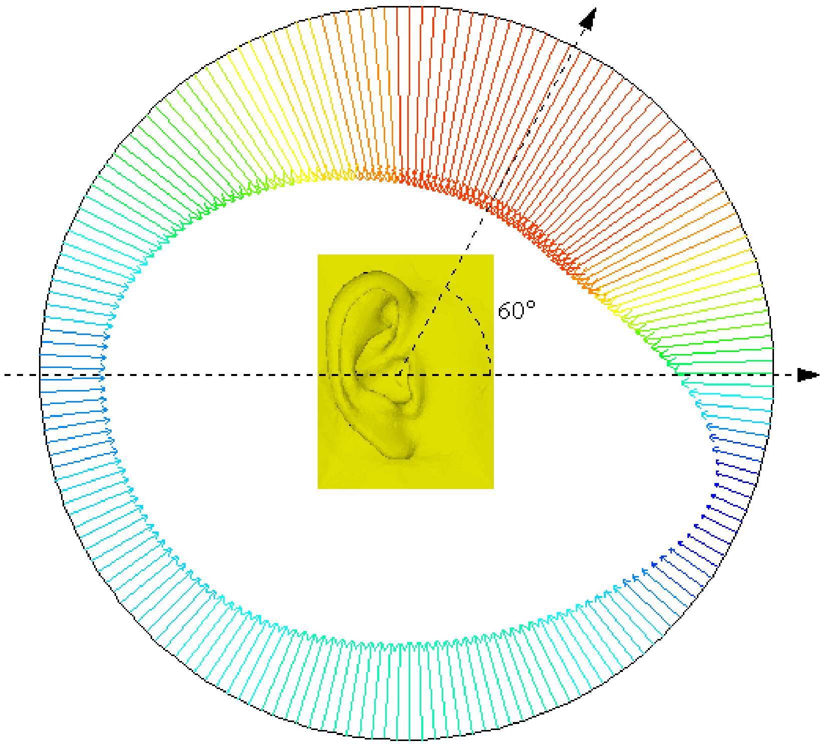
4.2 kHz

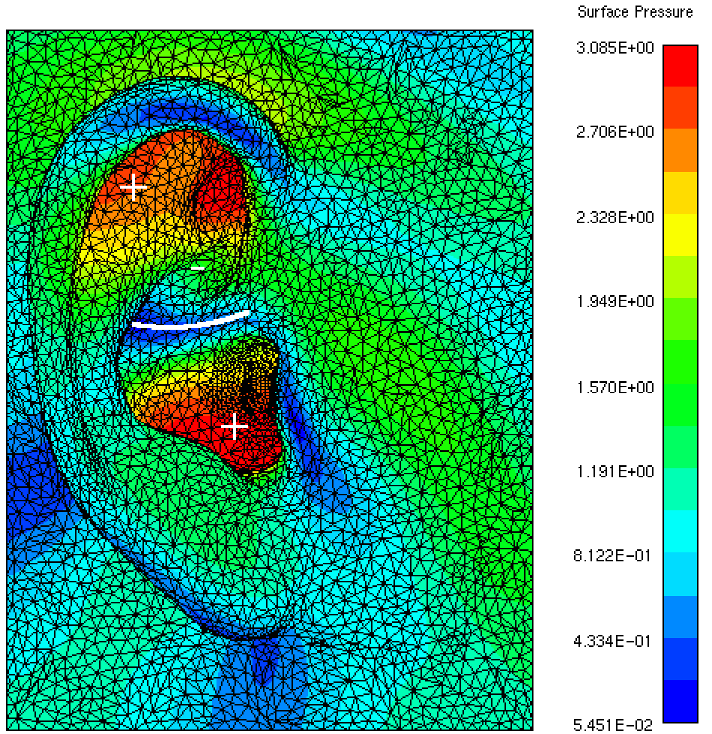




BEM SIMULATION OF THE MODE SHAPES OF THE DB65 PINNA – THE SECOND MODE

7.2 kHz





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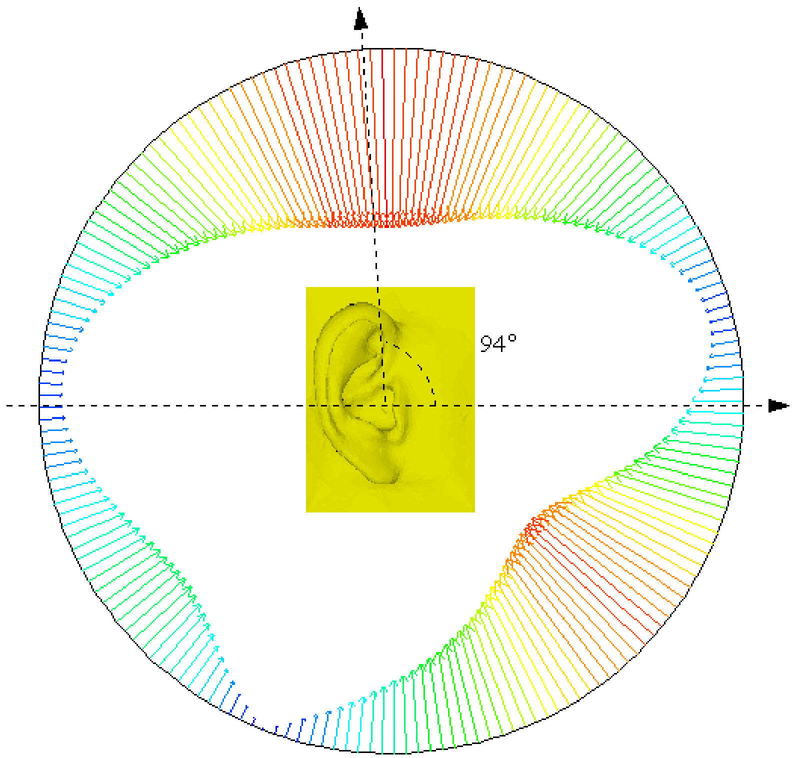
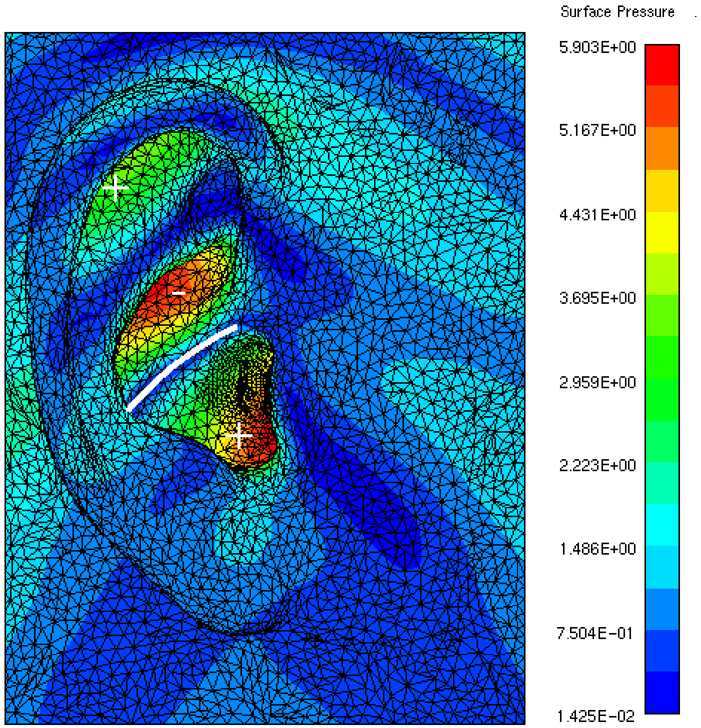
BEM SIMULATION OF THE MODE SHAPES OF THE DB65 PINNA – THE THIRD MODE

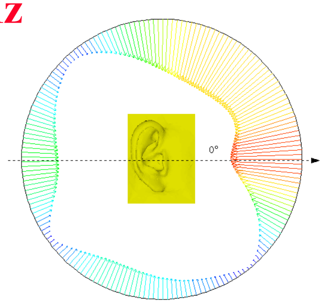
9.6 kHz

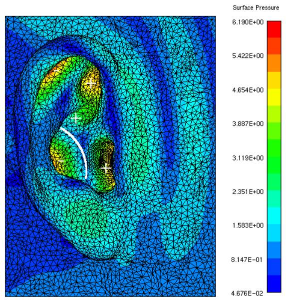




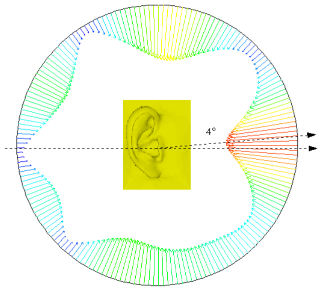
BEM SIMULATION OF THE MODE SHAPES OF THE DB65 PINNA – 'HORIZONTAL' MODES

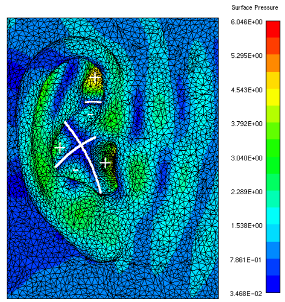
11.6 kHz



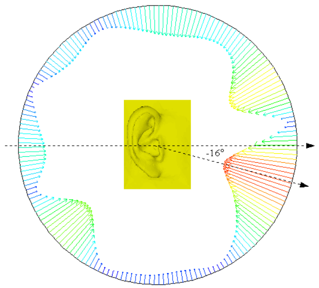


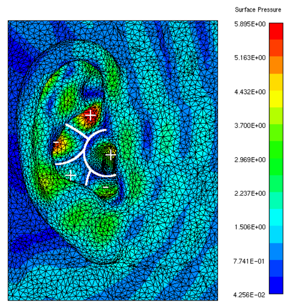
14.8 kHz





17.8 kHz





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Where are we?

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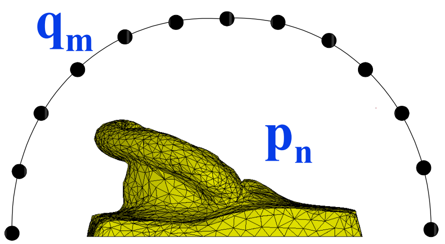
- Conclusions

SPATIAL PATTERNS IN ACOUSTIC SCATTERING



$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdots & \cdots & G_{1M} \\ G_{21} & G_{22} & \cdots & \cdots & G_{2M} \\ \vdots & \vdots & \ddots & & \\ \vdots & \vdots & & \ddots & \\ G_{N1} & G_{N2} & & & G_{NM} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ \vdots \\ q_M \end{bmatrix}$$

or

$$\mathbf{p} = \mathbf{G}\mathbf{q}$$

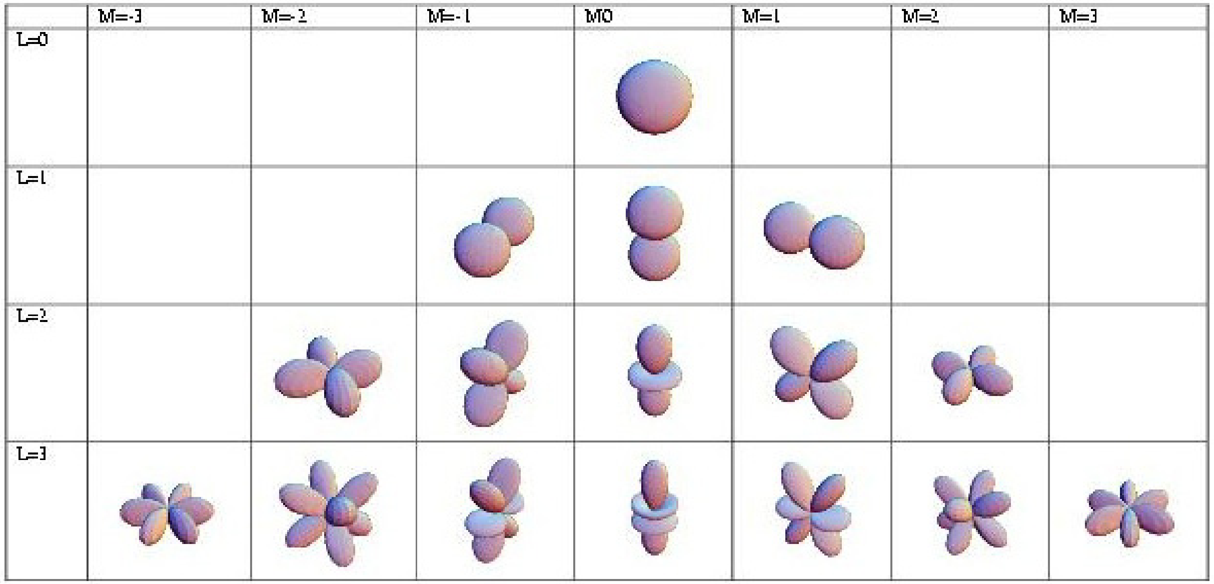
SINGULAR VALUE DECOMPOSITION ⟶ $\mathbf{G} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{H}}$

$$\mathbf{p} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{H}}\mathbf{q} \qquad (\mathbf{U}^{\mathbf{H}}\mathbf{U} = \mathbf{U}\mathbf{U}^{\mathbf{H}} = \mathbf{I})$$

$$\boxed{\mathbf{U}^{\mathbf{h}}\mathbf{p} = \boldsymbol{\Sigma}\mathbf{V}^{\mathbf{H}}\mathbf{q}}$$

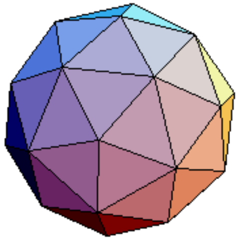
HRTF OF A RIGID SPHERE BASED ON SPHERICAL HARMONICS

$$Y_n^m(\theta,\phi) = (-1)^m \sqrt{\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{jm\phi}$$



$$p(\mathbf{r}) = j\rho_0 c_0 q(\hat{\mathbf{r}}) \sum_{n=0}^{\infty} \frac{h_n^{(2)}(kr)}{h_n^{(2)'}(ka)} \sum_{m=-n}^{n} Y_n^m(\theta,\phi) Y_n^{m*}(\hat{\theta},\hat{\phi})$$

GREEN FUNCTION MATRIX RELATING POINTS ON A RIGID SPHERE AND SOURCES IN THE FAR FIELD (LARGE SPHERE)



$$\mathbf{G}(\mathbf{r}\,|\,\hat{\mathbf{r}}) = \begin{bmatrix} \sum_{n=0}^{\infty} f_n \sum_{m=-n}^{n} Y_n^m(\theta_1,\phi_1) Y_n^{m*}(\hat{\theta}_1,\hat{\phi}_1) & \cdots & \sum_{n=0}^{\infty} f_n \sum_{m=-n}^{n} Y_n^m(\theta_1,\phi_1) Y_n^{m*}(\hat{\theta}_L,\hat{\phi}_L) \\ \sum_{n=0}^{\infty} f_n \sum_{m=-n}^{n} Y_n^m(\theta_2,\phi_2) Y_n^{m*}(\hat{\theta}_1,\hat{\phi}_1) & \cdots & \sum_{n=0}^{\infty} f_n \sum_{m=-n}^{n} Y_n^m(\theta_2,\phi_2) Y_n^{m*}(\hat{\theta}_L,\hat{\phi}_L) \\ \sum_{n=0}^{\infty} f_n \sum_{m=-n}^{n} Y_n^m(\theta_K,\phi_K) Y_n^{m*}(\hat{\theta}_1,\hat{\phi}_1) & \cdots & \sum_{n=0}^{\infty} f_n \sum_{m=-n}^{n} Y_n^m(\theta_K,\phi_K) Y_n^{m*}(\hat{\theta}_L,\hat{\phi}_L) \end{bmatrix}$$

$$\mathbf{G}_N(\mathbf{r}\,|\,\hat{\mathbf{r}}) = \begin{bmatrix} Y_0(\theta_1,\phi_1) & \cdots & Y_n^m(\theta_1,\phi_1) & \cdots & Y_N^N(\theta_1,\phi_1) \\ Y_0(\theta_2,\phi_2) & \cdots & Y_n^m(\theta_2,\phi_2) & \cdots & Y_N^N(\theta_2,\phi_2) \\ \vdots & & & & \\ Y_0(\theta_K,\phi_K) & \cdots & Y_n^m(\theta_K,\phi_K) & \cdots & Y_N^N(\theta_K,\phi_K) \end{bmatrix} \begin{bmatrix} f_0 & & & \\ & f_1 & & \\ & & \ddots & \\ & & & f_N \end{bmatrix} \begin{bmatrix} Y_0^*(\hat{\theta}_1,\hat{\phi}_1) & Y_0^*(\hat{\theta}_2,\hat{\phi}_2) & \cdots & Y_0^*(\hat{\theta}_L,\hat{\phi}_L) \\ \vdots & & & \\ Y_n^{m*}(\hat{\theta}_1,\hat{\phi}_1) & Y_n^{m*}(\hat{\theta}_2,\hat{\phi}_2) & \cdots & Y_n^{m*}(\hat{\theta}_L,\hat{\phi}_L) \\ \vdots & & & \\ Y_N^{N*}(\hat{\theta}_1,\hat{\phi}_1) & Y_N^{N*}(\hat{\theta}_2,\hat{\phi}_2) & \cdots & Y_N^{N*}(\hat{\theta}_L,\hat{\phi}_L) \end{bmatrix}$$

$$\mathbf{G}_N(\mathbf{r}\,|\,\hat{\mathbf{r}}) = \mathbf{Y}(\mathbf{r}_k)\mathbf{F}\mathbf{Y}^{\mathrm{H}}(\hat{\mathbf{r}}_l)$$

LINEAR TRANSFORMATION WITH UNITARY MATRICES

$$\mathbf{G}_N(\mathbf{r}\,|\,\hat{\mathbf{r}}) = \mathbf{Y}(\mathbf{r}_k)\mathbf{F}\mathbf{Y}^{\mathrm{H}}(\hat{\mathbf{r}}_l)$$

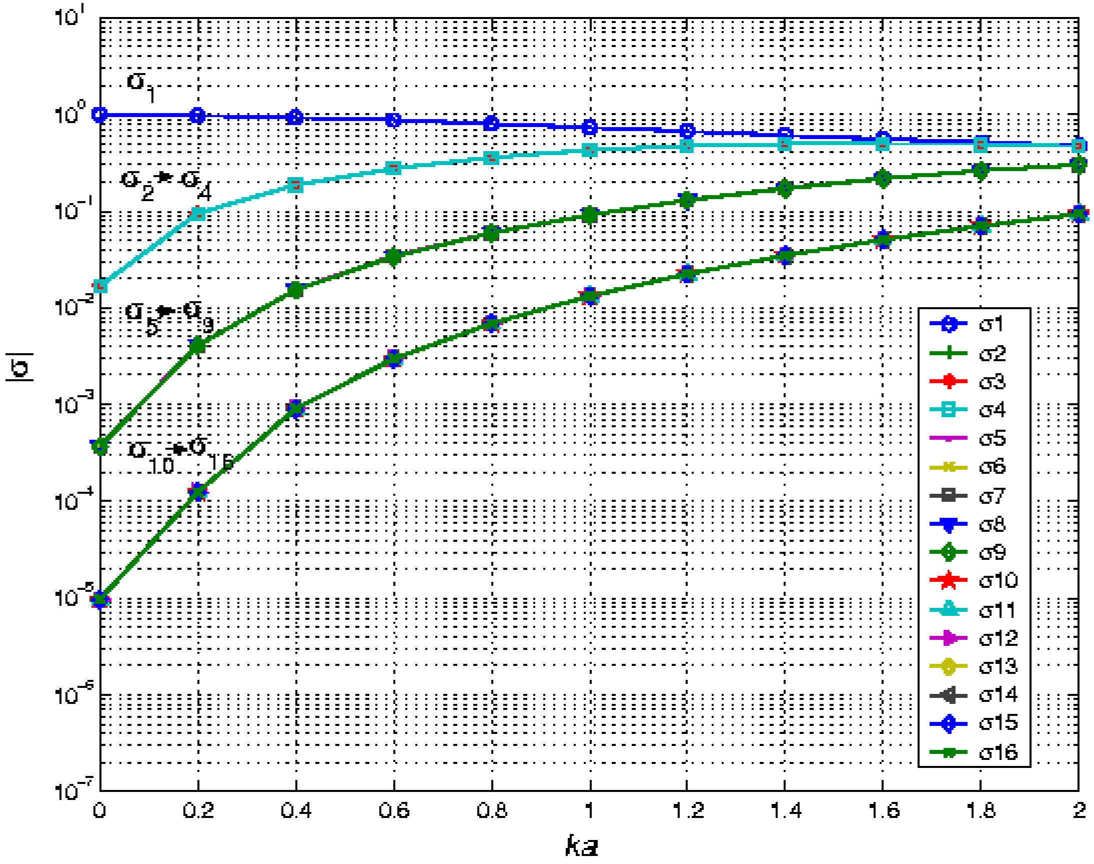
$$\mathbf{U}_N = \mathbf{Y}(\mathbf{r}_k)\mathbf{T}(\mathbf{r}_k)$$

$$\mathbf{V}_N = \mathbf{Y}(\hat{\mathbf{r}}_l)\mathbf{T}(\hat{\mathbf{r}}_l)$$

$$\mathbf{G}_N(\mathbf{r}\,|\,\hat{\mathbf{r}}) = \mathbf{Y}(\mathbf{r}_k)\underbrace{\mathbf{T}(\mathbf{r}_k)\Sigma_N\mathbf{T}^{\mathrm{H}}(\hat{\mathbf{r}}_l)}\mathbf{Y}^{\mathrm{H}}(\hat{\mathbf{r}}_l)$$

SVD: $$\mathbf{G}(\mathbf{r}\,|\,\hat{\mathbf{r}}) = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{H}}$$

THE SINGULAR VALUES OF A 32x32 GREEN FUNCTION MATRIX WITH UNIFORMLY SAMPLED SPHERES



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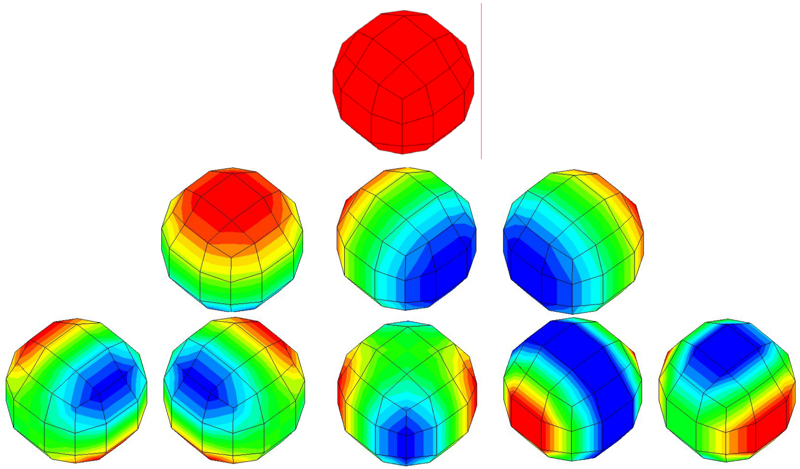
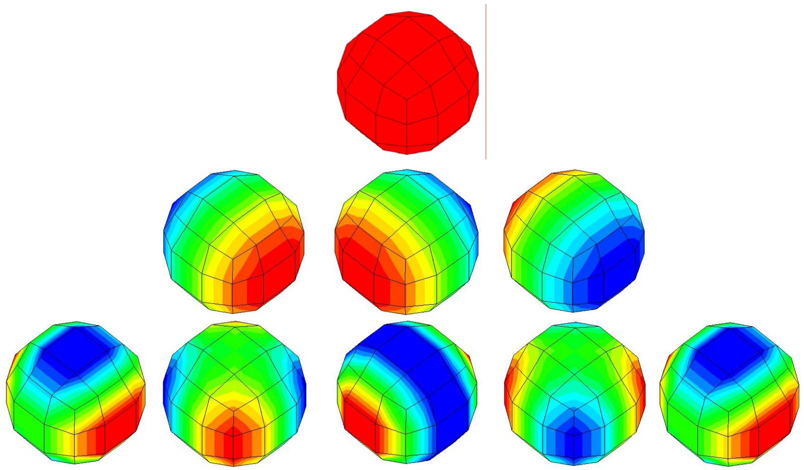
REAL PARTS OF SPHERICAL HARMONICS AND THE LEFT SINGULAR VECTORS OF THE GREEN FUNCTION MATRIX

$\mathrm{Re}\{\mathbf{Y}(\mathbf{r}_k)\}$

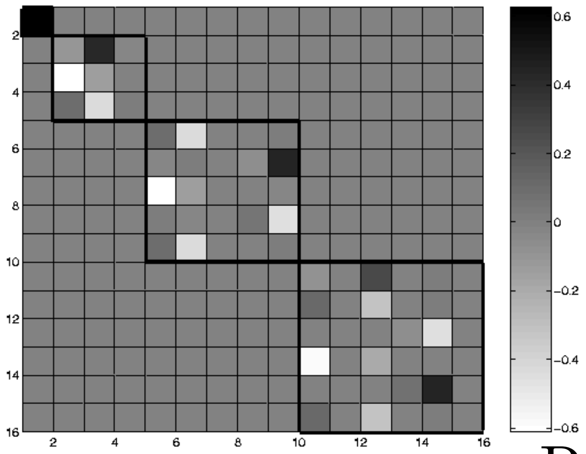


$\mathrm{Re}\{\mathbf{U}(\mathbf{r}_k)\}$

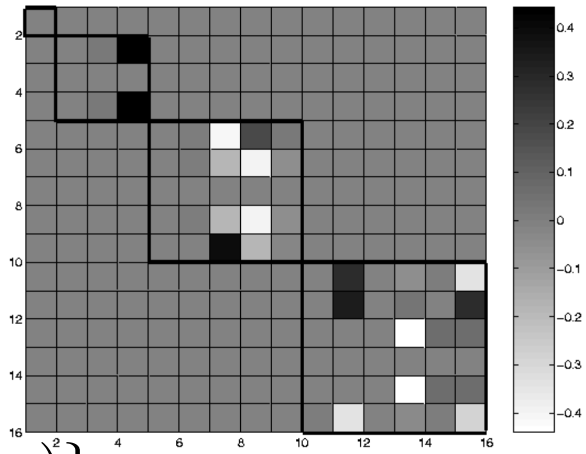


CALCULATION OF THE UNITARY TRANSFORMATION MATRICES

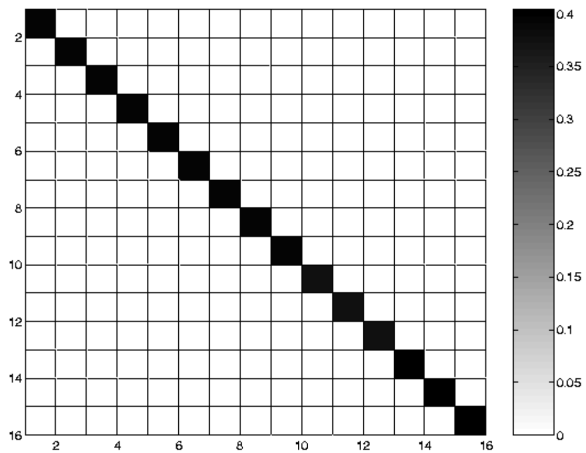
$\mathrm{Re}\{\mathbf{T}(\mathbf{r}_k)\}$



$\mathrm{Im}\{\mathbf{T}(\mathbf{r}_k)\}$



$\mathrm{Re}\{\mathbf{T}(\mathbf{r}_k)^{\mathrm{H}}\mathbf{T}(\mathbf{r}_k)\}$



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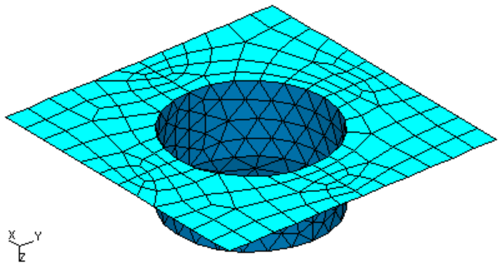
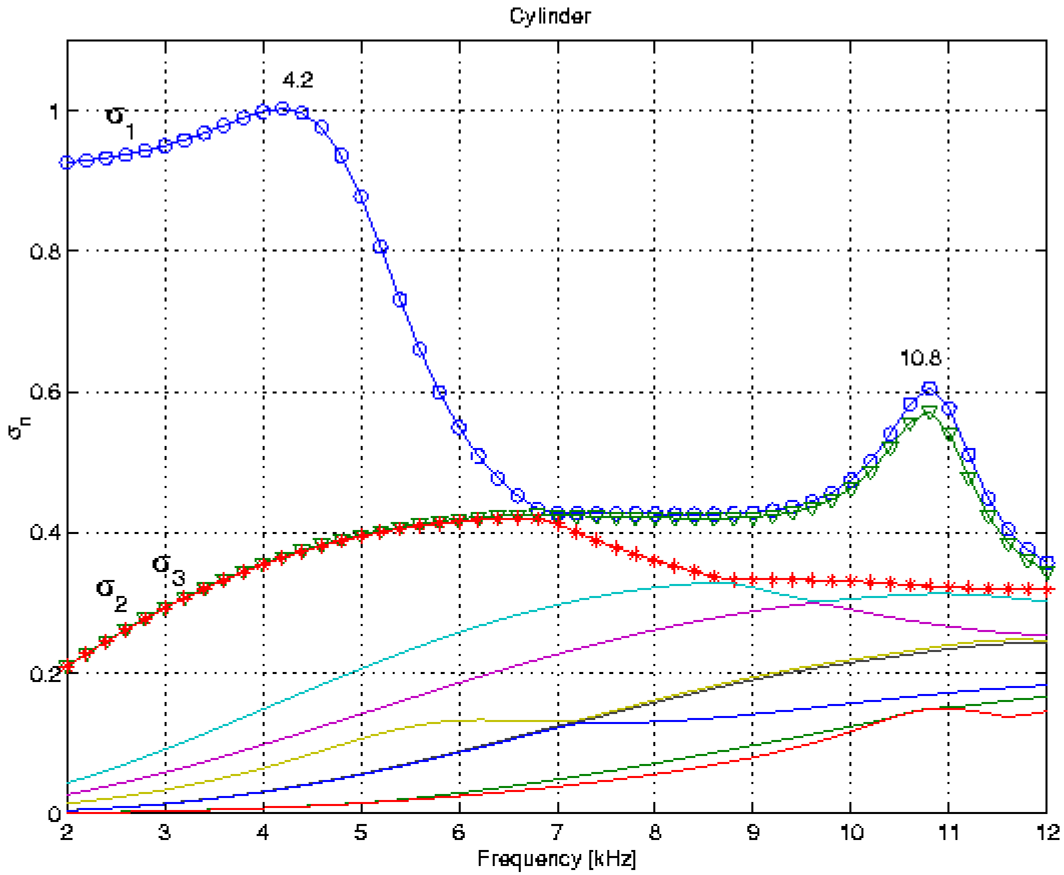
THE SINGULAR VALUES OF THE GREEN FUNCTION MATRIX RELATING A BAFFLED CYLINDER AND THE HEMISPHERE





356 'field' points
121 'source' points

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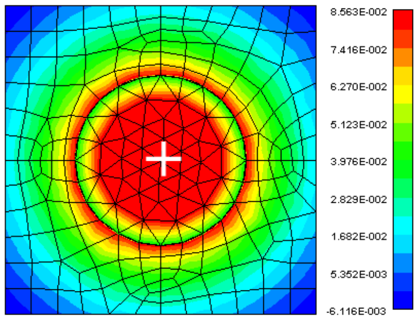
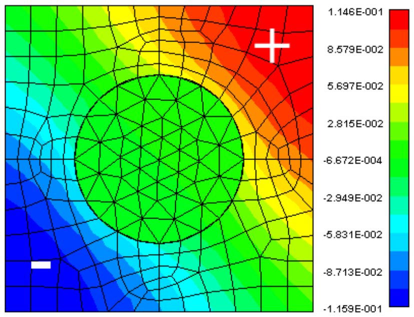
BAFFLED CYLINDER AND THE HEMISPHERE - COLOUR MAPS OF THE SINGULAR VECTORS

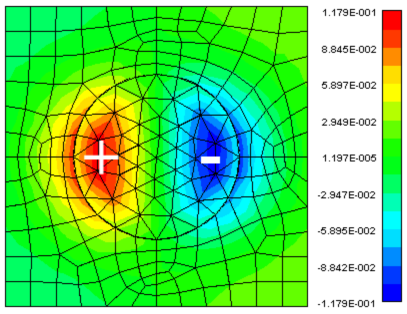
Re {v} σ_1 - 4.2 kHz



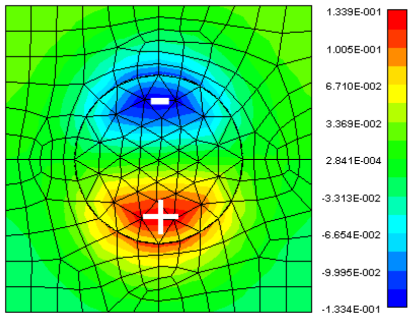
Re {v} σ_2 - 4.2 kHz



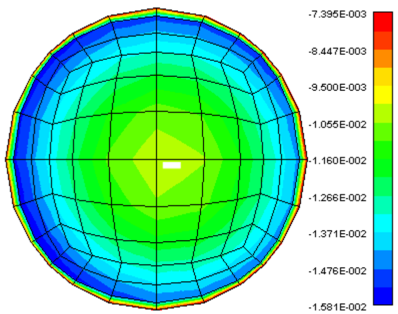
Re {v} σ_1 - 10.8 kHz



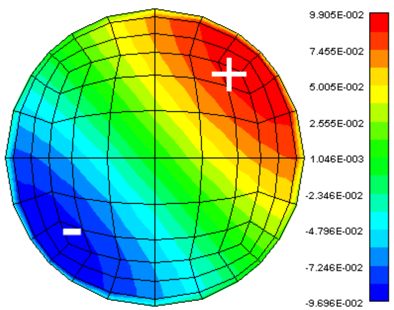
Re {v} σ_2 - 10.8 kHz



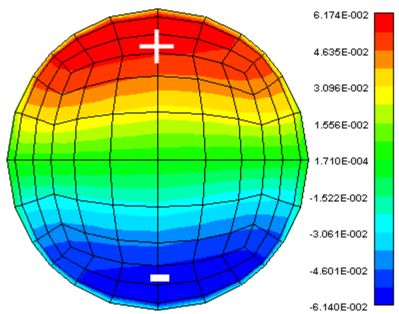
Re {u} σ_1 - 4.2 kHz



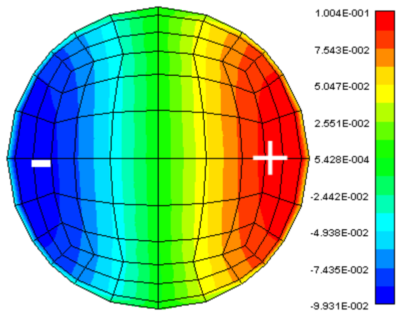
Re {u} σ_2 - 4.2 kHz



Re {u} σ_1 - 10.8 kHz



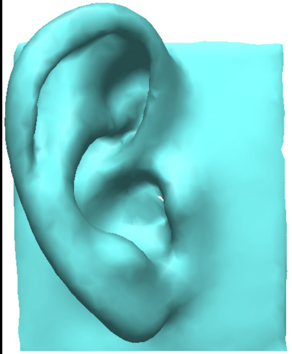
Re {u} σ_2 - 10.8 kHz



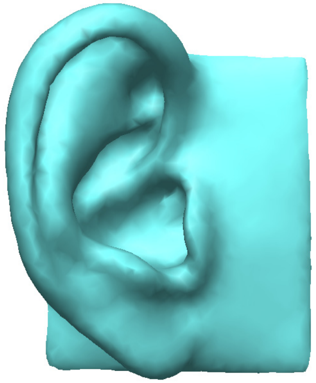
MODELLED PINNAE

KEMAR

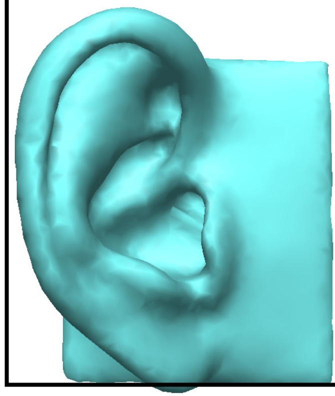
DB60



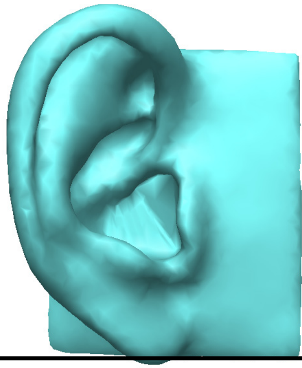
DB65



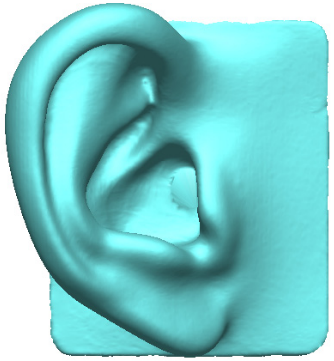
DB90



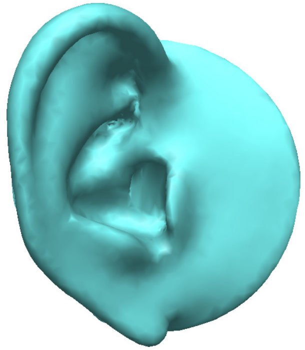
DB95



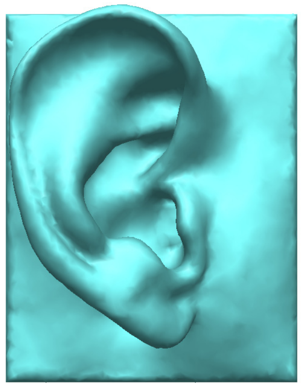
B&K



CORTEX



YK



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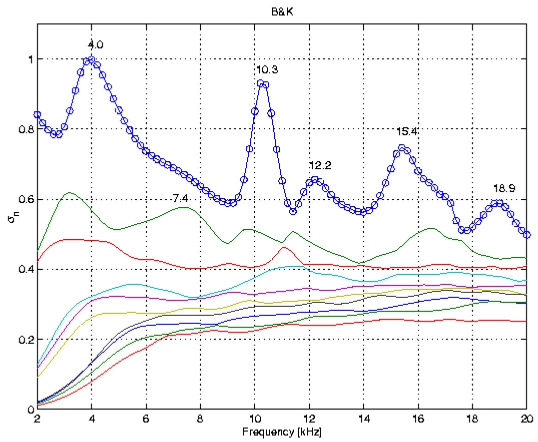
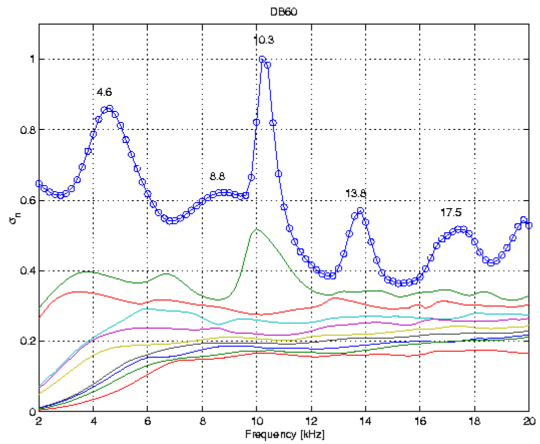
SINGULAR VALUES OF ACCURATE PINNAE

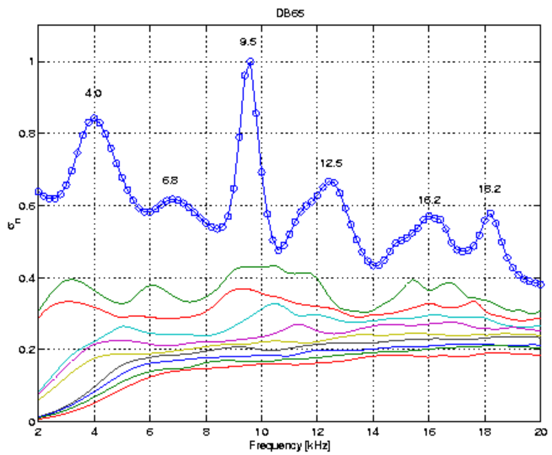
B&K (3906 × 209)



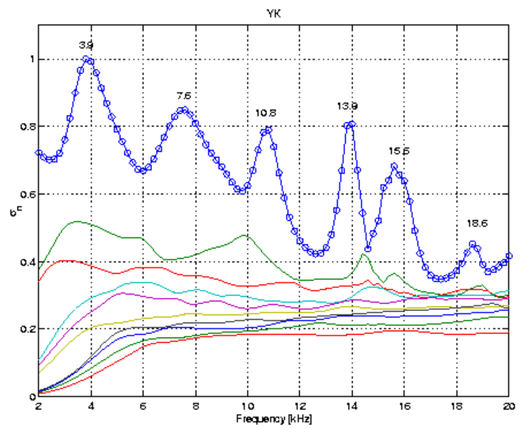
DB60 (2825 × 209)



DB65 (3389 × 209)

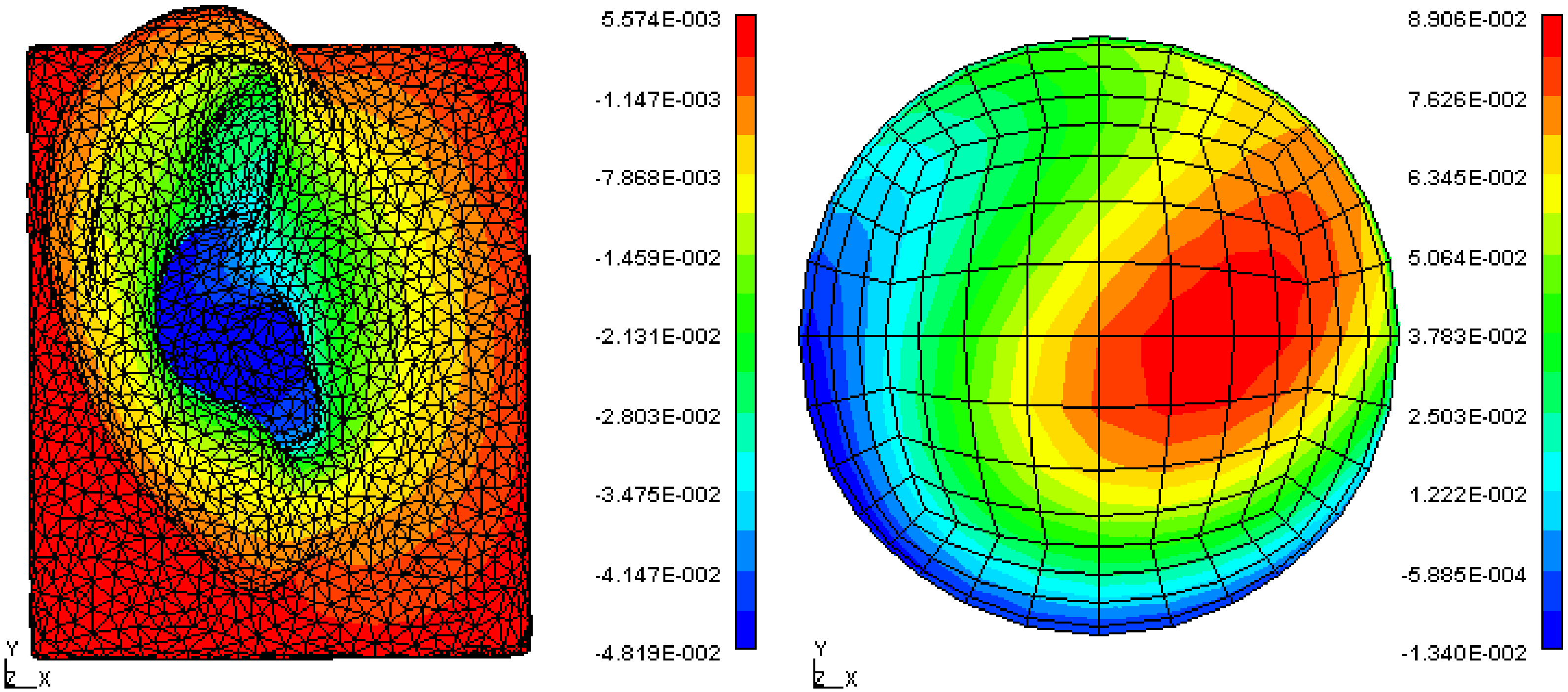


YK (3392 × 209)



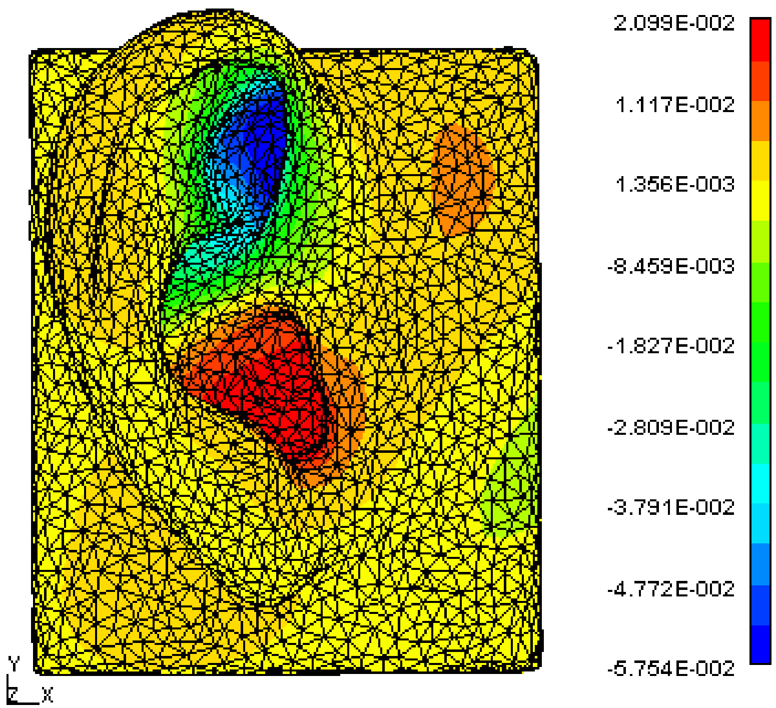
REAL PARTS OF THE SINGULAR VECTORS OF DB60

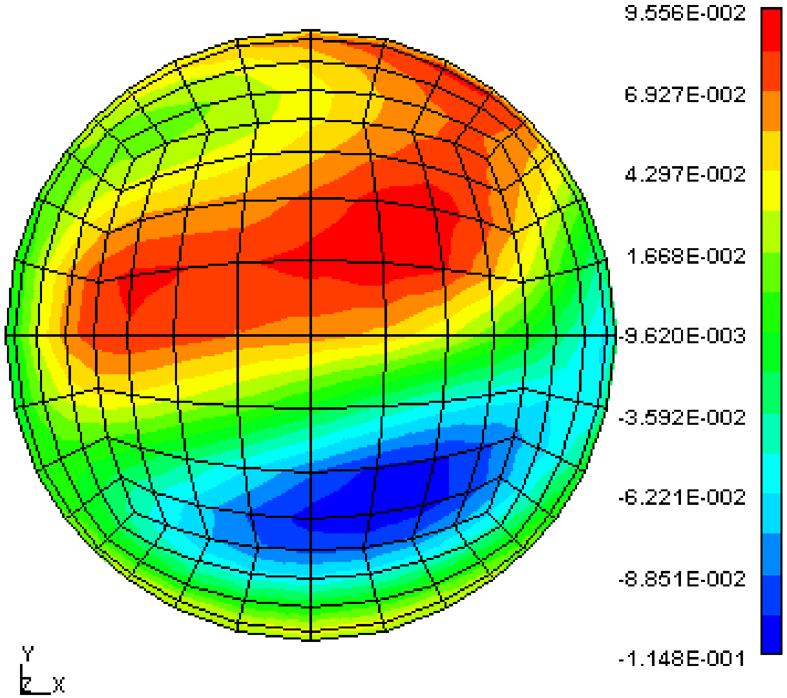
4.8 kHz



REAL PARTS OF THE SINGULAR VECTORS OF DB60

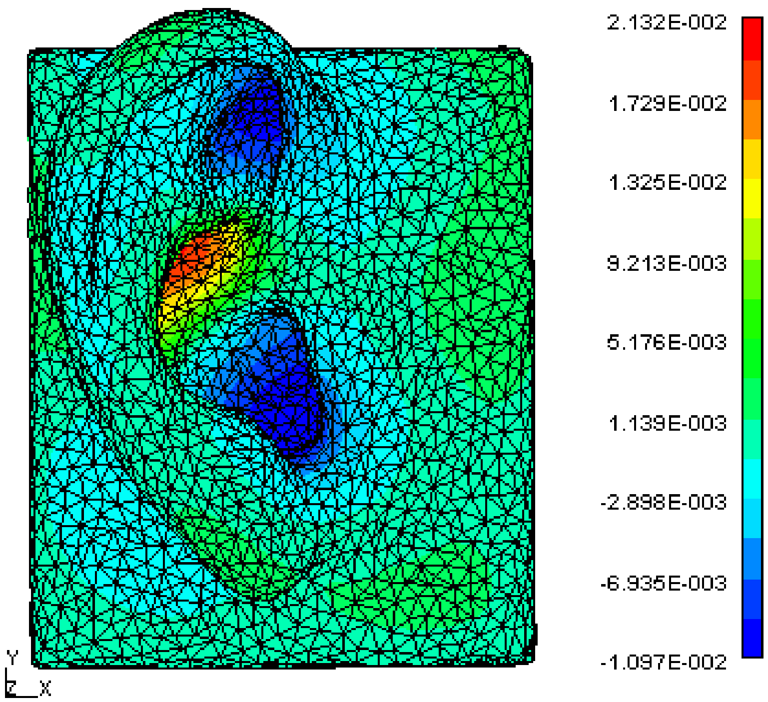
8.8 kHz

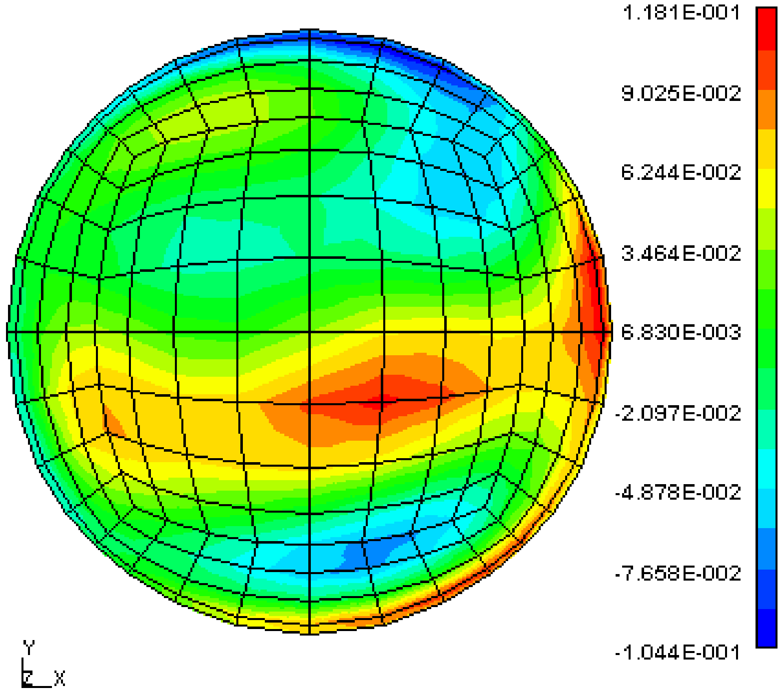




REAL PARTS OF THE SINGULAR VECTORS OF DB60

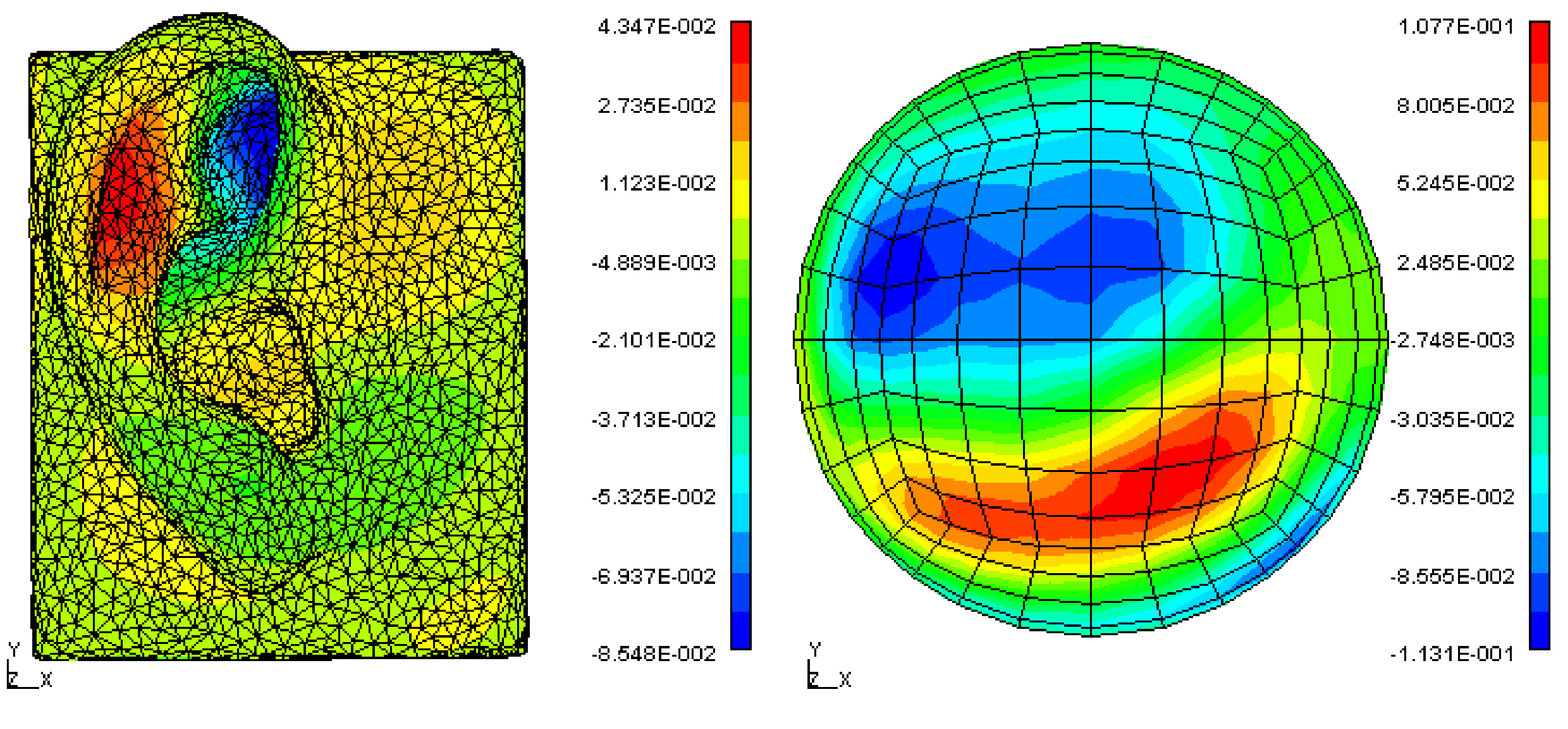
10.3 kHz / σ_1





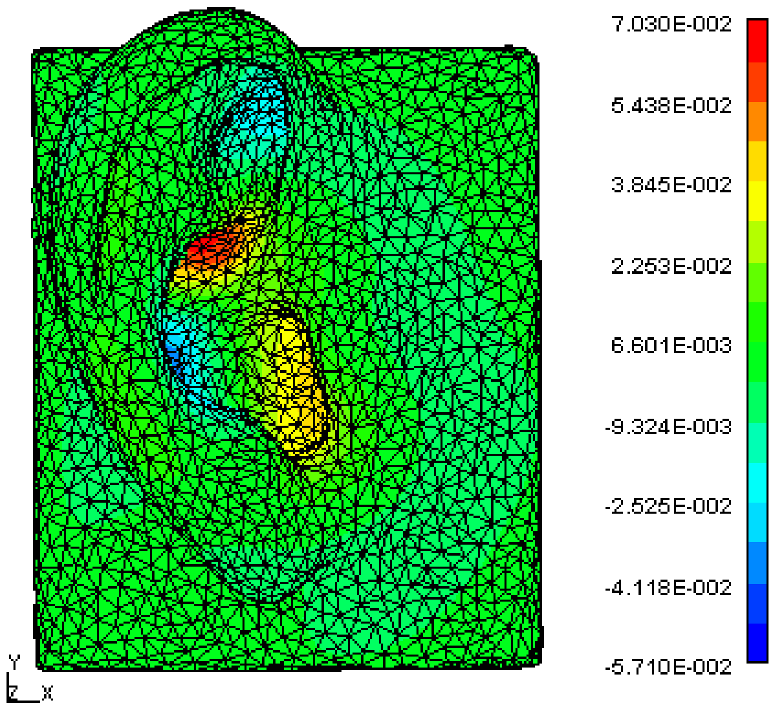
REAL PARTS OF THE SINGULAR VECTORS OF DB60

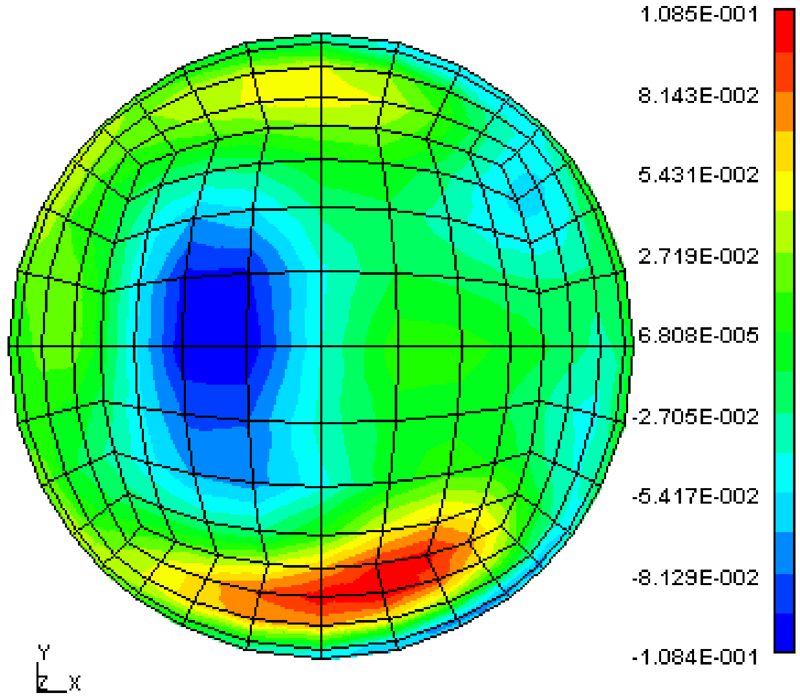
10.3 kHz / σ_2



REAL PARTS OF THE SINGULAR VECTORS OF DB60

13.8 kHz

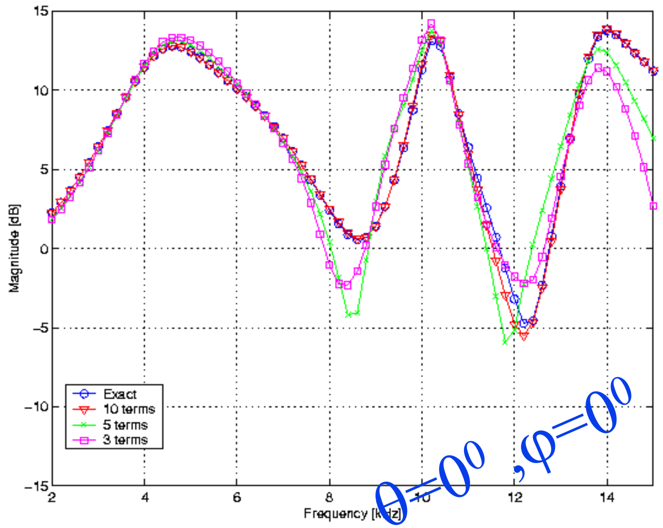


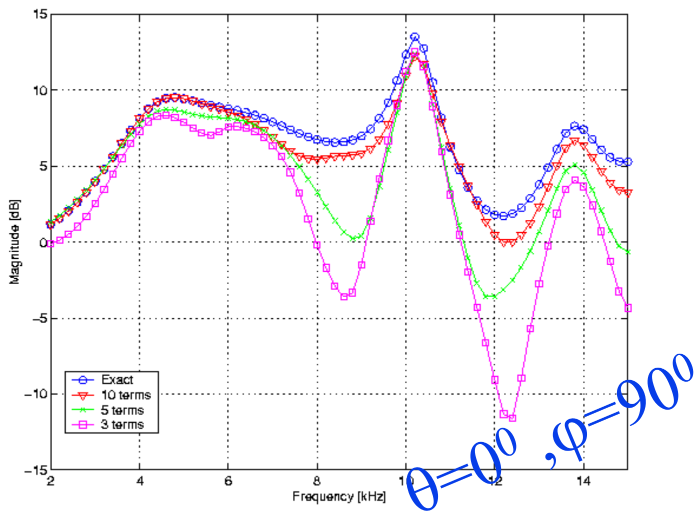


FREQUENCY RESPONSE DECOMPOSITION OF DB60 WITH TRUNCATED MATRICES

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} u_{11} & u_{21} & \cdots & u_{n1} & \cdots & u_{N1} \\ u_{12} & u_{22} & \cdots & u_{n2} & \cdots & u_{N2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ u_{1n} & u_{2n} & \cdots & u_{nn} & \cdots & u_{Nn} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{1N} & u_{2n} & \cdots & u_{nN} & \cdots & u_{NN} \end{bmatrix} \begin{bmatrix} \sigma_1 & & & & & \\ & \sigma_2 & & & & \\ & & \ddots & & & \\ & & & \sigma_n & & \\ & & & & \ddots & \\ & & & & & \sigma_N \end{bmatrix} \begin{bmatrix} v_{11}^* & v_{12}^* & \cdots & v_{1n}^* & \cdots & v_{1N}^* \\ v_{21}^* & v_{22}^* & \cdots & v_{2n}^* & \cdots & v_{2N}^* \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ v_{m1}^* & v_{m2}^* & \cdots & v_{mn}^* & \cdots & v_{mN}^* \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{M1}^* & v_{M2}^* & \cdots & v_{MN}^* & \cdots & v_{MN}^* \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \\ \vdots \\ q_M \end{bmatrix}$$

$$p_n = \sum_{n=1}^{N} \sigma_n u_{nn} v_{nm}^{\ *} q_m$$

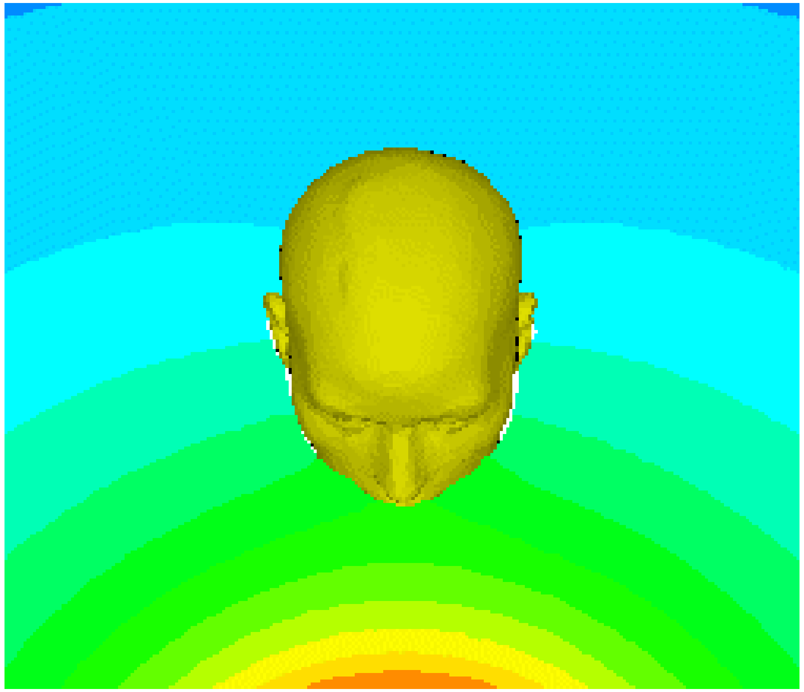


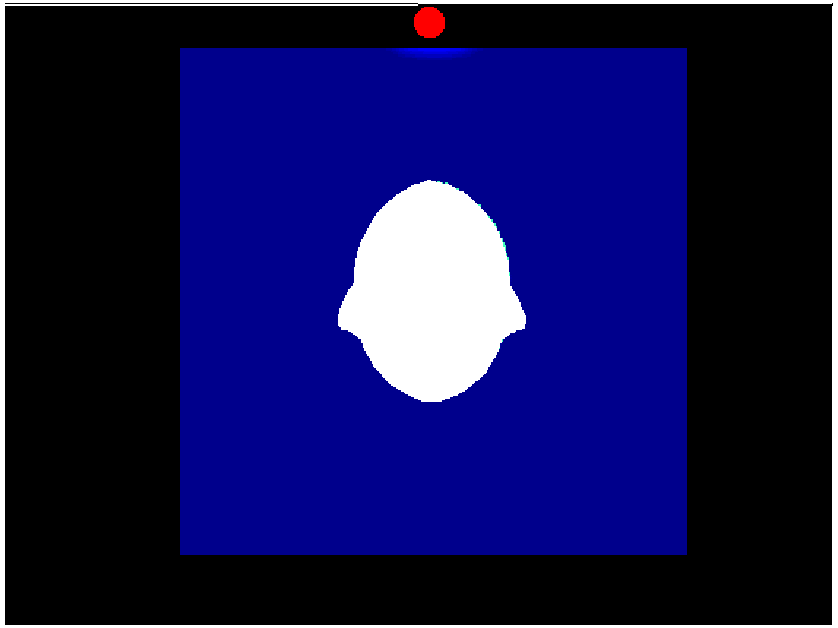


Where are we?

- Project description
- Overview of numerical modelling techniques in acoustics
- HRTFs and the principle of reciprocity (simple/complex models)
- Frequency response of baffled pinnae
- Acoustic modes of the external ear
- Spherical harmonics and mode shapes
- Extraction of HRTFs using the SVD and the BEM
- **Sound field animations**
- Conclusions

SCATTERED SOUND FIELD AROUND KEMAR DUE TO A MONOPOLE - FREQUENCY AND TIME DOMAINS

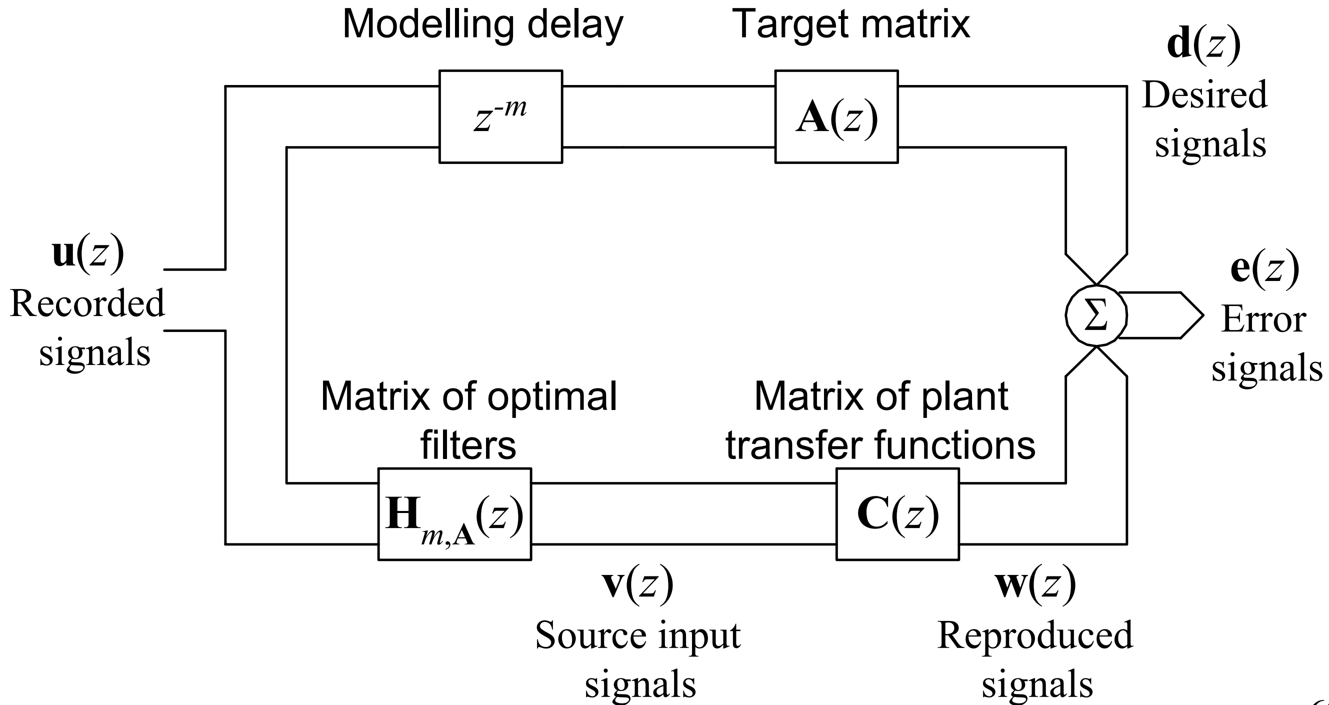




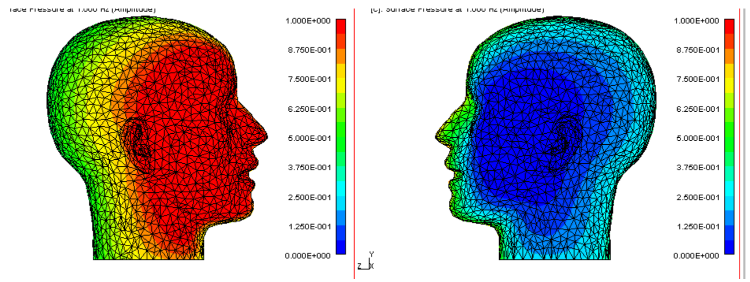
STEREO-DIPOLE
VIRTUAL ACOUSTIC IMAGING SYSTEM

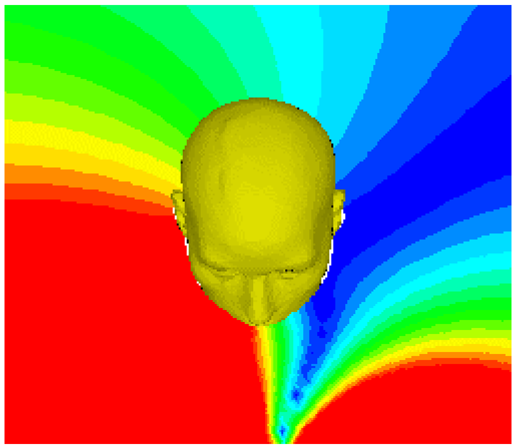
- Frequency domain - DC (1 Hz) to 6400 Hz, steps of 200 Hz
- Time domain - Digital Hanning pulse, impulse response of each field point
- Cross-talk cancellation - inverse problem

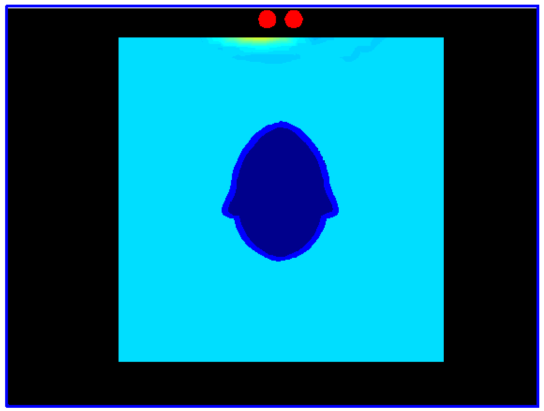
$[1\ 0\ ]^T$

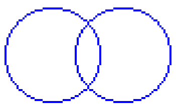


STEREO-DIPOLE
FREQUENCY AND TIME DOMAIN ANIMATIONS









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CONCLUSIONS

•Numerical modelling of HRTFs is NOT a trivial task.

•HRTFs can be modelled accurately to between10-15 kHz, and the response of baffled pinnae can be modelled accurately up to 20 kHz.

•The accuracy of the laser scanner appeared to be significant for the analysis at high frequency.

•The normal mode shapes, as found by Shaw, were validated and investigated with numerical techniques rather than measurements.

•A connection between orthogonal basis functions and the SVD has been shown.

•"Mode shapes" can be found for any defined Green function matrix.

•The spatial patterns (of the six investigated pinnae) have similar shapes although with differences in magnitude and a slight shift in resonance frequencies.

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CONCLUSIONS (cont.)

- It is possible to decompose a reduced order frequency response with only a few terms in the series for baffled pinnae.
- It is possible to visualise the sound field in the frequency and time domains for different arrangements of virtual acoustic imaging systems.